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FELIPE FREITAS DA ROCHA

THREE ESSAYS ON ENERGY ECONOMICS: BROADENING THE THEORETICAL BASES OF THE REBOUND EFFECT AND ENERGY SECURITY

RIO DE JANEIRO

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Orientador: Prof. Dr. Edmar Luiz Fagundes de Almeida

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Prof. Dr. Edmar Luiz Fagundes de Almeida,
(Universidade Federal do Rio de Janeiro)

Prof. Dr. Romero Cavalcanti Barreto da Rocha
(Universidade Federal do Rio de Janeiro)

Prof. Dr. Rodrigo Flora Calili
(Pontifícia Universidade Católica do Rio de Janeiro)

Prof. Dr. Luciano Dias Losekann, Doutor
(Universidade Federal Fluminense)

Prof. Dr. Francisco Javier Ramos Real
(Universidad de La Laguna)

I dedicate this thesis to all the scientists who came before me and those who keep the flame of knowledge alive

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ABSTRACT

ROCHA, Felipe Freitas da. Three essays on energy economics: broadening the theoretical bases of the rebound effect and energy security. 2021. 138 f. Tese (Doutorado em Economia) - Programa de Pós-Graduação em Economia, Instituto de Economia, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2021.

This Thesis presents three essays on energy economics, which aim to expand the theoretical bases of the rebound effect and energy security. The first essay attempts to contribute to broadening the theoretical foundation of the macroeconomic rebound effect. Economists recognize that energy efficiency improvements generate behavioral responses that reduce potential energy savings (rebound effect) and may even increase energy consumption (backfire). Much work has been done to explain rebound economics. Nevertheless, many of the important issues still do not have a clear answer, that is, under which circumstances the rebound is more powerful or weaker, whether the long-run effect is greater or less than the short-run effect, and in which situations backfire is definitely a problem. In order to answer these questions, the article expands the Wei (2010) general equilibrium model, including any number of energy services and non-energy inputs and endogenizing the output price. Furthermore, in order to analyze the effects of a neutral technical change on energy consumption, a parameter of non-energy inputs productivity is also included. The analysis corrects some results presented in Wei (2010) and provides several new findings. Regarding the energy-augmenting technical change, the main findings are the importance of the energy supply and the use of more than one energy service in the model for the rebound size. Moreover, in the simplest models, the long-run rebound effect is greater than the short-run effect. Regarding the neutral technical change, it is highlighted that the use of homogeneous production functions generates backfire. Moreover, we find that backfire is definitely a problem in terms of welfare only in situations where energy consumption is based on highly polluting energies and where output is highly energy-intensive. The second essay seeks to contribute to the broadening of the theoretical foundation of energy security. Although energy security has been an object of academic reflection since the 1960s, there are still two major gaps in the literature that must be addressed. Firstly, there is no consensus on its definition and whether it would be possible to define energy security universally. Secondly, the methodological framework that explains how its dimensions interact with each other, and consequently how they affect energy security, has not yet been properly developed. To clarify these gaps, the article develops a simplified energy security model that combines economic theory and the concept of security in a probabilistic framework. The analysis found that energy security is a universal concept, but it has several meanings. That

is, energy security is a subjective concept. This means that personal judgments are an integral part of its definition. However, energy security is not just a matter of opinion; there is consistency in its reasoning, ranging from premises to conclusions and so to prescriptions. Albeit our simplified model does not include all dimensions of energy security, when a change in one of its variables is identified, the model determines rationally how energy security will be affected. Therefore, the operationalization of the model can guide energy policies to improve energy security. The third essay analyzes the relationship between energy security and supplier diversity of energy imports. In the literature, there is a strong consensus that, ceteris paribus, the greater the supplier diversity of energy imports, the greater the energy security. In addition, the idea that that the greater the energy import dependency, the greater the relevance of the supplier diversity for energy security is also widespread. Nevertheless, as in general the definition of energy security is based on the enumeration of dimensions, the methodological frameworks for energy security assume in advance that, ceteris paribus, greater supplier diversity is equivalent, by definition, to greater energy security. That is, this positive relationship is not a result, or a conclusion, of these methodological frameworks, but rather an assumption. In this way, the article examines whether there is any theoretical basis that justifies the assumption of a positive relationship between energy security and supplier diversity of energy imports. Also, this article examines whether this positive relationship is more relevant when the energy import dependency is high. To do this, the definition of diversity proposed by Stirling (2010a, 2007) and the definition of energy security proposed in the second essay of this thesis are used. The analysis found that this relationship depends on the level of energy import dependency and the level of threats of each supplier. It is positive only when energy import dependency is small, otherwise, it is negative. Thus, supplier diversity of energy imports becomes less relevant to energy security when energy import dependency is high. Therefore, the policy recommendation is that energy imports should be widely diversified when the energy import dependency is low. On the other hand, when this dependency is high, energy imports should be concentrated, to some extent, on the most secure supplier and, as the energy import dependency increases, less secure suppliers should be replaced by more secure ones.

Keywords: Energy efficiency; Rebound effect; Backfire; Energy Security; Energy Supply Disruption; Energy Diversity; Import Diversification

RESUMO

ROCHA, Felipe Freitas da. Three essays on energy economics: broadening the theoretical bases of the rebound effect and energy security. 2021. 138 f. Tese (Doutorado em Economia) - Programa de Pós-Graduação em Economia, Instituto de Economia, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2021.

Esta Tese apresenta três ensaios sobre economia da energia, que visam ampliar as bases teóricas do efeito rebote e da segurança energética. O primeiro ensaio procura contribuir para ampliar a fundamentação teórica do efeito rebote macroeconômico. Os economistas reconhecem que as melhorias na eficiência energética geram respostas comportamentais que reduzem a economia potencial de energia (efeito rebote) e podem até aumentar o consumo de energia (*backfire*). Muitos trabalhos foram feitos para explicar a teoria economia do efeito rebote. No entanto, muitas das questões importantes ainda não têm uma resposta clara, ou seja, em quais circunstâncias o efeito rebote é mais forte ou mais fraco, se o efeito de longo prazo é maior ou menor do que o efeito de curto prazo e em quais situações o *backfire* é definitivamente um problema. Para responder a essas perguntas, o artigo expande o modelo de equilíbrio geral de Wei (2010), incluindo qualquer número de serviços de energia e insumos não-energéticos e endogenizando o preço de produto. Além disso, a fim de analisar os efeitos de uma mudança técnica neutra no consumo de energia, um parâmetro de produtividade dos insumos não-energéticos também é incluído. A análise corrige alguns resultados apresentados em Wei (2010) e fornece várias novas descobertas. Em relação à mudança técnica aumentadora de energia, os principais resultados são a importância da oferta de energia e a utilização de mais de um serviço de energia no modelo para o tamanho do efeito rebote. Ademais, nos modelos mais simples, o efeito rebote de longo prazo é maior do que o efeito de curto prazo. Em relação à mudança técnica neutra, destaca-se que o uso de funções de produção homogêneas gera *backfire*. Além disso, encontramos que o *backfire* é definitivamente um problema em termos de bem-estar apenas em situações onde o consumo de energia é baseado em energias altamente poluentes e onde a produção é altamente intensiva em energia. O segundo ensaio busca contribuir para a ampliação da fundamentação teórica da segurança energética. Embora a segurança energética venha sendo objeto de reflexão acadêmica desde a década de 1960, ainda existem duas lacunas importantes na literatura que devem ser abordadas. Em primeiro lugar, não existe consenso sobre a sua definição e se seria possível definir a segurança energética de forma universal. Em segundo lugar, o arcabouço metodológico que explica como suas dimensões interagem entre si e, consequentemente, como afetam a segurança energética, ainda não foi devidamente desenvolvido. Para esclarecer essas lacunas, o artigo desenvolve um modelo simplificado de

segurança energética que combina a teoria econômica e o conceito de segurança em uma estrutura probabilística. A análise concluiu que a segurança energética é um conceito universal, mas que possui vários significados. Ou seja, segurança energética é um conceito subjetivo. Isso significa que julgamentos pessoais são parte integrante de sua definição. No entanto, a segurança energética não é apenas uma questão de opinião; há consistência em seu raciocínio, que vai das premissas às conclusões e, portanto, às prescrições. Embora nosso modelo simplificado não incorpore todas as dimensões da segurança energética, quando uma mudança em uma de suas variáveis é identificada, o modelo determina racionalmente como a segurança energética será afetada. Portanto, a operacionalização do modelo pode guiar políticas energéticas que visem melhorar a segurança energética. O terceiro ensaio analisa a relação entre segurança energética e diversidade de fornecedores das importações de energia. Na literatura, existe um forte consenso de que, *ceteris paribus*, quanto maior a diversidade de fornecedores das importações de energia, maior a segurança energética. Além disso, é difundida a ideia de que quanto maior a dependência das importações de energia, maior a relevância da diversidade de fornecedores para a segurança energética. No entanto, como em geral a definição de segurança energética se baseia na enumeração de dimensões, os arcabouços metodológicos da segurança energética assumem em princípio que, *ceteris paribus*, uma maior diversidade de fornecedores equivale, por definição, a uma maior segurança energética. Ou seja, essa relação positiva não é um resultado, ou uma conclusão, desses arcabouços metodológicos, mas sim uma suposição. Dessa forma, o artigo examina se há algum embasamento teórico que justifique a suposição de uma relação positiva entre segurança energética e diversidade de fornecedores das importações de energia. Além disso, este artigo examina se essa relação positiva é mais relevante quando a dependência das importações de energia é alta. Para tanto, utiliza-se a definição de diversidade proposta por Stirling (2010a, 2007) e a definição de segurança energética proposta no segundo ensaio desta tese. A análise concluiu que esta relação depende do nível de dependência das importações de energia e do nível de ameaças de cada fornecedor. Ela é positiva apenas quando a dependência das importações de energia é pequena, caso contrário, é negativa. Assim, a diversidade de fornecedores das importações de energia torna-se menos relevante para a segurança energética quando a dependência das importações de energia é alta. Portanto, a recomendação de política é de que as importações de energia sejam amplamente diversificadas, quando a dependência das importações de energia for baixa. Por outro lado, quando essa dependência for alta, as importações de energia devem se concentrar, em certa medida, no fornecedor mais seguro e, conforme a dependência das importações de

energia aumenta, os fornecedores menos seguros devem ser substituídos por fornecedores mais seguros.

Palavras-chave: Eficiência energética; Efeito rebote; Tiro pela culatra; Seguranca energética; Interrupção no fornecimento de energia; Diversidade energética; Diversificação de importação

LIST OF FIGURES

CONTENTS

INTRODUCTION

The Paris Agreement, on 12 December 2015, is a milestone in recognizing the urgency of tackling global warming and subsequent climate change. Improvements in energy efficiency are widely recognized as an important tool for mitigating greenhouse gases emissions and, therefore, it is paramount for climate goals (i.e., preferentially limit global warming to 1.5ºC in relation to the pre-industrial period) (ACEEE; ALLIANCE; BCSE, 2020; IEA, 2019a; IPCC, 2018). This is because energy efficiency improvements are assumed to reduce energy consumption. According to IEA forecasts, energy efficiency improvements will provide approximately 40% of the reduction in energy-related greenhouse gas emissions over the next 20 years in the Sustainable Development Scenario, which fully incorporates the achievement of climate goals (IEA, 2020a, 2020b).

Nevertheless, economists have long recognized that energy efficiency improvements generate behavioral responses that reduce potential energy savings (rebound effect) and may even increase energy consumption (backfire). The emerge of the rebound effect literature can be traced back to Jevons (1865). Jevons (1865) noted that England's coal consumption increased considerably after energy efficiency improvements in the steam engine, rather than decreasing as expected. The modern era of rebound economics was initiated by Khazzoom (1980) and Brookes (1978, 1990, 2000), who also defended the backfire hypothesis.

Since then, academic literature on the rebound effect has been growing, following the debate on climate change. Nevertheless, although progress on the topic is evident, there is no consensus on the size of the rebound effect, which can range from negative rebound effects to backfire (CHAKRAVARTY; DASGUPTA; ROY, 2013; GILLINGHAM; RAPSON; WAGNER, 2016; JENKINS; NORDHAUS; SHELLENBERGER, 2011; MAXWELL et al., 2011; SORRELL, 2007; STERN, 2020; VAN DEN BERGH, 2011). Turner (2013) argues that one of the reasons for this is the lack of solid understanding of the theoretical foundation of the rebound effect. Turner (2013) further argues that the identification of this theoretical foundation is surely as, if not more, important than developing empirical studies. A solid theoretical basis on the rebound effect is paramount to understand the relationship between energy efficiency and energy consumption (and so greenhouse gas emissions).

In addition, there has been extensive discussion of the need to mitigate the rebound effect (FONT VIVANCO; KEMP; VAN DER VOET, 2016; FREIRE-GONZÁLEZ, 2020; FREIRE-GONZÁLEZ; PUIG-VENTOSA, 2015; MAXWELL et al., 2011; OUYANG; LONG;

HOKAO, 2010; VAN DEN BERGH, 2011). That is, since energy consumption contributes heavily to climate change, it is seen as a strong generator of external costs, which in turn decreases welfare. Nevertheless, several studies have described why such a perspective may be mistaken, since energy efficiency gains can increase economic output and so welfare (HANLEY et al., 2009; SAUNDERS, 1992; SAUNDERS; TSAO, 2012; TSAO et al., 2010; WEI; LIU, 2017). Therefore, looking at it from the perspective of welfare, the rebound effect may not be a concern and so there may not be a need to mitigate it. However, the lack of a solid theoretical basis for the rebound effect makes it difficult to identify the cases in which its mitigation is necessary or not.

The worsening of climate change has also generated a growing interest in the topic of energy security since the mid-2000s (ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018). It should be noted that other factors have also contributed to this, such as: the emergence of new large energy consumers, such as China and India, which increased competition for non-renewable resources; rising energy prices over the 2000s; the instability in some energy-exporting countries, such as Venezuela and the countries of the Middle East and North Africa that experienced the Arab Spring, and; increasing concern about new terrorist attacks since the events of 9/11 (AZZUNI; BREYER, 2018; CHERP; JEWELL, 2011; YERGIN, 2011). This growing interest on the topic of energy security has brought up a major problem. That is, despite six decades of academic debate on energy security (AZZUNI; BREYER, 2018; CHERP; JEWELL, 2014), there is no methodological framework that explains it reasonably. The lack of theoretical basis for energy security is reflected in the lack of consensus on its definition and in the lack of understanding about how its dimensions interact with each other and, consequently, how they affect energy security (ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018; CHESTER, 2010; KOULOURI; MOURAVIEV, 2019; PARAVANTIS et al., 2019; SOVACOOL, 2010; SOVACOOL; BROWN, 2010; WINZER, 2012).

In general, the concept of energy security is explained by drawing up lists of energy security concerns (i.e., dimensions). For example, Sovacool and Brown (2010, p. 81) state that "*energy security should be based on the interconnected factors of availability, affordability, efficiency, and environmental stewardship*", without presenting a formal definition and without developing a methodological framework with a solid theoretical basis. Nevertheless, there is no method that justifies the inclusion or omission of energy security dimensions (CHERP; JEWELL, 2011). Simply put, three methods of choosing dimensions can be identified: 1) choices based on a meta-analysis of previous studies (ABDULLAH et al., 2020; ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018; ERAHMAN et al., 2016; RAGHOO et al.,

2018; REN; SOVACOOL, 2014; SOVACOOL; BROWN, 2010); 2) choices based on interviews with experts (SOVACOOL, 2011, 2016; SOVACOOL; MUKHERJEE, 2011); and 3) arbitrary choices (APERC, 2007; HUGHES, 2009; LI; SHI; YAO, 2016; VIVODA, 2010; VON HIPPEL et al., 2011). Meta-analysis and interviews are relatively systematic methods, but their values are diminished by the fact that the underlying studies and experts can be based on the method of arbitrary choice (CHERP, 2012; CHERP; JEWELL, 2011).

Therefore, despite the clear utility of the method of drawing up lists of energy security dimensions, this categorization remains only as a technical exercise in taxonomy and it does not provide a theoretical basis for the concept of energy security. Furthermore, defining energy security and measuring it are two sides of the same coin, since the measurement only has meaning if it quantifies a clearly defined entity (AXON; DARTON; WINZER, 2013). However, the lack of a rigorous methodological framework that explains energy security results in a highly inconsistent measurement (GASSER, 2020; VALDÉS, 2018). Depending on the definition and subsequent choice of dimensions, energy security measurements can present the most diverse results (BÖHRINGER; BORTOLAMEDI, 2015; CHERP; JEWELL, 2010; NARULA; REDDY, 2015; VALENTINE, 2010; WINZER, 2012).

Thus, the ability of energy security studies to inform energy policy has so far been limited, since it is only possible to improve energy security if policy-makers know what it really means. In particular, the policy recommendation, widely suggested in the literature, that suppliers of energy imports must be diversified to improve energy security may be a mistake. Since there is no methodological framework that rigorously explains energy security, there is no theoretical basis to support the existence of a positive relationship between it and supplier diversity of energy imports. In other words, the methodological frameworks based on drawing up lists of energy security concerns assume in advance that greater supplier diversity is equivalent, by definition, to greater energy security. This means that this positive relationship is not a result, or a conclusion, of these methodological frameworks, but rather an assumption.

All of these points stress the need to develop a rigorous theoretical basis for the rebound effect and energy security. Therefore, the goal of this Thesis is to broaden the theoretical foundations of the rebound effect and energy security. This Thesis is composed of three essays. While the first and second essays are independent, the third essay depends on reading the second one.

The first essay – "A general equilibrium model of macroeconomic rebound effect: a broader view" – addresses the topic of the rebound effect, aiming to contribute to the broadening of its theoretical foundation. For this, the first essay expands the general equilibrium model developed by Wei (2010). The first essay seeks to answer the following questions:

- What are the theoretical mechanisms that govern the rebound effect?
- Under what circumstances is the long-term rebound effect greater (or less) than the short-term rebound effect?
- Under what circumstances is the rebound more powerful (or weaker)?
- In what situations is the backfire a concern in terms of welfare?

The second – "An economic model of energy security: a proposal to unify the concept" – and third essays – "Analyzing the relationship between energy security and supplier diversity of energy imports using an economic model of energy security" – address the topic of energy security. The second essay seeks to provide the theoretical basis for the concept of energy security, allowing the unification of such a divergent concept. To do this, the second essay develops a simplified model of energy security that combines economic theory and the concept of security in a probabilistic framework. The second essay seeks to answer the following questions:

- What does energy security mean?
- How do their dimensions interact with each other and how do they affect energy security?

The third essay analyzes the relationship between energy security and supplier diversity of energy imports. To do this, it is necessary to rigorous define what "energy security" and "diversity" mean. In this way, the third essay uses the definition of diversity proposed by Stirling (2007, 2010a) and the definition of energy security proposed in the second essay. Thus, a simulation of the energy security model presented in the second essay is carried out. This makes it possible to analyze: the correlation between energy security and the diversity index; as changes in the share of suppliers in energy imports affect energy security and diversity index, and; the optimal value for the share of suppliers in energy imports which maximizes energy security. In this way, the third essay seeks to test the following hypotheses:

- Whether there is a positive relationship between energy security and the supplier diversity of energy import.

- Whether this positive relationship is more relevant when the dependence on energy imports is greater.

It is noteworthy that the first essay is published in the Energy Economics Journal (ROCHA; ALMEIDA, 2021), while the second essay is submitted to the same journal (at the time of publication of this Thesis, there was still no final decision on the second essay). The third essay is yet to be submitted.

1 A GENERAL EQUILIBRIUM MODEL OF MACROECONOMIC REBOUND EFFECT: A BROADER VIEW*

1.1 Introduction

Energy efficiency improvements are often seen as an important tool for reducing energy consumption. However, economists have long recognized that energy efficiency improvements generate behavioral responses that reduce potential energy savings (rebound effect) and may even increase energy consumption (backfire). The emerge of the rebound effect literature can be traced back to Jevons (1865). Jevons (1865) noted that England's coal consumption increased considerably after energy efficiency improvements in the steam engine, rather than decreasing as expected. The modern era of rebound economics was initiated by Khazzoom (1980) and Brookes (1978, 1990, 2000), who also defended the backfire hypothesis. Saunders (1992) was one of the first authors to use the Neoclassical Growth theory to explain rebound economics, arguing that economic theory allows backfire in some cases.

Since then, much work has been done to explain rebound economics. Simply put, we can divide the theoretical literature on the rebound effect into macroeconomic rebound models and microeconomic rebound models (GILLINGHAM; RAPSON; WAGNER, 2016). Macroeconomic rebound models are derived through the Neoclassical Growth theory or Neoclassical Production theory, such as Saunders (1992, 2000a, 2008), Howarth (1997), Wei (2007, 2010), Sorrell (2014), Zhang and Lawell (2017), Brockway et al. (2017), and Lemoine (2020). Microeconomic rebound models are derived through Neoclassical Consumer theory, such as Borenstein (2015), Ghosh and Blackhurst (2014), Chan and Gillingham (2015), and Sorrell and Dimitropoulos (2008). This article focuses exclusively on the macroeconomic rebound effect.

Since the beginning of the rebound theory, there has been a debate about its magnitude (DIMITROPOULOS, 2007). Several literature reviews have been carried out and they all argue that there are the most diverse empirical results, ranging from negative rebound effects (super-conservation) to backfire (CHAKRAVARTY; DASGUPTA; ROY, 2013; GILLINGHAM; RAPSON; WAGNER, 2016; JENKINS; NORDHAUS; SHELLENBERGER, 2011;

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MAXWELL et al., 2011; SORRELL, 2007; STERN, 2020; VAN DEN BERGH, 2011). Turner (2013) argues that one of the reasons for this is the lack of solid understanding of the theoretical foundations of the wide range of mechanisms that govern the rebound effect.[1] Turner (2013) further argues that the identification of these mechanisms is surely as, if not more, important than developing empirical studies. As we will show in this paper, the assumptions about the model and, consequently, about the rebound mechanisms that are included in it, can change the rebound size.

The main issue would be to identify under which circumstances energy efficiency gains would lead to backfire. Empirical evidence shows that the rebound effect is particularly large when the energy efficiency gains are accompanied by improvements in the productivity of non-energy inputs (neutral technical change), tending to generate backfire (SAUNDERS, 1992, 2005, 2013, 2015). As Sorrell (2007, 2009) points out, rebound effects may be particularly large when energy efficiency improvements are associated with general-purpose technologies, which have potential for use in a wide variety of products and processes and have strong complementarities with existing or potential new technologies. Sorrell (2007, 2009) also argues that this energy efficiency improvement was used by Jevons and Brookes, that is, repetitively steam engines and electric motors, in order to support the backfire hypothesis. In fact, the neutral technical change, also known as the innovation rebound effect (GILLINGHAM; RAPSON; WAGNER, 2016), is the rationale behind many of the backfire claims in the literature (GILLINGHAM; RAPSON; WAGNER, 2016; JENKINS; NORDHAUS; SHELLENBERGER, 2011; SORRELL, 2007, 2009). However, the conditions under which the innovation effect results in backfire are not yet clear.

In addition, there has been extensive discussion of the need to mitigate the rebound effect (FONT VIVANCO; KEMP; VAN DER VOET, 2016; FREIRE-GONZÁLEZ, 2020; FREIRE-GONZÁLEZ; PUIG-VENTOSA, 2015; MAXWELL et al., 2011; OUYANG; LONG; HOKAO, 2010; VAN DEN BERGH, 2011). This perspective is based on the fact that energy consumption contributes heavily to several of the most important environmental problems, especially climate change. That is, energy consumption is seen as a strong generator of external costs, which in turn decreases welfare. Nevertheless, several authors have described why such

[1] Despite the lack of solid understanding of the theoretical foundations of the wide range of mechanisms that govern rebound effect, there are several typologies of rebound effect in the literature (GILLINGHAM; RAPSON; WAGNER, 2016; GREENING; GREENE; DIFIGLIO, 2000; JENKINS; NORDHAUS; SHELLENBERGER, 2011; SORRELL, 2007; VAN DEN BERGH, 2011). However, as Turner (2013) points out, although these typologies are pedagogically useful, they may lead to confusion and neglect of potentially important mechanisms that influence the nature and magnitude of the rebound effect.

a perspective may be mistaken, since energy efficiency gains can increase economic output and so welfare (HANLEY et al., 2009; SAUNDERS, 1992; SAUNDERS; TSAO, 2012; TSAO et al., 2010; WEI; LIU, 2017). Therefore, looking at it exclusively from the perspective of welfare, backfire may not be a concern. As far as we know, no study has aimed to analyze the innovation rebound effect from a perspective of welfare economics.[2]

The literature has also been debated whether the short-run rebound would be greater or less than the long-run rebound. Some empirical works find that the long-run effect is greater than the short-run effect, such as Small and Van Dender (2007), Odeck and Johansen (2016), Wang, Han, and Lu (2016), Yang et al. (2019), and Belaïd, Bakaloglou, and Roubaud (2018). Some theoretical works corroborate these findings, such as Saunders (2008) and Wei (2007). However, other empirical works, such as Allan et al. (2007), Turner (2009), Saunders (2013), Lu, Liu, and Zhou (2017), Adetutu, Glass, and Weyman-Jones (2016), and Yan et al. (2019), found that short-run effect can be greater than long-run effect, which seems to contradict previous theoretical studies. In this sense, Wei (2010) develops a theoretical model where the short-run effect can be both greater and lesser than the long-run effect. However, as will be shown in this article, the comparison between the long-run and short-run effects made by Wei (2010) only allows the long-run rebound to be greater than the short-run rebound. Thus, the conditions under which the long-run rebound effect is greater or less than the short-run effect are not yet clear.

Therefore, many of the important theoretical issues on the macroeconomic rebound effect still do not have a clear answer. One of the main reasons for this is that the existing theoretical models have a limited capacity to explain the mechanisms that govern macroeconomic rebound. First, because most macroeconomic rebound models focus exclusively on the direct rebound effect (BROCKWAY et al., 2017; SAUNDERS, 1992, 2000a, 2008; SORRELL, 2014; ZHANG; LAWELL, 2017). That is, the models are generally partial equilibrium models, being composed of a single energy service and with all exogenous prices. Only a few theoretical works incorporate some indirect rebound effects, such as the general equilibrium models developed by Wei (2007, 2010) and Lemoine (2020). Furthermore, according to Stern (2020), most empirical studies using econometric methods are partial equilibrium approaches (e.g., Adetutu, Glass, and Weyman-Jones (2016) and Yan et al. (2019)) that do not include all the mechanisms that can influence the rebound effect.

[2] It should be noted that Chan and Gillingham (2015) analyzes the relationship between welfare and the microeconomic rebound effect.

Second, because most of the macroeconomic rebound models incorporate only a single representative energy service (no work that using more than one energy service to explain the economics of the macroeconomic rebound effect was found).[3] However, as we will see in this article, the use of a single energy service limits the rebound size, that is, in this case, super-conservation is not allowed.

Third, most of the macroeconomic rebound models use specific production functions, such as Leontief, Cobb-Douglas, Constant Elasticity of Substitution (CES), and many others (BROCKWAY et al., 2017; LEMOINE, 2020; SAUNDERS, 2000a, 2008; SORRELL, 2014; WEI, 2007; ZHANG; LAWELL, 2017). In addition, as highlighted by Broberg, Berg, and Samakovlis (2015), in general, empirical studies using computable general equilibrium models employ CES production function or one of their special cases (ALLAN et al., 2007; HANLEY et al., 2009; LU; LIU; ZHOU, 2017). Nevertheless, one of the main contributions of Saunders' work (2000a, 2000b, 2008) was to show that the specification of the production function considerably influences the rebound size. One of the few works to use generic production functions is Wei (2010). However, this author bases his analysis on the functions of marginal product of inputs, rather than on the functions of inputs demand and of output supply, which makes it difficult to draw some important conclusions.

Therefore, the objective of the article is to propose a macroeconomic rebound effect model that allows identifying:

- the theoretical mechanisms that govern the rebound effect;
- under what circumstances the long-term rebound effect is greater or less than the short-term rebound effect.
- under what circumstances the rebound is more powerful or weaker;
- in which situations the backfire is a concern in terms of welfare.

For this, this article expands the general equilibrium model developed by Wei (2010), including any number of energy services and non-energy inputs and endogenizing the output price. Furthermore, in order to analyze (in a simplified way) the innovation rebound effect, a parameter of non-energy inputs productivity is also included. However, in order to facilitate the understanding of the rebound concept, we will describe the macroeconomic rebound effect

[3] It is worth mentioning that Chan and Gillingham (2015) and Ghosh and Blackhurst (2014) use more than one energy service in their microeconomic rebound models.

through generics demand and supply functions (i.e., generic functional forms), and not through the functions of marginal product of inputs as in Wei (2010).

The paper is structured as follows. Section 1.2 presents the energy–energy services relationship. Section 1.3 expands the general equilibrium model developed by Wei (2010). Section 1.4 defines the rebound effect. Section 1.5 shows the macroeconomic rebound effect from energy-augmenting technical change (called the reallocation rebound effect). Section 1.6 shows in a simplified way the innovation rebound effect. Section 1.7 provides cautions and limitations related to the analysis. Section 1.8 draws conclusions. Furthermore, five appendices give a detailed description of the model and in-depth proof of some important results.

1.2 The energy–energy services relationship

1.2.1 Energy services

Consumers do not demand energy per se, but rather the services generated by this energy, called energy services.[4] For example, it is the consumption of transportation service that will require some kind of energy to be generated (e.g., gasoline or electricity). In this sense, energy demand is a derived demand, that is, it is a byproduct of energy service demand. Moreover, in order to energy consumption to occur, it is necessary to use equipment that converts energy into energy services, called conversion equipment. For example, one way to take advantage of petroleum fuels is through vehicles that convert this energy into transportation services. The consumer does not sit in a barrel of crude oil and is transported. Each conversion equipment has an associated energy efficiency, which uses a certain amount of energy inputs to generate a certain amount of energy services.

1.2.2 Energy efficiency

Energy efficiency (ε) is measured as the ratio of useful output (UO) to energy input (E) for a system: $\varepsilon = \frac{UO}{E}$. However, as Patterson (1996) and Sorrell (2007) point out, this definition depends on how useful is defined and how outputs and inputs are measured. There are different ways used to define energy efficiency: thermodynamic, physical, or economic (PATTERSON, 1996; SORRELL, 2007). For our purpose, the important definition is the physical definition.

[4] In the literature, there is no consensus on energy services definition (FELL, 2017). However, the definition proposed by Fell (2017, p. 137) will be followed: "*Energy services are those functions performed using energy which are means to obtain or facilitate desired end services or states*". This definition encompasses examples of energy services that are commonly used (e.g., lighting, cooking, heating, cooling, transport, etc.).

Physical energy efficiency is defined as: $\varepsilon_i = \frac{S_i}{E_i}$, where ε_i is the energy efficiency, S_i is the consumption of energy services and E_i is the energy consumption needed to generate the amount of energy service i. This definition can be broken down into two other measures: $\varepsilon_i = \frac{S_i}{E_i} = \frac{U_i}{E_i}\frac{S_i}{U_i}$, where U_i is the useful energy consumption. The first element (thermodynamic energy efficiency) is the ratio of useful energy outputs to the heat content of fuel inputs, while the second is the ratio of energy service outputs to useful energy inputs. Therefore, variations in physical energy efficiency may result from factors other than improvements in thermodynamic energy efficiency. For example, changes in the aerodynamics, size, or weight of a vehicle can alter physical energy efficiency, measured in kilometers per liter, without any modification in the combustion engine. This means that, often, new technologies that aim to improve the productivity of non-energy inputs (e.g., capital) can result in an energy efficiency improvement and vice versa.

1.2.3 Quantity relationship

Using the energy efficiency definition, a relationship between the consumption of energy service and energy can be obtained:

$$E_i = \frac{S_i}{\varepsilon_i} \quad \textbf{(1.1)}$$

For example, if a passenger travels 50 kilometers (energy service) and the vehicle has an energy efficiency of 10 kilometers per liter of fuel, the passenger's energy consumption is 5 liters of fuel. Also, it is recognized that energy services have broader attributes (GILLINGHAM; RAPSON; WAGNER, 2016; SORRELL; DIMITROPOULOS, 2008). For example, all cars deliver passenger kilometers, but they may vary widely in terms of features such as speed, comfort, acceleration, and prestige (SORRELL; DIMITROPOULOS, 2008). However, disaggregation is a strong tool for dealing with this problem. For example, the energy services of kilometers traveled per passenger can be disaggregated as kilometers traveled per passenger with high, medium, or low comfort or with high, medium, or low speed, or some combination of these two attributes. This means that, now, there is not only a single energy service, but several different types of energy services. The disaggregation can be done until the attributes are homogeneous for each energy service. Each energy service will have an associated energy efficiency.

1.2.4 Price relationship

A simplified relationship between energy service price and energy price can also be obtained:

$$P_{S_i} = \frac{P_{E_i}}{\varepsilon_i} \qquad \textbf{(1.2)}$$

Where P_{S_i} is the implicit price of energy services i and $P_{E_i}$is the price of energy used by the energy service i. As energy services are functions performed using energy (see footnote 4), they are somehow produced by the consumers themselves using a combination of conversion equipment (i.e., capital), energy, and other inputs – such as in the model proposed by Sorrell and Dimitropoulos (2008). That is, consumers do not pay directly for the consumption of energy services. If they do so, services generate by energy will no longer be energy services, but end services. As Turner (2013) points out, as energy services are not directly marketed commodities, their prices are derived rather than market ones. Equation (1.2) shows that the energy service price is indirect, and it is given by the energy price adjusted by the physical energy efficiency.[5]

1.3 The model

Let a representative firm that produces a global output (Y) using m non-energy inputs (q_i) and n energy inputs (E_i), which are associated with n energy services (S_i): $Y = f(\overrightarrow{\tau q}, \overrightarrow{\varepsilon E}\,)$, where $f(.)$ is the production function[6], $\overrightarrow{\tau q}$ represents the vector $[\tau_1 q_1, \dots, \tau_m\, q_m]$, τ_i is the productivity of non-energy input q_i and $\overrightarrow{\varepsilon E}$ represents the vector $[\varepsilon_1 E_1, \dots, \varepsilon_n E_n]$. It is assumed that all energy services use the same type of energy. This hypothesis results in an equal energy price (P_E) for all energy services. The firm's cost is given by: $\sum_{i=1}^{m} P_{q_i} q_i + \sum_{i=1}^{n} P_E E_i$, where P_{q_i} is the price of respective non-energy input. Thus, the firm's profit maximization problem is given by:

$$\max_{\vec{q},\vec{E}} P_Y f(\overrightarrow{\tau q}, \overrightarrow{\varepsilon E}\,) - \left(\sum_{i=1}^{m} P_{q_i} q_i + \sum_{i=1}^{n} P_E E_i \right) \qquad \textbf{(1.3)}$$

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[5] It is noteworthy that equation (1.2) is an oversimplification, since the price of the energy service is also made up of the capital cost (i.e., conversion equipment cost), labor and other inputs used.
[6] It is always assumed that the production function is well-behaved: strictly increasing, continuous, differentiable and strictly concave. It is also assumed that the firm operates in a perfectly competitive market.

where P_Y is the output price, $\vec{q}$ represents the vector $[q_1, \dots, q_m]$ and $\vec{E}$ represents the vector $[E_1, \dots, E_n]$.

Equation (1.3) implicitly uses equations (1.1) and (1.2). The use of these equations, despite being a simple step, is extremely important for our explanation of the macroeconomic rebound economics. The simple understanding that the energy demand is a function derived from the energy services demand, will enable us to obtain some important results. Thus, it is not necessary to have a theoretical framework as complex as the one developed by Saunders (2005).

First, note that the firm's cost can be rewritten as $\sum_{i=1}^{m} \frac{P_{q_i}}{\tau_i} \tau_i\, q_i + \sum_{i=1}^{n} \frac{P_E}{\varepsilon_i} \varepsilon_i E_i$. Denoting $Q_i = \tau_i\, q_i$ and $P_{Q_i} = \frac{P_{q_i}}{\tau_i}$, and using equations (1.1) and (1.2), the firm's cost is transformed into: $\sum_{i=1}^{m} P_{Q_i} Q_i + \sum_{i=1}^{n} P_{S_i} S_i$. Second, note that production function can be rewritten as $Y = f(\vec{Q}, \vec{S})$, where $\vec{Q}$ represents the vector $[Q_1, \dots, Q_m]$ and $\vec{S}$ represents the vector $[S_1, \dots, S_n]$. Therefore, the firm's profit maximization problem in equation (1.3) is summed up to:

$$\max_{\vec{Q},\vec{S}} P_Y f(\vec{Q}, \vec{S}) - \left(\sum_{i=1}^{m} P_{Q_i} Q_i + \sum_{i=1}^{n} P_{S_i} S_i \right) \qquad \textbf{(1.4)}$$

The first-order conditions from equation (1.3) require that $P_Y \frac{\partial f}{\partial \varepsilon_i E_i} \varepsilon_i = P_E$ and $P_Y \frac{\partial f}{\partial \tau_i q_i} \tau_i = P_{q_i}$, and from equation (1.4) that $P_Y \frac{\partial f}{\partial S_i} = P_{S_i}$ and $P_Y \frac{\partial f}{\partial Q_i} = P_{Q_i}$. The first-order conditions from equation (1.3) can be rewritten as $P_Y \frac{\partial f}{\partial \varepsilon_i E_i} = \frac{P_E}{\varepsilon_i}$ and $P_Y \frac{\partial f}{\partial \tau_i q_i} = \frac{P_{q_i}}{\tau_i}$. This is exactly the first-order conditions from equation (1.4). Therefore, in order to find the energy demand function, just find the demand function for energy services and then use the equations (1.1) and (1.2).[7] The analogous is also true for the non-energy inputs demands. Thus, in the long run, the unconditional energy demands are equal to $E_i = \frac{S_i\left(\overrightarrow{\frac{P_E}{\varepsilon}}, \overrightarrow{\frac{P_q}{\tau}}, P_Y\right)}{\varepsilon_i}$, where $\overrightarrow{\frac{P_E}{\varepsilon}}$ represents the vector $\left[\frac{P_E}{\varepsilon_1}, \dots, \frac{P_E}{\varepsilon_n}\right]$, $\overrightarrow{\frac{P_q}{\tau}}$ represents the vector $\left[\frac{P_{q_1}}{\tau_1}, \dots, \frac{P_{q_m}}{\tau_m}\right]$ and $S_i\left(\overrightarrow{\frac{P_E}{\varepsilon}}, \overrightarrow{\frac{P_q}{\tau}}, P_Y\right)$ is the

[7] Using the same process, it is possible to demonstrate duality for the cost function (C). It is possible to show that the cost function is equal to $C = \sum_{i=1}^{n} \frac{P_E}{\varepsilon_i} S_i^{\,c}\left(\overrightarrow{\frac{P_E}{\varepsilon}}, \overrightarrow{\frac{P_q}{\tau}}, Y\right) + \sum_{i=1}^{m} \frac{P_{q_i}}{\tau_i} Q_i^{\,c}\left(\overrightarrow{\frac{P_E}{\varepsilon}}, \overrightarrow{\frac{P_q}{\tau}}, Y\right)$, where the subscript c denotes the respective functions of conditional inputs demands. Thus, $C\left(\overrightarrow{\frac{P_E}{\varepsilon}}, \overrightarrow{\frac{P_q}{\tau}}, Y\right)$.

unconditional energy service demands. This means that the total energy demand in the long run is equal to:

$$E = \sum_{i=1}^{n} \frac{S_i\left(\frac{\overrightarrow{P_E}}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)}{\varepsilon_i} \tag{1.5}$$

In the long run, the unconditional demands for the non-energy inputs are equal to $q_i = \frac{Q_i\left(\frac{\overrightarrow{P_E}}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)}{\tau_i}$ and the output supply is equal to $Y\left(\frac{\overrightarrow{P_E}}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)$.

The short-run will be defined when only the energy input (E_i) associated with the energy service i (S_i) is variable and all other energy inputs ($E_j\ \forall j \neq i$) and non-energy inputs (q_i) are fixed. In this way, the output supply and the total energy demand in the short run are equal, respectively, to $Y^r\left(\frac{P_E}{\varepsilon_i}, P_Y, \overrightarrow{\tau q}, \overrightarrow{\varepsilon E_{-\iota}}\right)$ and

$$E^r = \frac{S_i^{\,r}\left(\frac{P_E}{\varepsilon_i}, P_Y, \overrightarrow{\tau q}, \overrightarrow{\varepsilon E_{-\iota}}\right)}{\varepsilon_i} + \sum_{j \neq i} E_j \tag{1.6}$$

where the subscript r identifies the short-run functions and $\overrightarrow{\varepsilon E_{-\iota}}$ represents the vector $[\varepsilon_1 E_1, \dots, \varepsilon_{i-1} E_{i-1}, \varepsilon_{i+1} E_{i+1}, \dots, \varepsilon_n E_n]$ (i.e., the vector $\overrightarrow{\varepsilon E}$ without the i-th element).

As in Wei (2010), we will assume that all markets have a perfect competition equilibrium, for both inputs and output markets. Mathematically, this means $Y = Y^D$, $E^S = E$ and ${q_h}^S = q_h\ \forall\ h = 1,2, \dots, m$, where ${q_h}^S$ is the supply for the input q_h, E^S is the energy supply and Y^D is the output demand. As in Wei (2010), we will also use the hypothesis that Y^D, E^S and ${q_h}^S$ are affected only by their respective prices. Simply put, this means $Y^D(P_Y)$, $E^S(P_E)$ and ${q_h}^S(P_{q_h})$.[8] That is, energy efficiency improvements not affect (directly or indirectly) Y^D, E^S and ${q_h}^S$.

1.4 Rebound effect definition

[8] Despite that all of these functions (${q_h}^S$, E^S and Y^D) have not been developed further, it is easy to see that the supply functions can be developed using the Production theory and the demand function can be developed using both the Consumer theory and the Production theory (if the output is an input for another production). However, the hypothesis that we need in the model is that the energy efficiency improvements not affect (directly or indirectly) these functions.

First, it should be noted that, in this section, we will deal only with the long-run effect, since the short-run effect is analogous (in Appendix 1.A and Appendix 1.B the short-run effect is developed). In addition, as the rebound effect is generally defined through elasticities, then we will denote $\dot{\eta}_j(i) = \frac{di}{dj}\frac{j}{i}$ and $\eta_j(i) = \frac{\partial i}{\partial j}\frac{j}{i}$ as the elasticity of the function "i" with respect to the variable "j".

The rebound effect is the difference between the actual energy savings (AES) and the potential energy savings (PES). AES is the energy savings that actually occurs after economic variables adjust to energy efficiency improvements. PES is the energy savings that is expected to occur without behavioral response to energy efficiency improvements. Therefore, the rebound definition is a residual definition, which arises from the definition of the PES and AES. Thereby, in order to define the rebound effect, it is first necessary to find the elasticity of energy demand expressed in equation (1.5) with respect to energy efficiency i (ε_i), that is, it is necessary to find the $\dot{\eta}_{\varepsilon_i}(E) = -\text{AES}$ (see Appendix 1.A):

$$\dot{\eta}_{\varepsilon_i}(E) = -\sigma_i \dot{\eta}_{\varepsilon_i}(\varepsilon_i) + \sum_{j=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(S_j) \qquad \textbf{(1.7)}$$

where $\sigma_j = \frac{E_j}{E}$ is the share of energy consumption associated with energy service j in the total energy consumption. The term "$\sigma_i \dot{\eta}_{\varepsilon_i}(\varepsilon_i)$" represents the variation in energy consumption when there is no behavioral response to energy efficiency improvements. That is, $\text{PES} = \sigma_i$, since $\dot{\eta}_{\varepsilon_i}(\varepsilon_i) = 1$. Therefore, equation (1.7) shows that the rebound effect (R_{ε_i}) is equal to:

$$R_{\varepsilon_i} = \sigma_i + \dot{\eta}_{\varepsilon_i}(E) = \sum_{j=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(S_j) \qquad \textbf{(1.8)}$$

In particular, when the model incorporates only a single energy service[9], equation (1.8) yields $R_\varepsilon = 1 + \dot{\eta}_\varepsilon(E) = \dot{\eta}_\varepsilon(S)$. This is the commonly used definition of rebound effect (SAUNDERS, 2000a, 2008; SORRELL, 2014; SORRELL; DIMITROPOULOS, 2008; WEI, 2010).

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[9] It should be noted that the case of a single energy service is equivalent to analyzing how energy consumption responds to an improvement in the average energy efficiency of the economy. On the other hand, the case of multi-energy services is equivalent to analyzing how energy consumption responds to improvement in a single energy efficiency while the others are kept constant. Note that this case is a simplification of the case where the different energy efficiencies are improved at different rates. If we assume that all energy efficiencies are improved at the same rate, that is, if we assume that they are all equal, the case of a single energy service will be replicated in the model with multi-energy services.

The rebound definition in equation (1.8) assumes an isolated variation in a single energy efficiency. Although an isolated change in energy efficiency is an important theoretical tool, it is somewhat artificial. As already mentioned, often, new technologies can improve several energy efficiencies at the same time, as well as the productivity of non-energy inputs. Despite this, we will only deal with two extreme cases. The first is the aforementioned case of an isolated improvement in a single energy efficiency, which will demonstrate the energy-augmenting technical change, known as the reallocation effect (GILLINGHAM; RAPSON; WAGNER, 2016). The second is the case of neutral technical change, known as the innovation effect (GILLINGHAM; RAPSON; WAGNER, 2016). This means that all energy efficiencies and the productivity of all non-energy inputs vary at the same rate. That is, $\varepsilon_i = \varepsilon_j = \tau_h \; \forall j, h$, then $\dot{\eta}_{\varepsilon_i}(\varepsilon_i) = \dot{\eta}_{\varepsilon_i}(\varepsilon_j) = \dot{\eta}_{\varepsilon_i}(\tau_h) = 1 \; \forall \; j, h$. In this case, in order to determine the value of $\dot{\eta}_{\varepsilon_i}(\varepsilon_j)$ and $\dot{\eta}_{\varepsilon_i}(\tau_h)$, there is no behavioral response. Appendix 1.B shows that the innovation effect (R_N) is equal to:

$$R_N = 1 + \dot{\eta}_{\varepsilon_i}(E) = \sum_{J=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(S_j) \tag{1.9}$$

Following Saunders (2008), five rebound conditions can be defined:

- Backfire: $\dot{\eta}_{\varepsilon_i}(E) > 0 \Leftrightarrow R > PES$
- Full rebound: $\dot{\eta}_{\varepsilon_i}(E) = 0 \Leftrightarrow R = PES$
- Partial rebound: $-PES < \dot{\eta}_{\varepsilon_i}(E) < 0 \Leftrightarrow 0 < R < PES$
- Zero rebound: $\dot{\eta}_{\varepsilon_i}(E) = PES \Leftrightarrow \; R = 0$
- Super-conservation: $\dot{\eta}_{\varepsilon_i}(E) < -PES \Leftrightarrow \; R < 0$

where R is the rebound effect, that is, for the reallocation effect $R = R_{\varepsilon_i}$ and $PES = \sigma_i$, and for the innovation effect $R = R_N$ and $PES = 1$.

It is noteworthy that the definition of PES can be modified. For example, Guerra and Sancho (2010) argues that the usual definition of PES (i.e., definition we used above) is inappropriate and that PES should rather be defined as those energy savings that occurred when prices are held constant. This definition means that the new PES will be formed by adding our usual definition of PES with the direct effect and the cross-price effect (which will be defined in section 1.5). In this case, the rebound effect would occur through changes in relative prices (i.e., it is the sum of the price effects, which will be defined in section 1.5). As Tuner (2013) points out, since the rebound definition is a residual definition, the size of the rebound effect may change depending on the definition of PES. However, what matters, in the end, is how energy efficiency gains affect energy consumption, that is, what really matters is the AES ($\dot{\eta}_{\varepsilon_i}(E)$). The usual definition of PES and, consequently, the usual rebound definition, is just an intuitive way to decompose the effects that energy efficiency gains have on energy consumption. PES represents the technological effects that energy efficiency improvements cause on energy consumption, while the rebound effect represents the economic effects. As highlighted by Sorrel (2009, 2014), the rebound effect is just an umbrella term for a variety of economic mechanisms that reduce the energy savings from improved energy efficiency. That is, as shown in equation (1.7), $\dot{\eta}_{\varepsilon_i}(E) \coloneqq R_{\varepsilon_i} - \sigma_i$, which means that there is a direct relationship between the rebound effect and AES, as demonstrated by the five rebound conditions above.

1.5 Reallocation rebound effect

Decomposing the effects of equation (1.8) ($\dot{\eta}_{\varepsilon_i}(S_j)$), we find the long-run rebound effect (see Appendix 1.A):

$$\begin{aligned} R_{\varepsilon_i} = -\sigma_i \eta_{P_{S_i}}(S_i) - \sum_{j \neq i} \sigma_j \eta_{P_{S_i}}(S_j) + \sum_{j=1}^{n} \sum_{x=1}^{n} \sigma_j \eta_{P_{S_x}}(S_j) \dot{\eta}_{\varepsilon_i}(P_E) \\ + \sum_{h=1}^{m} \sum_{j=1}^{n} \sigma_j \eta_{P_{Q_h}}(S_j) \dot{\eta}_{\varepsilon_i}(P_{q_h}) + \sum_{j=1}^{n} \sigma_j \eta_{P_Y}(S_j) \dot{\eta}_{\varepsilon_i}(P_Y) \end{aligned} \quad \textbf{(1.10)}$$

where $R_{\varepsilon_i}(S_i) = -\sigma_i \eta_{P_{S_i}}(S_i)$ is the direct effect, $R_{\varepsilon_i}(S_j) = -\sum_{j \neq i} \sigma_j \eta_{P_{S_i}}(S_j)$ is the cross-price effect, $R_{\varepsilon_i}\left(P_{E_j}\right) = \sum_{j=1}^{n} \sum_{x=1}^{n} \sigma_j \eta_{P_{S_x}}(S_j) \dot{\eta}_{\varepsilon_i}(P_E)$ is the energy price effect, $R_{\varepsilon_i}(P_{q_h}) = \sum_{h=1}^{m} \sum_{j=1}^{n} \sigma_j \eta_{P_{Q_h}}(S_j) \dot{\eta}_{\varepsilon_i}(P_{q_h})$ is the input price effect and $R_{\varepsilon_i}(P_Y) = \sum_{j=1}^{n} \sigma_j \eta_{P_Y}(S_j) \dot{\eta}_{\varepsilon_i}(P_Y)$ is the output price effect. All rebound effects, other than the direct effect, will be referred to as indirect effects.

Given the perfect competition hypothesis, it is possible to obtain the values for $\dot{\eta}_{\varepsilon_i}(P_E)$, $\dot{\eta}_{\varepsilon_i}(P_{q_h})$ and $\dot{\eta}_{\varepsilon_i}(P_Y)$, thus, obtaining the rebound effect expressed in equation (1.10) with endogenous prices. Appendix 1.C shows that long-run rebound is equal to:

$$R_{\varepsilon_i} = \sigma_i + \frac{\eta_{P_E}(E^S) \det\left({}_{(2+m)}^{\;b}\Lambda - \sigma_i \mathrm{H}\right)}{\det(\Lambda)} \quad \textbf{(1.11)}$$

where Λ and $\left[{}_{(2+m)}^{b}\Lambda - \sigma_i \mathrm{H}\right]$ are matrices that include the price elasticities of unconditional demands and supplies for inputs, as well as the price elasticities of supply and demand for output (see Appendix 1.C for a formal definition). These matrices are obtained from the Hessian matrix of second-order partials derivatives of the profit function (hereinafter referred to as the "HMS").

1.5.1 Direct round effect

The intuition behind the direct effect is that the improvement in energy efficiency will reduce the implicit price of energy service, causing the firm to use more of this energy service, thereby increasing energy consumption. Whenever the hypotheses about the production function (i.e., convexity of the profit function) are respected, the price elasticity of demand for energy service is always negative ($\eta_{P_{S_i}}(S_i) \leq 0$). Thus, the direct effect will always be positive ($R_{\varepsilon_i}(S_i) = -\sigma_i \eta_{P_{S_i}}(S_i) \geq 0$). If the demand for energy service i is price-elastic ($-\eta_{P_{S_i}}(S_i) > 1$), then the direct effect will more than offset the PES (σ_i). That is, if $-\eta_{P_{S_i}}(S_i) > 1$, then $\sigma_i \left[-1 - \eta_{P_{S_i}}(S_i)\right] > 0$. Thus, the greater the price-elasticity of demand for energy service i, the greater the chances of backfire. Nevertheless, due to the indirect effects, however great this elasticity is, the rebound effect can still fit into any of the five rebound conditions (backfire, full rebound, partial rebound, zero rebound, or super-conservation). Equation (1.11) shows that the backfire condition will occur when $\det\left({}_{(2+m)}^{b}\Lambda - \sigma_i \mathrm{H}\right) > 0$.[10]

The direct effect can be subdivided into substitution and output effects. The Production theory requires that $\eta_{P_{S_i}}(S_i) = \eta_{P_{S_i}}(S_i^c) + \eta_Y(S_i^c)\eta_{P_{S_i}}(Y)$, where S_i^c is the conditional demand for energy service i. Thus, $R_{\varepsilon_i}(S_i)_S = -\sigma_i \eta_{P_{S_i}}(S_i^c)$ is the substitution effect and $R_{\varepsilon_i}(S_i)_Y = -\sigma_i \eta_Y(S_i^c)\eta_{P_{S_i}}(Y)$ is the output effect.[11] Both effects are positive ($R_{\varepsilon_i}(S_i)_S, R_{\varepsilon_i}(S_i)_Y \geq 0$), since $\eta_{P_{S_i}}(S_i^c)$ and $\eta_Y(S_i^c)\eta_{P_{S_i}}(Y)$ are both negative.[12]

1.5.2 Indirect rebound effects

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[10] Since $\eta_{P_E}(E^S) \geq 0$ and $\det(\Lambda) > 0$, as shown in Appendix 1.D.

[11] Given the impact of energy efficiency on implicit price of energy service, the substitution effect captures the impact on energy consumption caused by the substitution of energy service i by other energy services and other non-energy inputs, keeping the output constant. On the other hand, the output effect captures the impact on energy consumption caused by changes on the output level.

[12] For proof of this statement, see Shishko (1974).

1.5.2.1 Cross-price effect

The cross-price effect captures the impact on energy consumption caused by the change in demand for other energy services, given the variation in the implicit price of energy service i. That is, the reduction in the implicit price of an energy service will modify the other energy services demands, which may lead the firm to use more or less of these energy services. The sign of the cross-price effect ($R_{\varepsilon_i}(S_j) = -\sum_{j\neq i}\sigma_j\eta_{P_{S_i}}(S_j)$) is undetermined at first. When all other energy services are gross substitutes for energy service i ($\eta_{P_{S_i}}(S_j) > 0$), then the cross-price effect is negative ($R_{\varepsilon_i}(S_j) < 0$), while all other energy services are gross complementary ($\eta_{P_{S_i}}(S_j) < 0$), this effect is positive ($R_{\varepsilon_i}(S_j) > 0$).

It is noteworthy that, similarly to the direct effect, the cross-price effect can be divided into substitution effect ($R_{\varepsilon_i}(S_j)_S$) and output effect ($R_{\varepsilon_i}(S_j)_Y$). It is easy to see that $R_{\varepsilon_i}(S_j) = -\sum_{j\neq i}\sigma_j\left[\eta_{P_{S_i}}(S_j^c) + \eta_Y(S_j^c)\eta_{P_{S_i}}(Y)\right] = -\sum_{j\neq i}\sigma_j\eta_{P_{S_i}}(S_j^c) - \eta_{P_{S_i}}(Y)\sum_{j\neq i}\sigma_j\eta_Y(S_j^c)$, where $R_{\varepsilon_i}(S_j)_S = -\sum_{j\neq i}\sigma_j\eta_{P_{S_i}}(S_j^c)$ and $R_{\varepsilon_i}(S_j)_Y = -\eta_{P_{S_i}}(Y)\sum_{j\neq i}\sigma_j\eta_Y(S_j^c)$.

1.5.2.2 Input price effect and output price effect

The input price effect and output price effect (as well as the energy price effect) are effects on the market equilibrium. Appendix 1.C shows that output price effect and input price effect are equal, respectively, to:

$$R_{\varepsilon_i}(P_Y) = \frac{\eta_{P_E}(E^S)}{\det(\Lambda)}\eta_{P_{S_i}}(Y)(-1)^{1+(2+m)}\det\left({}_{(2+m)}^{\ b}\Lambda_{1\times(2+m)}\right) \quad \textbf{(1.12)}$$

$$R_{\varepsilon_i}(P_{q_h}) = -\frac{\eta_{P_E}(E^S)}{\det(\Lambda)}\sum_{h=1}^{m}\eta_{P_{S_i}}(Q_h)(-1)^{(1+h)+(2+m)}\det\left({}_{(2+m)}^{\ b}\Lambda_{(1+h)\times(2+m)}\right) \quad \textbf{(1.13)}$$

where ${}_{(2+m)}^{\ b}\Lambda_{l\times k}$ is the submatrix of ${}_{(2+m)}^{\ b}\Lambda$ obtained by suppressing its l-th row and its k-th column (see Appendix 1.C for a formal definition).

As highlighted in equations (1.12) and (1.13), these effects depend on complex interactions between different markets, represented by the determinant terms. Changes in the relative prices of output and all inputs (including energy) will affect each other, having complex effects on equilibrium prices. The input price effect can be separated into different effects from each specific input (for each h), such as capital or labor.

Despite the complexity, the intuition behind these effects is quite simple. The intuition behind the input price effect is that the energy efficiency improvement alters the relative prices, which, in turn, displaces the demands for non-energy inputs, and may increase or decrease them. The displacement of demands changes the equilibrium prices of non-energy inputs, which, in turn, has a cross effect on the consumption of all energy services. This could result in greater (less) use of energy services and, consequently, greater (less) use of energy. Similarly, the output price effect is explained because the change in relative prices, displaces the output supply function (increase or decrease), which, in turn, changes the output equilibrium price. This change in the equilibrium output price affects the unconditional demands of energy services, which may generate greater or lesser use of them and, consequently, of energy.

The signs of both of these effects are undetermined at first, for two reasons First, the sign of $\eta_{P_{S_i}}(Q_h)$ depends on whether non-energy inputs and energy service i are gross substitute ($\eta_{P_{S_i}}(Q_h) > 0$) or gross complementary ($\eta_{P_{S_i}}(Q_h) < 0$). Likewise, the sign of $\eta_{P_{S_i}}(Y)$ depends on whether energy service i are inferior ($\eta_Y(S_j^c) < 0$) or normal inputs ($\eta_Y(S_j^c) > 0$), since $\eta_{P_{S_i}}(Y) = -\theta_{S_i}\eta_Y(S_i^c)\eta_{P_Y}(Y)$, where $\theta_{S_i} = \frac{S_i P_{S_i}}{Y P_Y}$. Second, the signs of $\det\left({}_{(2+m)}^{\ b}\Lambda_{l\times k}\right)$ $\forall l \neq k$ can be positive or negative, as argued in Appendix 1.D.

1.5.2.3 Energy price effect

Appendix 1.C shows that the energy price effect can be expressed as:

$$\begin{aligned} R_{\varepsilon_i}\left(P_{E_j}\right) = \sigma_i - \left[R_{\varepsilon_i}(S_i) + R_{\varepsilon_i}(S_j) + R_{\varepsilon_i}\left(P_{q_h}\right) + R_{\varepsilon_i}(P_Y)\right] \\ + \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)}{\det(\Lambda)} \end{aligned} \quad \textbf{(1.14)}$$

As Wei (2010) points out, the supply side of the energy market is of equivalent importance to the demand side, but most studies pay insufficient attention to the supply side. The price-elasticity of the energy supply is extremely important for the rebound size. Appendix 1.E shows that when the energy price energy is endogenous (i.e., when energy supply function is introduced), the magnitude of the AES (i.e., $\dot{\eta}_{\varepsilon_i}(E)$) decreases. That is, when the energy price is endogenous, energy consumption will react less to energy efficiency shocks. Moreover, the more price-inelastic is the energy supply, the smaller the magnitude of the AES. In the extreme case, when the energy supply is fixed ($\eta_{P_E}(E^S) = 0$), the energy consumption does not respond to shocks in energy efficiency ($\dot{\eta}_{\varepsilon_i}(E) = 0$). Therefore, the energy supply is a counterweight to variations in energy consumption. In terms of the rebound effect, this means that the more price-inelastic is the energy supply, the rebound effect will be closer to the full rebound condition and, in the extreme case when energy supply is fixed, this condition will be checked ($R_{\varepsilon_i} = \sigma_i$).

The energy price effect is illustrated in Figure 1.1. Point A shows the initial market equilibrium. When energy efficiency improves, it shifts the energy demand function from E_0^D to E_1^D. Thus, energy consumption is expected to change to point B (i.e., it expected to be reduced by PES $= \sigma_i$). However, rebound effect caused when the energy price is fix displaces the energy demand function to E_2^D, modifying energy consumption to point C. This new energy demand (E_2^D) will generate a new energy market equilibrium, where the energy price effect will adjust energy consumption to point D. The difference between point A and point D is the AES. If the rebound effect with fixed energy price is so strong that it displaces the energy demand to $E_{2'}^D$, the energy consumption will be shifted to point C'. However, the energy price effect will adjust the energy consumption to point D', and the difference between point D' and A is the actual energy augmentation. If energy supply is fixed, the rebound effect must be a full rebound, thus making the energy savings equal to zero, yields the market equilibrium to point E or E'.

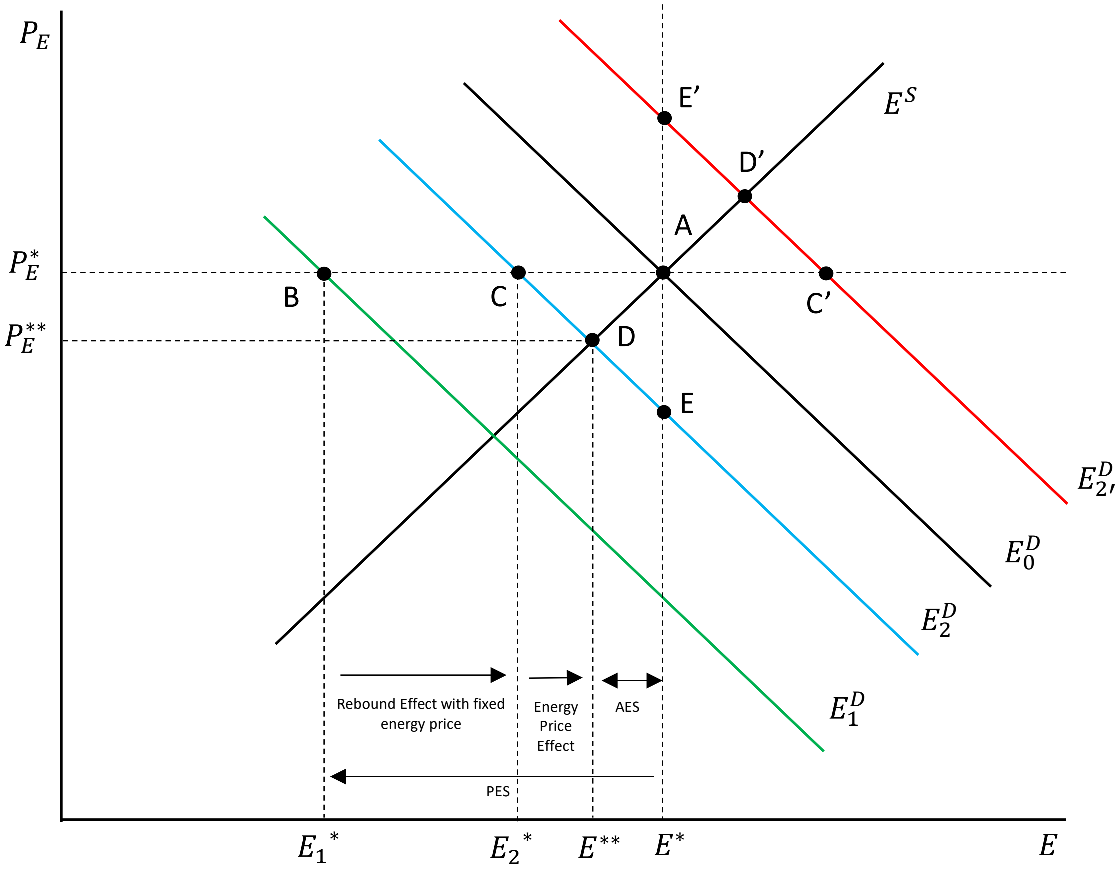

Figure 1.1 – Rebound Effect
Source: Authors' own elaboration.

The results found here differ from what Wei (2010) points out. The author states that the rebound magnitude is always smaller when the energy supply is incorporated, and if it is fixed, the rebound effect must be zero (WEI, 2010, p. 664). However, the author is confused by the rebound definition. Wei (2010), in his equation 2, defines the rebound effect as $R_{\varepsilon} = 1 + \dot{\eta}_{\varepsilon}(E)$. Nevertheless, in order to verify the importance of the energy supply, the author uses $\dot{\eta}_{\varepsilon}(E)$ (his equation 23 and 42). In other words, what the author finds is that the magnitude of the variation in energy consumption is smaller when the energy supply is incorporated, and not the rebound effect itself. Therefore, Wei (2010), in his model, obtains the same result that is being generalized here.[13] The results obtained here are also different from those highlighted by Gillingham, Rapson, and Wagner (2016) and Wei and Liu (2017). The authors argue that, *ceteris paribus*, the more price-inelastic is the energy supply, the greater the rebound effect (GILLINGHAM; RAPSON; WAGNER, 2016; WEI; LIU, 2017). This is incorrect, because if $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right) > 0$ in equation (1.11) (this is equivalent to the shift in energy demand to $E_{2'}^{D}$, in Figure 1.1), then the more price-inelastic is the energy supply, the lower the rebound effect (see Appendix 1.E).

1.5.3 Super-conservation

Super-conservation occurs when the rebound effect is negative, amplifying the PES. This condition is considered to be a counter-intuitive phenomenon (SAUNDERS, 2008; WEI, 2010). The main reason for this is because the macroeconomic rebound models have only a single energy service. In this case, as shown in Appendix 1.D, when the hypotheses are well-behaved, the positive semidefinite property of the HMS makes super-conservation impossible. In other words, when there is a single energy service in the model, the different rebound effects can be positive or negative (except the direct effect which is always positive), but their sum is always greater than or equal to zero ($R_{\varepsilon_i} \geq 0$).

For example, in a model that has only one energy service and all exogenous prices (hereinafter referred to as the "simplest model"), the rebound effect will always be positive, since the rebound effect comes down to direct effect: $R_{\varepsilon_i} = -\eta_{P_S}(S) \geq 0$. This explains why, in general, the energy demand models developed in Saunders (2008) are not compatible with super-conservation. If the production function does not respect the well-behaved hypotheses (i.e., convexity of the profit function), the direct effect can be negative, this would explain how

[13] It is possible to see from equations 3 and 5 of the Wei's (2010) article that when the price elasticity of energy supply tends to zero, both the short-run and long-run rebound effects tend to 1, that is, they tend to full rebound.

some functions expressed in Saunders (2008) (i.e., the Gallant-Fourier cost function) allows super-conservation.[14]

Although Wei (2010) argues that his model allows super-conservation in the long run, this is incorrect. This is because his substitution parameter "θ" in well-behaved situations cannot assume any value, as defended by the author. To see this, assume the same hypotheses as in Wei (2010), that is, the model has a single energy service and a single non-energy input, and the output price is fixed, thus equation (1.11) yields:

$$R_\varepsilon = \frac{\left[1 + \eta_{P_E}(E^S)\right]\left[-\eta_{P_S}(S)\eta_{P_q} - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\right]}{\eta_{P_E}\eta_{P_q} - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})} \qquad \textbf{(1.15)}$$

where $\eta_{P_E} = \left[\eta_{P_E}(E^S) - \eta_{P_S}(S)\right]$, $\eta_{P_q} = \left[\eta_{P_q}(q^S) - \eta_{P_Q}(Q)\right]$ and there is no longer a need for h and i subscriptions.

It can be seen in equation (1.15) that, in order to the rebound effect to be less than zero ($R_\varepsilon < 0$), it is necessary that $-\eta_{P_S}(S)\eta_{P_q} < \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S}) < \eta_{P_E}\eta_{P_q}$. However, the positive semidefinite property of the HMS imposes $\eta_{P_S}(S)\eta_{P_Q}(Q) \geq \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})$. Since $\eta_{P_q}(q^S) \geq 0$, then $-\eta_{P_S}(S)\eta_{P_q} \geq \eta_{P_S}(S)\eta_{P_Q}(Q) \geq \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})$. Therefore, the rebound effect cannot be negative.[15] In other words, the positive semidefinite property of the HMS requires $\eta_{P_E}\eta_{P_q} -$

[14] As Wei (2010) points out for the short-run rebound, the super-conservation expressed in the Gallant-Fourier cost function must be incorrect in Saunders (2008). Here, we are expanding and claiming that the result of super-conservation in the long-run rebound must also be incorrect. However, further explanations are needed to clarify how the specification of the Gallant-Fourier cost function parameters resulted in super-conservation in Saunders (2008). It is speculated that there must be problems with the concavity of the cost function.

[15] If Wei's (2010) production function is concave, we can show that his long-run rebound is always positive. Using Wei's (2010) notation, his long-run effect (i.e. his equation 5) can be rewritten as $R^l = \frac{\left(1+\frac{1}{\sigma^S}\right)\left(\frac{1}{\sigma_k^S}-\sigma_k^k\right)}{\left[\left(\frac{1}{\sigma^S}-\sigma_e^e\right)\left(\frac{1}{\sigma_k^S}-\sigma_k^k\right)-\sigma_e^k\sigma_k^e\right]} = \frac{\left(1+\frac{1}{\sigma^S}\right)\left(\frac{1}{\sigma_k^S}-\sigma_k^k\right)}{\det[f'(\sigma-f'')X]}$, where $f'' = \begin{bmatrix} f_{kk} & f_{ke} \\ f_{ek} & f_{ee} \end{bmatrix}$, $\sigma = \begin{bmatrix} \frac{1}{dK^S/dP_k} & 0 \\ 0 & \frac{1}{\tau^2}\frac{1}{dE^S/dP_e} \end{bmatrix}$, $f' = \begin{bmatrix} \frac{1}{f_k} & 0 \\ 0 & \frac{1}{f_e} \end{bmatrix}$ and $X = \begin{bmatrix} K & 0 \\ 0 & \tau E \end{bmatrix}$. Since the production function is concave, the Hessian matrix of second-order partials derivatives of the production function (f'') is negative semidefinite. Then, minus this Hessian matrix ($-f''$) is positive semidefinite. Note that σ is positive definite, since the supplies functions (K^S and E^S) are increasing in their prices (i.e. $\frac{dK^S}{dP_k}, \frac{dE^S}{dP_e} > 0$ and $\tau > 0$).Therefore, $\sigma + (-f'')$ is a positive definite matrix (Proof: let $\vec{x} \neq 0$ nonsingular vector, then $\vec{x}^t[\sigma + (-f'')]\vec{x} = \vec{x}^t\sigma\vec{x} + \vec{x}^t(-f'')\vec{x} > 0$, since $\vec{x}^t\sigma\vec{x} > 0$ – i.e. positive definite – and $\vec{x}^t(-f'')\vec{x} \geq 0$ – i.e. positive semidefinite). Thus, $\det(\sigma - f'') > 0$. In addition, f' and X are also positive definite, since the production function is increasing ($f_k, f_e > 0$) and $K, \tau E > 0$. Thus, $\det(f'), \det(X) > 0$. This implies $\det[f'(\sigma - f'')X] = \det(f')\det(\sigma - f'')\det(X) > 0$. Thus, Wei's (2010) long-run effect is always positive ($R^l > 0$), since $\frac{1}{\sigma^S}, \frac{1}{\sigma_k^S}, -\sigma_k^k$

$\eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S}) > 0$ and $-\eta_{P_S}(S)\eta_{P_q} - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S}) \geq 0$. Thus, the super-conservation is not allowed by the model.

However, Production theory is fully compatible with super-conservation, even in the presence of well-behaved hypotheses. In order to the rebound effect to be negative, it is necessary (but not sufficient) to include more than one energy service in the model (i.e., it is necessary to incorporate the cross-price effect). For example, assume that $Y = f(S_1, S_2)$. Also, assume that P_E and P_Y are exogenous. Thus, equation (1.10) yields: $R_{\varepsilon_i} = -\sigma_i \eta_{P_{S_i}}(S_i) - \sigma_j \eta_{P_{S_i}}(S_j)$, where $\sigma_i = \frac{E_i}{E} = \frac{S_i/\varepsilon_j}{S_1/\varepsilon_1 + S_2/\varepsilon_2} = \frac{P_{S_i} S_i}{P_{S_1} S_1 + P_{S_2} S_2}$, since $E_j = \frac{S_j}{\varepsilon_j}$ and $P_{S_j} = \frac{P_E}{\varepsilon_j}$. The Production theory requires that the unconditional input demands are homogeneous of degree zero in prices: $S_i(P_{S_1}, P_{S_2}, P_Y)$. Using the Euler's homogeneous function theorem is possible to identify: $\eta_{P_{S_1}}(S_i) + \eta_{P_{S_2}}(S_i) + \eta_{P_Y}(S_i) = 0$. Therefore, $\eta_{P_Y}(S_i) = -\eta_{P_{S_1}}(S_i) - \eta_{P_{S_2}}(S_i)$. Due to the symmetry property of the HMS, we have $\frac{\partial S_i}{\partial P_{S_j}} = \frac{\partial S_j}{\partial P_{S_i}}$ and then $\eta_{P_{S_j}}(S_i) = \frac{\sigma_j}{\sigma_i} \eta_{P_{S_i}}(S_j)$. Thus, it is easy to see that $\sigma_i \eta_{P_Y}(S_i) = -\sigma_i \eta_{P_{S_i}}(S_i) - \sigma_j \eta_{P_{S_i}}(S_j) = R_{\varepsilon_i}$. Therefore, $R_{\varepsilon_i} = \sigma_i \eta_{P_Y}(S_i) = \sigma_i \eta_Y(S_i^c) \eta_{P_Y}(Y)$. In this way, when S_i is an inferior input ($\eta_Y(S_i^c) < 0$), then $R_{\varepsilon_i} \leq 0$, since $\sigma_i, \eta_{P_Y}(Y) \geq 0$. It is noteworthy that inferior inputs, although unusual, exist in well-behaved situations (BERTOLETTI; RAMPA, 2013; EPSTEIN; SPIEGEL, 2000).

1.5.4 Short-run versus long-run rebound effects

The short-run rebound effect is similar to the long-run effect and is equal to (see Appendix 1.A):

$$R_{\varepsilon_i}^r = -\sigma_i^r \eta_{P_{S_i}}({S_i}^r) + +\sigma_i^r \eta_{P_{S_i}}({S_i}^r) \dot{\eta}_{\varepsilon_i}(P_E) + \sigma_i^r \eta_{P_Y}({S_i}^r) \dot{\eta}_{\varepsilon_i}(P_Y) \qquad \textbf{(1.16)}$$

where the subscript r identifies the short-run variables. The equivalent short-run effects are: $R_{\varepsilon_i}^r(S_i) = -\sigma_i^r \eta_{P_{S_i}}({S_i}^r)$ is the direct effect, $R_{\varepsilon_i}^r\left(P_{E_j}\right) = \sigma_i^r \eta_{P_{S_i}}({S_i}^r) \dot{\eta}_{\varepsilon_i}(P_E)$ is the energy price effect and $R_{\varepsilon_i}^r(P_Y) = \sigma_i^r \eta_{P_Y}({S_i}^r) \dot{\eta}_{\varepsilon_i}(P_Y)$ is the output price effect.

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and $-\sigma_e^e$ are all positive. Nevertheless, Wei (2010) states that the rebound effect can be negative when $\theta = \frac{\sigma_e^k \sigma_k^e}{\left(\frac{1}{\sigma_k^S} - \sigma_k^k\right)} > \left(\frac{1}{\sigma^S} - \sigma_e^e\right)$, which means $\det[f'(\sigma - f'')X] < 0$, which is impossible.

Appendix 1.C shows that the short-run effect with endogenous prices is equivalent to the long-run effect with a single energy service and without non-energy inputs (multiplied by σ_i^r):

$$R_{\varepsilon_i}^r = \frac{[1 + \eta_{P_E}(E^{S^r})]\sigma_i^r \det(\widehat{\Lambda}^r)}{\det(\Lambda^r)} \quad \textbf{(1.17)}$$

where Λ^r is the matrix equivalent to the matrix Λ in the short run, and the matrix $\widehat{\Lambda}^r$ is the matrix obtained from Λ^r by subtracting $\eta_{P_E}(E^{S^r})$ from its element $a_{(2+m)\times(2+m)}$ (see Appendix 1.C for a formal definition). Therefore, the same results presented in the long-run effect are also true for the short-run effect. That is, in the short run, the energy supply is also a counterweight to variations in energy consumption and the super-conservation is not allowed (i.e., $R_{\varepsilon_i}^r \geq 0$).[16]

The big question is whether the short-run effect is greater or less than the long-run effect. In the simplest model, the long-run effect will always be greater than or equal to the short-run effect ($R_{\varepsilon_i} \geq R_{\varepsilon_i}^r$). This explains why all the functional forms presented in Saunders (2008) and the Cobb-Douglas function presented in Wei (2007) have greater long-run effects compared to short-run effects.

This statement derives from the Le Chatelier principle. Given these hypotheses, it is possible to identify from equations (1.10) and (1.16) that $R_{\varepsilon_i} = -\eta_{P_S}(S)$ and $R_{\varepsilon_i}^r = -\eta_{P_S}(S^r)$, that is, the rebound effect comes down to the direct effect. We can express the long-run unconditional demand for energy service ($S\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)$) through the short-run demand ($S^r\left(\frac{P_E}{\varepsilon}, P_Y, \vec{Q}\right)$. For this, just replace the long-run demand for non-energy inputs ($Q_h\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)$) in their respective short-run value (Q_h). Thus, $S\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right) = S^r\left(\frac{P_E}{\varepsilon}, P_Y, \overrightarrow{Q\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)}\right)$, where $\overrightarrow{Q\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)}$ is the vector $\left[Q_1\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right), \dots, Q_m\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)\right]$. In this way, we can obtain a relationship between short-run and long-run price elasticities. It is easy to see that $\eta_{P_S}(S) = \eta_{P_S}(S^r) + \sum_{h=1}^{m} \eta_{P_{Q_h}}(S)\eta_{P_S}(Q_h)\left[\eta_{P_{Q_h}}(Q_h)\right]^{-1}$. Multiplying both sides by minus one yields:

[16] This is why Wei (2010) needs to include the externalities to generate super-conservation in the short-run model.

$$-\eta_{P_S}(S) = -\eta_{P_S}(S^r) + \sum_{h=1}^{m} \eta_{P_{Q_h}}(S)\eta_{P_S}(Q_h)\left[-\eta_{P_{Q_h}}(Q_h)\right]^{-1} \quad \mathbf{(1.18)}$$

Therefore, equation (1.18) yields: $R_{\varepsilon_i} - R_{\varepsilon_i}^r = \sum_{h=1}^{m} \eta_{P_{Q_h}}(S)\eta_{P_S}(Q_h)\left[-\eta_{P_{Q_h}}(Q_h)\right]^{-1}$. The symmetry and positive semidefinite properties of the HMS will imply that $\eta_{P_{Q_h}}(S)\eta_{P_S}(Q_h) \geq 0$ and $-\eta_{P_{Q_h}}(Q_h) \geq 0$. Thus, $\sum_{h=1}^{m} \eta_{P_{Q_h}}(S)\eta_{P_S}(Q_h)\left[-\eta_{P_{Q_h}}(Q_h)\right]^{-1} \geq 0$, then $R_{\varepsilon_i} \geq R_{\varepsilon_i}^r$.

When we have the same hypotheses as in Wei's (2010) model, the short-run rebound is equal to:

$$R_{\varepsilon_i}^r = \frac{[1 + \eta_{P_E}(E^{S^r})][-\eta_{P_S}(S^r)]}{\eta_{P_E}^r} \quad \mathbf{(1.19)}$$

where $\eta_{P_E}^r = \left[\eta_{P_E}\left(E^{S^r}\right) - \eta_{P_S}(S^r)\right]$. If we make the same comparison made by Wei (2010) between the short-run effect and the long-run effect, that is, if the comparison is made assuming that price-elasticity of energy supply is the same in both the short and long run ($\eta_{P_E}(E^S) = \eta_{P_E}\left(E^{S^r}\right)$)[17], the result will indicate that the long-run effect is always greater than or equal to the short-run effect. To see this, just subtract equation (1.19) from equation (1.15), which yields:

$$R_{\varepsilon_i} - R_{\varepsilon_i}^r = \frac{\eta_{P_E}(E^S)[1 + \eta_{P_E}(E^S)]\left\{\eta_{P_q}\left[\eta_{P_S}(S^r) - \eta_{P_S}(S)\right] - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\right\}}{\eta_{P_E}^r\left[\eta_{P_E}\eta_{P_q} - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\right]} \quad \mathbf{(1.20)}$$

As already seen, the positive semidefinite property of the HMS imposes that $\left[\eta_{P_E}\eta_{P_q} - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\right]$, $\eta_{P_E}^r$, $\eta_{P_E}(E^S)$ are all non-negative. Therefore, the result will depend on the sign of $\left\{\eta_{P_q}\left[\eta_{P_S}(S^r) - \eta_{P_S}(S)\right] - \eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\right\}$. However, if we replace equation (1.18) (with $h = 1$) in equation (1.20), we get: $R_{\varepsilon_i} - R_{\varepsilon_i}^r =$

[17] The hypothesis used by Wei (2010) is that the values of all elasticities are the same in both the short run and long run. However, as Wei (2010) is using the elasticities of the production function, the values of these elasticities will not change if the production function is the same for both periods (although some elasticities may appear in the long run, but may not appear in the short run). Essentially, the only hypothesis that Wei (2010) makes, in order to compare the rebound effects, is that the elasticity of the energy supply is equal in both the short run and long run.

$\frac{\eta_{P_E}(E^S)[1+\eta_{P_E}(E^S)]\eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\left[-\eta_{P_{Q_h}}(Q_h)\right]^{-1}\eta_{P_q}(q^S)}{\eta^r_{P_E}\left[\eta_{P_E}\eta_{P_q}-\eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\right]}$. The symmetry and positive semidefinite properties of the HMS require that $\eta_{P_S}(Q)\eta_{P_Q}(\mathrm{S})\left[-\eta_{P_{Q_h}}(Q_h)\right]^{-1}\eta_{P_q}(q^S) \geq 0$, then $R_{\varepsilon_i} - R^r_{\varepsilon_i} \geq 0$.[18]

Nevertheless, supposing that the price elasticities in the short run and the long run are equal is a very restrictive hypothesis, since they will often be different.[19] Thus, if the price elasticity of energy supply in the short run and the long run is different, Wei's (2010) model also allows the long-run effect to be less than the short-run effect. Generalizing through the model presented here, we can affirm, when comparing equation (1.17) with equation (1.11), that whenever the model includes more than one energy service, endogenizes the output price (P_Y) or energy price (P_E), the long-run effect may be greater or less than the short-run effect.

1.6 A note on the innovation rebound effect

1.6.1 The backfire hypothesis

Since the innovation effect is the rationale behind many of the backfire claims in the literature (GILLINGHAM; RAPSON; WAGNER, 2016; JENKINS; NORDHAUS; SHELLENBERGER, 2011; SORRELL, 2007, 2009), it is important to determine under what circumstances this effect results in backfire. In order to reduce complexity, we will use the simplest model to analyze the innovation rebound effect. It is noteworthy that this model has been used in most works that examine the innovation effect (SAUNDERS, 1992, 2005, 2013, 2015). Given this hypothesis, Appendix 1.B shows that the long-run innovation effect expressed in equation (1.9) is equal to: $R_N = -\eta_{P_S}(S) - \sum_{h=1}^{m}\eta_{P_{Q_h}}(S)$. The Production theory requires

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[18] We can show this fact through Wei's (2010) model. Wei's (2010) equation 6 can be written as: $R^l - R^S = \frac{\sigma_e^k\sigma_k^e\left(1+\frac{1}{\sigma^S}\right)}{\left(\frac{1}{\sigma^S}-\sigma_e^e\right)\left[\left(\frac{1}{\sigma^S}-\sigma_e^e\right)\left(\frac{1}{\sigma_k^S}-\sigma_k^k\right)-\sigma_e^k\sigma_k^e\right]}$. Using the footnote 15 notations, is possible to see that this equation is equal to: : $R^l - R^S = \frac{\sigma_e^k\sigma_k^e\left(1+\frac{1}{\sigma^S}\right)}{\left(\frac{1}{\sigma^S}-\sigma_e^e\right)\det[f'(-f''+\sigma)X]}$. Essentially, what Wei (2010) is using to show that the long-run effect may be less than the short-run effect is to assume that the $\det[f'(-f''+\sigma)X] < 0$ (i.e. $\theta = \frac{\sigma_e^k\sigma_k^e}{\left(\frac{1}{\sigma_k^S}-\sigma_k^k\right)} > \left(\frac{1}{\sigma^S}-\sigma_e^e\right)$ in Wei's (2010) words), since $(\sigma_e^k\sigma_k^e)$, $\frac{1}{\sigma^S}$ and $-\sigma_e^e$ are all non-negative. However, as shown in footnote 15, $\det[f'(-f''+\sigma)X] > 0$. Therefore, the long-run rebound effect will always be greater than or equal to the short-run rebound effect ($R^l \geq R^S$).

[19] For example, the Le Chatelier Principle states that firms will respond more to price change in the long run than in the short run, that is, $\frac{\partial y}{\partial P_y} \geq \frac{\partial y^r}{\partial P_y}$ and $-\frac{\partial X}{\partial P_X} \geq -\frac{\partial X^r}{\partial P_X}$, where y represents the supply functions (Y, E^S and q_h^S) and X represents the inputs (unconditional) demands functions (S_j and Q_h).

that the unconditional input demands are homogeneous of degree zero in prices. Using the Euler's homogeneous function theorem is possible to identify: $\eta_{P_S}(S) + \sum_{h=1}^{m} \eta_{P_{Q_h}}(S) + \eta_{P_Y}(S) = 0$. Therefore, the long-run innovation effect can be rewritten as:

$$R_N = \eta_{P_Y}(S) = \eta_Y(S^c)\eta_{P_Y}(Y) \qquad \textbf{(1.21)}$$

Hence, the sign of long-run innovation effect will depend on whether the energy service is normal input ($\eta_Y(S^c) > 0$) or inferior inputs ($\eta_Y(S^c) < 0$), since Production theory requires $\eta_{P_Y}(Y) \geq 0$. In this way, although improbable, the super-conservation is possible for the innovation effect in the long-run. For that, it is sufficient that the energy service is an inferior input. Nevertheless, when the production function is homothetic (a property widely used in economic theory), the innovation effect cannot be negative. This is because homothetic production functions ensure that all inputs will be normal ($\eta_Y(S^c) = \eta_Y(Q_h^c) > 0\ \forall h$).[20]

Although the homotheticity hypothesis is not a sufficient condition to generate backfire, when the hypothesis is restricted, that is, when the production function is homogeneous, the innovation effect always leads to backfire. When the production function (strictly concave) is homogeneous of degree $0 < \gamma < 1$, then $\eta_Y(S^c) = \frac{1}{\gamma}$ and $\eta_{P_Y}(Y) = \frac{\gamma}{(1-\gamma)}$ (see footnote 20). Therefore, equation (1.21) yields $R_N = \frac{1}{1-\gamma}$ and so $R_N > 1$. That is, the homogeneity hypothesis is sufficient to generate backfire.

Similar to section 1.5.4, we can find a relationship between the short-run innovation effect and the long-run effect. Appendix 1.B shows that the short-run innovation effect is equal

[20] When the production function is homothetic, the conditional input demands and the cost function are multiplicatively separable: $X^c\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, Y\right) = h(Y)X^c\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, 1\right)$ and $C\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, Y\right) = h(Y)C\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, 1\right)$, where $h(.)$ is a strictly increasing function ($\frac{\partial h(Y)}{\partial Y} > 0$), $X^c\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, 1\right)$ is the conditional input demand ($X = S, Q_h$) for 1 unit of output and $C\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, 1\right)$ is the unit cost function (JEHLE; RENY, 2011). Therefore, for any X^c we have: $\eta_Y(X^c) = \eta_Y\big(h(Y)\big) > 0\ \forall h$, since $h(Y)$ is strictly increasing. Given the perfect competition hypothesis, the Production theory requires that $\frac{\partial C\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, Y\right)}{\partial Y} = P_Y$. Thus, through the derivative of the inverse function, we can easily find that $\eta_{P_Y}(Y) = \frac{1}{\eta_Y(h'(Y))} > 0$, where $h'(Y) = \frac{\partial h(Y)}{\partial Y}$. Therefore, for any input: $\eta_{P_Y}(X) = \eta_Y(X^c)\eta_{P_Y}(Y) = \frac{\eta_Y(h(Y))}{\eta_Y(h'(Y))}$. When the production function (strictly concave) is homogeneous of degree $0 < \gamma < 1$, then $h(Y) = Y^{\frac{1}{\gamma}}$. Thus, $\eta_Y(X^c) = \frac{1}{\gamma}$ and $\eta_{P_Y}(Y) = \frac{\gamma}{(1-\gamma)}$.

to: $R_{\mathrm{N}}^{r} = -\eta_{P_{S_i}}(S_i{}^r) + \sum_{h=1}^{m} \eta_{Q_h}(S_i{}^r)$. Using the identity $S\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, P_Y\right) = S^r\left(\frac{P_E}{\varepsilon}, P_Y, \overrightarrow{Q\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, P_Y\right)}\right)$, it's easy to see that:

$$R_{\mathrm{N}}^{r} = R_N + \sum_{h=1}^{m} \frac{\eta_{P_{Q_h}}(S)}{-\eta_{P_{Q_h}}(Q_h)} \left[\eta_{P_Y}(Q_h) - 1\right] \quad \textbf{(1.22)}$$

When the production function is homogeneous, then $\eta_{P_Y}(Q_h) = \eta_{P_Y}(S) = R_N > 1 \; \forall h$ (see footnote 20). Thus, in order to the short-run innovation effect to result in backfire ($R_{\mathrm{N}}^{r} > 1$), it is necessary that $R_N\left[1 + \sum_{h=1}^{m} \frac{\eta_{P_{Q_h}}(S)}{-\eta_{P_{Q_h}}(Q_h)}\right] > \left[1 + \sum_{h=1}^{m} \frac{\eta_{P_{Q_h}}(S)}{-\eta_{P_{Q_h}}(Q_h)}\right]$. As $R_N > 1$, it is necessary that $\sum_{h=1}^{m} \frac{\eta_{P_{Q_h}}(S)}{-\eta_{P_{Q_h}}(Q_h)} > -1$ and it is sufficient that $\eta_{P_{Q_h}}(S) \geq 0 \; \forall h$ (since $-\eta_{P_{Q_h}}(Q_h) > 0 \; \forall h$), in order to the short-run effect to generate backfire. In other words, when the production function is homogeneous, it is sufficient that the non-energy inputs are gross substitutes for the energy service ($\eta_{P_{Q_h}}(S) \geq 0$) to short-run effect leads to backfire. In this case, it is possible to see using equation (1.22) that the short-run innovation effect also will be greater than the long-run effect. That is, if $\eta_{P_{Q_h}}(S) \geq 0 \; \forall h$, then $R_{\mathrm{N}}^{r} \geq R_N > 1$. On the other hand, if the non-energy inputs are gross complements for the energy service ($\eta_{P_{Q_h}}(S) < 0 \; \forall h$), the short-run effect will be less than the long-run effect and may not generate backfire.

It is noteworthy that when the neutral technical change is defined as the variation in the total factor productivity (TFP), that is, $Y = \varepsilon f(\vec{q}, E)$, all the results obtained above using the homogeneity hypothesis, will be true only with the weakest hypothesis of homotheticity. This is because the long-run innovation effect, in this case, will be equal to: $R_N = 1 + \eta_Y(E^c)\eta_{P_Y}(Y)$.[21] Thereby, the innovation effect always leads to backfire if the energy is a

[21] As $Y = \varepsilon f(\vec{q}, E)$, the first-order condition of profit maximization requires $\varepsilon P_Y \frac{\partial f}{\partial E} = P_E$ and $\varepsilon P_Y \frac{\partial f}{\partial q_h} = P_{q_h}$. Thus, the unconditional inputs demands are equal to $E\left(P_E, \overrightarrow{P_q}, \varepsilon P_Y\right)$ and $q_h\left(P_E, \overrightarrow{P_q}, \varepsilon P_Y\right)$. Therefore, $\dot{\eta}_\varepsilon(E) = \eta_{P_Y}(E) = \eta_Y(E^c)\eta_{P_Y}(Y)$ and then, by the innovation effect definition in equation (1.9), $R_N = 1 + \eta_{P_Y}(E) = 1 + \eta_Y(E^c)\eta_{P_Y}(Y)$. It is easy to see that in short run the unconditional inputs demand is equal to $E^r\left(\frac{P_E}{\varepsilon}, \vec{q}, P_Y\right)$ and so $R_{\mathrm{N}}^{r} = 1 + \dot{\eta}_\varepsilon(E^r) = 1 - \eta_{P_E}(E^r)$. Using the identity $E\left(P_E, \overrightarrow{P_q}, \varepsilon P_Y\right) = E^r\left(\frac{P_E}{\varepsilon}, P_Y, \overrightarrow{q\left(P_E, \overrightarrow{P_q}, \varepsilon P_Y\right)}\right)$, it's easy to see that: $R_{\mathrm{N}}^{r} = R_N + \sum_{h=1}^{m} \frac{\eta_{P_{q_h}}(E)}{-\eta_{P_{q_h}}(q_h)} \eta_{P_Y}(q_h)$. Since we are assuming that the production function is homothetic, then $\eta_{P_Y}(q_h) = \eta_{P_Y}(E) = R_N - 1$. Thereby, $R_{\mathrm{N}}^{r} = R_N + \sum_{h=1}^{m} \frac{\eta_{P_{q_h}}(E)}{-\eta_{P_{q_h}}(q_h)} [R_N - 1]$.

normal input ($\eta_Y(E^C) > 0$) and so the homotheticity hypothesis is a sufficient condition to generate backfire. Furthermore, when the non-energy inputs are gross substitutes for the energy input ($\eta_{P_{q_h}}(E) \geq 0$), the homotheticity hypothesis is sufficient for the short-run effect to be greater than the long-run effect and therefore the short-run effect also leads to backfire (see footnote 21).

These results explain why, in all of Saunders' works, the long-run innovation effect leads to backfire (SAUNDERS, 1992, 2005, 2013, 2015). This is because these works use homogeneous production functions (Cobb-Douglas, CES, and others functions with homogeneous of degree 1). The results also could explain why Saunders (2013) finds that the short-run innovation effect leads to more powerful backfires than the long-run effects.

1.6.2 Backfire and welfare

It is important to identify in which situations the innovation rebound effect results in a loss of welfare. In other words, it is important to identify in which situations the backfire is definitely a problem. For this, we will assume that the global welfare (W) is proportional to output Y and that there are external costs arising from environmental problems generated by energy consumption. Furthermore, we will assume that the welfare function is linearly separable: $W = Y - g(E)$, where $g(.)$ is the external costs function and $g' = \frac{\partial g(E)}{\partial E} > 0$.[22] Given the hypotheses about the simplest model, it is easy to see that $Y(P_E, \overrightarrow{P_q}, \varepsilon P_Y)$.[23] Thereby, we have:

$$\frac{\partial W}{\partial \varepsilon} = \eta_{P_Y}(Y)\frac{Y}{\varepsilon} - g'\frac{E}{\varepsilon}[R_N - 1] \qquad \textbf{(1.23)}$$

On the one hand, equation (1.23) shows that the innovation rebound effect increases the output (since $\eta_{P_Y}(Y) \geq 0$), resulting in welfare gains. On the other hand, it shows that the innovation rebound effect can generate welfare loss if it results in backfire ($R_N > 1$).

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[22] This hypothesis ($g' = \frac{\partial g(E)}{\partial E} > 0$) means that consuming an additional unit of energy results in a higher total external cost. That is, *ceteris paribus*, a greater energy consumption results in a greater environmental damage and consequently in loss of welfare. Note that g' can be large if the energy consumed is highly polluting (e.g., fossil fuels), or it can be small if the energy consumed is clean (e.g., renewable energy).

[23] In fact, we have that $Y\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)$. However, as in the case of the innovation rebound effect, we are assuming $\varepsilon = \tau_h \ \forall h$, hence $Y\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, P_Y\right)$. Thus, since the supply function is homogeneous of degree zero in prices, we have: $Y\left(\frac{P_E}{\varepsilon}, \frac{\overrightarrow{P_q}}{\varepsilon}, \frac{\varepsilon}{\varepsilon}P_Y\right) = \left(\frac{1}{\varepsilon}\right)^0 Y(P_E, \overrightarrow{P_q}, \varepsilon P_Y) = Y(P_E, \overrightarrow{P_q}, \varepsilon P_Y)$.

Notwithstanding, if the innovation rebound effect does not lead to backfire ($R_N \leq 1$), energy efficiency gains necessarily result in welfare gains ($\frac{\partial W}{\partial \varepsilon} > 0$). Thus, for the innovation rebound effect to be a concern in terms of welfare, it must generate backfire.

Thereby, assume that the production function is homogeneous of degree $0 < \gamma < 1$, so the innovation rebound effect results in backfire: $R_N = \frac{1}{1-\gamma} > 1$. In this case, we have that $R_N - 1 = \eta_{P_Y}(Y) = \frac{\gamma}{(1-\gamma)}$ (see footnote 20). Therefore, equation (1.23) yields $\frac{\partial W}{\partial \varepsilon} = \frac{\gamma}{(1-\gamma)} \frac{1}{\varepsilon}(Y - g'E)$. Thus, in order to the backfire to result in a loss of welfare ($\frac{\partial W}{\partial \varepsilon} < 0$), it is necessary that $g' > \frac{1}{EI}$, where $EI = \frac{E}{Y}$ is the energy intensity. Therefore, backfire is more likely to result in loss of welfare in situations where the innovation rebound effect drives the consumption of highly polluting fossil energies (i.e., when g' is large) and where output is highly energy-intensive (i.e., when $\frac{1}{EI}$ is small). On the other hand, if innovation rebound effect drives the consumption of low carbon energies (i.e., when g' is small) and the energy intensity is low (i.e., when $\frac{1}{EI}$ is large), then the backfire can be beneficial in terms of welfare. Thus, whether the backfire will be a problem, or not, will largely depend on the type of energy (e.g., fossil fuels or renewable energies) that will be driven by the innovation rebound effect.

1.7 Cautions and limitations

Several cautions and limitations were already exposed in Wei (2010), such as the use of perfect competition hypotheses and the static comparison between market equilibriums. However, since our model does not incorporate all the economic mechanisms that govern the rebound effect, some caveats are still needed.

First, we assume a global economy or a closed economy. However, Koesler, Swales, and Turner (2016) find that the rebound effect on a global scale is less than the rebound effect of the economy in which energy efficiency gains occur. The authors argue that this is because improving energy efficiency modifies comparative advantages, that is, it makes domestic production of energy-intensive commodities more productive compared to the rest of the world (KOESLER; SWALES; TURNER, 2016). Thus, encouraging the production of these goods within the domestic economy and discouraging it in the rest of the world.

Second, our model assumes that energy efficiency improvements are exogenous and occur at zero-cost. Nevertheless, as highlighted in several studies, if energy efficiency improvements are introduced at a cost, the rebound size is significantly lower than if the energy efficiency gain comes free of charge (ALLAN et al., 2007; BROBERG; BERG; SAMAKOVLIS, 2015; FULLERTON; TA, 2020; LEMOINE, 2020; PENG et al., 2019). In general, the energy efficiency cost is modeled as a reduction in the productivity of some non-energy input (e.g., capital or labor).[24] If we included this hypothesis in our model, we would have $\dot{\eta}_{\varepsilon_i}(\tau_h) < 0$ for some h, and not $\dot{\eta}_{\varepsilon_i}(\tau_h) = 0\ \forall h$ as assuming in our model. As Lemoine (2020) points out, the impact that the energy efficiency cost has on the rebound size depends on the magnitude of $\dot{\eta}_{\varepsilon_i}(\tau_h)$. In other words, although in general the energy efficiency cost reduces the rebound effect, in some situations it can amplify the rebound size (LEMOINE, 2020).

Third, our model assumes that energy efficiency does not affect (directly or indirectly) the functions ${q_h}^S$, E^S and Y^D. Nevertheless, several studies incorporate the indirect effect that energy efficiency gains have on the energy supply function: some introduce this effect by endogenizing capital (ALLAN et al., 2007; LU; LIU; ZHOU, 2017; TURNER, 2009), while others by endogenizing labor (LEMOINE, 2020; WEI; LIU, 2017).

For example, Turner (2009) assumes that the level of capital (K) in energy production depends on the energy price, that is, capital is endogenous. If this hypothesis were introduced in our model, we would have $E^S\big(P_E, K(P_E)\big)$.[25] However, energy price is endogenous and hence it is affected by energy efficiency. Thereby, in the model developed by Turner (2009) energy efficiency indirectly affects the energy supply function, through the energy price and the subsequent adjustment in the level of capital. The rationale of what Turner (2009) called the divestment effect is as follows. After the initial energy efficiency improvement, the energy demand is reduced in the short run, since the economy cannot take full advantage of improved productivity. In turn, reduced demand puts downward pressure on energy prices, reducing profit and discouraging investments to expand energy production. Thus, in the long run, production capacity decreases, and the energy price increases, which mitigates the long-run rebound.

Turner (2009) argued that the disinvestment effect can lead to a lower long-run rebound effect than the short-run effect, a result that contradicts Saunders (2008) and Wei (2007). Nevertheless, as shown in Section 1.5.4, if the energy price is endogenous, the long-run rebound effect may be less than the short-run effect. Thereby, in the model developed by Turner (2009)

[24] It should be noted that Fullerton and Ta (2020) models the energy efficiency cost in a different way. In a scenario focused on household energy use, Fullerton and Ta (2020) specify a production function for appliance efficiency, thus considering the extra cost of requiring more efficient appliances.

[25] This hypothesis allows our energy supply function expressed in Figure 1.1 (E^S) to be displaced to the left, thus mitigating the rebound effect. As highlighted in Lemoine (2020), this effect could also shift the energy supply function to the right in Figure 1.1, amplifying the rebound effect.

the divestment effect (i.e., endogenous capital) is not necessary for the long-run effect to be less than the short-run effect, although this effect may contribute to this. As highlighted in Lemoine (2020), depending on the parameter values, the divestment effect can amplify the rebound size in the long run. Notwithstanding, the importance of the divestment effect for the contradiction of our results is because it allows the rebound effect to be negative even in a model that uses only one energy service (TURNER, 2009). Lemoine (2020) shows in which situations the divestment effect can cause super-conservation.

1.8 Conclusion

In expanding Wei's (2010) general equilibrium model, this article has attempted to contribute to broadening the theoretical foundation of macroeconomic rebound effects. The article raised several new findings and corrects some results presented in Wei (2010). These findings can serve to assist the intuitive understanding of results generated from empirical studies.

The article demonstrated several theoretical mechanisms that govern the rebound effect (i.e., direct effect, cross-price effect, input price effect, output price effect, and energy price effect), highlighting when they can amplify or mitigate the rebound size. Regarding the reallocation effect, we point out the importance of the direct effect. That is, the greater the price elasticity of demand for energy service i (direct effect), the greater the chances of backfire. Nevertheless, due to the indirect effects, however great this elasticity is, the rebound effect can still fit into any of the five rebound conditions (backfire, full rebound, partial rebound, zero rebound, or super-conservation). We also show the importance of the energy supply for the rebound magnitude. That is, the more price-inelastic is the energy supply, the rebound effect will be closer to the full rebound condition and, in the extreme case when energy supply is fixed, this condition will be checked. Furthermore, we find that the number of energy services is relevant to the rebound size. This is because, when the model includes only a single energy service, the super-conservation is not allowed.

The article also showed several findings under what circumstances the long-run effect is greater or less than the short-run effect. Regarding the reallocation effect, we show that, in the simplest model, the long-run effect will always be greater than or equal to the short-run effect. This is explained by the Le Chatelier principle. When the model includes more than one energy service and/or endogenizes the output price or energy price, the long-run effect may be greater or less than the short-run effect.

Our simplified analysis of the innovation rebound effect showed that, whenever the production function is homogeneous, this effect will generate backfire. In addition, we found that when the production function is homogeneous and the non-energy inputs are gross substitutes for the energy service, the short-run innovation effect will generate a more powerful backfire than the long-run effect. On the other hand, if the non-energy inputs are gross complements for the energy service, the short-run effect will be less than the long-run effect and may not generate backfire. Finally, we point out that backfire is definitely a problem in terms of welfare in situations where energy consumption is based on highly polluting energies (e.g., fossil fuels) and where output is highly energy-intensive.

Appendix 1.A Reallocation effect definition

Equations (1.5) and (1.6) are energy demand functions equal to $E = \sum_{j=1}^{n} E_j$, where $E_j = \frac{S_j}{\varepsilon_j}$. Denoting $\dot{\eta}_j(i) = \frac{di}{dj}\frac{j}{i}$ and $\eta_j(i) = \frac{\partial i}{\partial j}\frac{j}{i}$ as the elasticity of the function "i" with respect to the variable "j" and obtaining the elasticity of the energy demand function with respect to energy efficiency i ($\dot{\eta}_{\varepsilon_i}(E)$), we have:

$$\dot{\eta}_{\varepsilon_i}(E) = \frac{dE}{d\varepsilon_i}\frac{\varepsilon_i}{E} = \sum_{j=1}^{n} \frac{dE_j}{d\varepsilon_i}\frac{\varepsilon_i}{E} \tag{1.24}$$

Multiplying and dividing the right side of equation (1.24) by E_j and denoting $\sigma_j = \frac{E_j}{E}$, we get:

$$\dot{\eta}_{\varepsilon_i}(E) = \sum_{j=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(E_j) \tag{1.25}$$

Decomposing the elasticity $\dot{\eta}_{\varepsilon_i}(E_j)$ for any j and using the identity $E_j = \frac{S_j}{\varepsilon_j}$, we have:

$$\dot{\eta}_{\varepsilon_i}(E_j) = \frac{dS_j}{d\varepsilon_i}\frac{\varepsilon_i}{E_j}\frac{\partial E_j}{\partial S_j} + \frac{d\varepsilon_j}{d\varepsilon_i}\frac{\varepsilon_i}{E_j}\frac{\partial E_j}{\partial \varepsilon_j} = \frac{dS_j}{d\varepsilon_i}\frac{\varepsilon_i}{S_j} - \frac{d\varepsilon_j}{d\varepsilon_i}\frac{\varepsilon_i}{\varepsilon_j} = \dot{\eta}_{\varepsilon_i}(S_j) - \dot{\eta}_{\varepsilon_i}(\varepsilon_j) \tag{1.26}$$

Substituting equation (1.26) in equation (1.25) yields:

$$\dot{\eta}_{\varepsilon_i}(E) = \sum_{j=1}^{n} \sigma_j \left[\dot{\eta}_{\varepsilon_i}(S_j) - \dot{\eta}_{\varepsilon_i}(\varepsilon_j) \right] \tag{1.27}$$

Regarding energy-augmenting technical change, $\dot{\eta}_{\varepsilon_i}(\varepsilon_j) = 0\ \forall j \neq i$. Thus, in the long run, equation (1.27) yields:

$$\dot{\eta}_{\varepsilon_i}(E) = -\sigma_i \dot{\eta}_{\varepsilon_i}(\varepsilon_i) + \sum_{j=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(S_j) \tag{1.28}$$

In the short run, since $\dot{\eta}_{\varepsilon_i}(E_j) = 0\ \forall j \neq i$, then equation (1.25) is equal to $\dot{\eta}_{\varepsilon_i}(E) = \sigma_i \dot{\eta}_{\varepsilon_i}(E_i)$. Thus, in the short run, equation (1.27) yields:

$$\dot{\eta}_{\varepsilon_i}(E^r) = -\sigma_i^r \dot{\eta}_{\varepsilon_i}(\varepsilon_i) + \sigma_i^r \dot{\eta}_{\varepsilon_i}({S_i}^r) \tag{1.29}$$

where the subscript r identifies the respective variables in the short run. Since $\dot{\eta}_{\varepsilon_i}(\varepsilon_i) = 1$, equations (1.28) and (1.29) shows that the long-run (R_{ε_i}) and short-run ($R_{\varepsilon_i}^r$) reallocation effects are, respectively, equal to:

$$R_{\varepsilon_i} = \sigma_i + \dot{\eta}_{\varepsilon_i}(E) = \sum_{j=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(S_j) \tag{1.30}$$

$$R_{\varepsilon_i}^r = \sigma_i^r + \dot{\eta}_{\varepsilon_i}(E^r) = \sigma_i^r \dot{\eta}_{\varepsilon_i}({S_i}^r) \tag{1.31}$$

Decomposing the elasticity $\dot{\eta}_{\varepsilon_i}(S_j)$ for any $S_j\left(\frac{\overrightarrow{P_E}}{\varepsilon}, \frac{\overrightarrow{P_q}}{\tau}, P_Y\right)$ in the long run, we have:

$$\begin{aligned} \dot{\eta}_{\varepsilon_i}(S_j) = {} & \frac{\mathrm{d}P_E}{\mathrm{d}\varepsilon_i}\frac{\varepsilon_i}{S_j}\left(\sum_{\mathrm{x}=1}^{n}\frac{\partial S_j}{\partial P_{S_x}}\frac{1}{\varepsilon_x}\right) + \sum_{h=1}^{m}\frac{\mathrm{d}P_{q_h}}{\mathrm{d}\varepsilon_i}\frac{\varepsilon_i}{S_j}\frac{\partial S_j}{\partial P_{Q_h}}\frac{1}{\tau_h} + \frac{\mathrm{d}P_Y}{\mathrm{d}\varepsilon_i}\frac{\varepsilon_i}{S_j}\frac{\partial S_j}{\partial P_Y} \\ & - \sum_{h=1}^{m}\frac{\mathrm{d}\tau_h}{\mathrm{d}\varepsilon_i}\frac{\varepsilon_i}{S_j}\frac{\partial S_j}{\partial P_{Q_h}}\frac{P_{q_h}}{{\tau_h}^2} - \sum_{x=1}^{n}\frac{\mathrm{d}\varepsilon_x}{\mathrm{d}\varepsilon_i}\frac{\varepsilon_i}{S_j}\frac{\partial S_j}{\partial P_{S_x}}\frac{P_E}{{\varepsilon_x}^2} \end{aligned} \tag{1.32}$$

Multiplying and dividing the first three elements of the equation (1.32) by their respective price (P_E, P_{q_h} and P_Y), reordering the terms and writing them as elasticities, we have:

$$\dot{\eta}_{\varepsilon_i}(S_j) = \dot{\eta}_{\varepsilon_i}(P_E)\sum_{x=1}^{n}\eta_{P_{S_x}}(S_j) + \sum_{h=1}^{m}\dot{\eta}_{\varepsilon_i}(P_{q_h})\eta_{P_{Q_h}}(S_j) + \dot{\eta}_{\varepsilon_i}(P_Y)\eta_{P_Y}(S_j)$$
$$-\sum_{h=1}^{m}\dot{\eta}_{\varepsilon_i}(\tau_h)\eta_{P_{Q_h}}(S_j) - \sum_{x=1}^{n}\dot{\eta}_{\varepsilon_i}(\varepsilon_x)\eta_{P_{S_x}}(S_j) \quad \textbf{(1.33)}$$

In the case of the reallocation effect ($\dot{\eta}_{\varepsilon_i}(\varepsilon_j) = 0 = \dot{\eta}_{\varepsilon_i}(\tau_h)\ \forall h, j \neq i$), substituting equation (1.33) into equation (1.30) and rearranging the terms (and note that $\dot{\eta}_{\varepsilon_i}(\varepsilon_i) = 1$), we have:

$$R_{\varepsilon_i} = -\sigma_i\eta_{P_{S_i}}(S_i) - \sum_{j\neq i}\sigma_j\eta_{P_{S_i}}(S_j) + \sum_{j=1}^{n}\sum_{x=1}^{n}\sigma_j\eta_{P_{S_x}}(S_j)\dot{\eta}_{\varepsilon_i}(P_E)$$
$$+\sum_{h=1}^{m}\sum_{j=1}^{n}\sigma_j\eta_{P_{Q_h}}(S_j)\dot{\eta}_{\varepsilon_i}(P_{q_h}) + \sum_{j=1}^{n}\sigma_j\eta_{P_Y}(S_j)\dot{\eta}_{\varepsilon_i}(P_Y) \quad \textbf{(1.34)}$$

The same procedures can be applied to decompose $\eta_{\varepsilon_i}(S_i{}^r)$, where $S_i{}^r\left(\frac{P_E}{\varepsilon_i}, P_Y, \overrightarrow{\tau q}, \overrightarrow{\varepsilon E_{-\iota}}\right)$. It is easy to see that:

$$\dot{\eta}_{\varepsilon_i}(S_i{}^r) = -\eta_{P_{S_i}}(S_i{}^r) + \sum_{j\neq i}\eta_{S_j}(S_i{}^r)\dot{\eta}_{\varepsilon_i}(\varepsilon_j) + \sum_{h=1}^{m}\eta_{Q_h}(S_i{}^r)\dot{\eta}_{\varepsilon_i}(\tau_h)$$
$$+\eta_{P_{S_i}}(S_i{}^r)\dot{\eta}_{\varepsilon_i}(P_E) + \eta_{P_Y}(S_i{}^r)\dot{\eta}_{\varepsilon_i}(P_Y) \quad \textbf{(1.35)}$$

Regarding reallocation effect ($\dot{\eta}_{\varepsilon_i}(\varepsilon_j) = 0 = \dot{\eta}_{\varepsilon_i}(\tau_h)\ \forall h, j \neq i$), substituting equation (1.35) into equation (1.31) yields:

$$R^r_{\varepsilon_i} = -\sigma_i^r\eta_{P_{S_i}}(S_i{}^r) + \sigma_i^r\eta_{P_{S_i}}(S_i{}^r)\dot{\eta}_{\varepsilon_i}(P_E) + \sigma_i^r\eta_{P_Y}(S_i{}^r)\dot{\eta}_{\varepsilon_i}(P_Y) \quad \textbf{(1.36)}$$

Appendix 1.B Innovation effect definition

In order to find the innovation effect (R_N), we assume a neutral technical change, i.e., $\varepsilon_i = \varepsilon_j = \tau_h\ \forall j, h$, then $\dot{\eta}_{\varepsilon_i}(\varepsilon_i) = \dot{\eta}_{\varepsilon_i}(\varepsilon_j) = \dot{\eta}_{\varepsilon_i}(\tau_h) = 1\ \forall\ j, h$. Thus, it is possible to see by equation (1.27) that long-run (R_{N}) and short-run (R^r_N) innovation effect (since $\dot{\eta}_{\varepsilon_i}(E_j) = 0\ \forall j \neq i$) are, respectively, equal to:

$$R_{\mathrm{N}} = 1 + \dot{\eta}_{\varepsilon_i}(E) = \sum_{j=1}^{n} \sigma_j \dot{\eta}_{\varepsilon_i}(S_j) \tag{1.37}$$

$$R_N^r = 1 + \dot{\eta}_{\varepsilon_i}(E^r) = \dot{\eta}_{\varepsilon_i}({S_i}^r) \tag{1.38}$$

Also, we assume that all prices are exogenous ($\dot{\eta}_{\varepsilon_i}(P_E) = \dot{\eta}_{\varepsilon_i}(P_{q_h}) = \dot{\eta}_{\varepsilon_i}(P_Y) = 0$) and that there is only one energy service. Thus, substituting equation (1.33) into equation (1.37) and substituting equation (1.35) into equation (1.38) we obtain, respectively, the long-run and the short-run innovation effects:

$$R_N = -\eta_{P_S}(S) - \sum_{h=1}^{m} \eta_{P_{Q_h}}(S) \tag{1.39}$$

$$R_{\mathrm{N}}^r = -\eta_{P_{S_i}}({S_i}^r) + \sum_{h=1}^{m} \eta_{Q_h}({S_i}^r) \tag{1.40}$$

Appendix 1.C Reallocation effect with endogenous prices

First, we will find the long-run effect. Multiplying both sides of (1.34) by E and using the equality $S_j = \varepsilon_j E_j$ (i.e., writing the equation (1.34) in terms of derivatives) yields:

$$\begin{aligned} R_{\varepsilon_i} E = &\sum_{j=1}^{n} \frac{\mathrm{d}P_Y}{\mathrm{d}\varepsilon_i} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_Y} + \sum_{j=1}^{n} \sum_{\mathrm{h}=1}^{m} \frac{\mathrm{d}P_{q_h}}{\mathrm{d}\varepsilon_i} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{Q_h}} \frac{1}{\tau_h} + \sum_{j=1}^{n} \sum_{x=1}^{n} \frac{\mathrm{d}P_E}{\mathrm{d}\varepsilon_i} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{S_x}} \frac{1}{\varepsilon_x} \\ &- \frac{P_E}{{\varepsilon_i}^2} \sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{S_i}} \end{aligned} \tag{1.41}$$

Writing equation (1.41) in matrix terms, we have:

$$R_{\varepsilon_i} E = (\Upsilon \vec{g})^t \vec{p} - \frac{P_E}{{\varepsilon_i}^2} \sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{S_i}} \tag{1.42}$$

where Υ is a diagonal matrix of order $(2+m)$, where the main diagonal is equal to $\begin{bmatrix} 1 & \frac{1}{\tau_1} & \cdots & \frac{1}{\tau_m} & \frac{1}{\varepsilon_i} \end{bmatrix}$, $\vec{g}$ and $\vec{p}$ are column vectors of order $(2+m)$, where $\vec{g}^t =$

$\left[\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_Y} \quad \sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{Q_1}} \quad \cdots \quad \sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{Q_m}} \quad \sum_{j=1}^{n}\sum_{\mathrm{x}=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_x}}\frac{\varepsilon_i}{\varepsilon_x}\right]$ and $\vec{p}^t = \left[\frac{\mathrm{d}P_Y}{\mathrm{d}\varepsilon_i} \quad \frac{\mathrm{d}P_{q_1}}{\mathrm{d}\varepsilon_i} \quad \cdots \quad \frac{\mathrm{d}P_{q_m}}{\mathrm{d}\varepsilon_i} \quad \frac{\mathrm{d}P_E}{\mathrm{d}\varepsilon_i}\right]$.

Now, it is necessary to find the endogenous value of the vector $\vec{p}$. With the perfect competition hypothesis, we have: $Y\left(\frac{\overrightarrow{P_E}}{\varepsilon},\frac{\overrightarrow{P_q}}{\tau},P_Y\right) = Y^D(P_Y)$, $E^S(P_E) = \sum_{j=1}^{n}\frac{S_j\left(\frac{\overrightarrow{P_E}}{\varepsilon},\frac{\overrightarrow{P_q}}{\tau},P_Y\right)}{\varepsilon_j}$ and ${q_h}^S\left(P_{q_h}\right) = \frac{Q_h\left(\frac{\overrightarrow{P_E}}{\varepsilon},\frac{\overrightarrow{P_q}}{\tau},P_Y\right)}{\tau_h}$ $\forall h = 1,2,\dots,m$. Totally differentiating the equilibrium relations, assuming constant τ_h, ε_x for all h and $x \neq i$, and multiplying both sides by $\frac{1}{\mathrm{d}\varepsilon_i}$, we obtain:

$$\left(\frac{\partial Y}{\partial P_Y}-\frac{\partial Y^D}{\partial P_Y}\right)\frac{\mathrm{d}P_Y}{\mathrm{d}\varepsilon_i}+\sum_{h=1}^{m}\frac{\partial Y}{\partial P_{Q_h}}\frac{1}{\tau_h}\frac{\mathrm{d}P_{q_h}}{\mathrm{d}\varepsilon_i}+\sum_{j=1}^{n}\frac{\partial Y}{\partial P_{S_j}}\frac{1}{\varepsilon_j}\frac{\mathrm{d}P_E}{\mathrm{d}\varepsilon_i}=\frac{P_E}{{\varepsilon_i}^2}\frac{\partial Y}{\partial P_{S_i}} \tag{1.43}$$

$$\begin{aligned}-\frac{\partial Q_h}{\partial P_Y}\frac{\mathrm{d}P_Y}{\mathrm{d}\varepsilon_i}+\left(\tau_h\frac{\partial {q_h}^S}{\partial P_{q_h}}-\frac{\partial Q_h}{\partial P_{Q_h}}\frac{1}{\tau_h}\right)\frac{\mathrm{d}P_{q_h}}{\mathrm{d}\varepsilon_i}-\sum_{x\neq h}\frac{\partial Q_h}{\partial P_{Q_x}}\frac{1}{\tau_x}\frac{\mathrm{d}P_{q_x}}{\mathrm{d}\varepsilon_i}-\sum_{j=1}^{n}\frac{\partial Q_h}{\partial P_{S_j}}\frac{1}{\varepsilon_j}\frac{\mathrm{d}P_E}{\mathrm{d}\varepsilon_i} \\ = -\frac{P_E}{{\varepsilon_i}^2}\frac{\partial Q_h}{\partial P_{S_i}}\end{aligned} \tag{1.44}$$

$$\begin{aligned}-\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_Y}\frac{\mathrm{d}P_Y}{\mathrm{d}\varepsilon_i}-\sum_{h=1}^{m}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{Q_h}}\frac{1}{\tau_h}\frac{\mathrm{d}P_{q_h}}{\mathrm{d}\varepsilon_i}+\left(\varepsilon_i\frac{\partial E^S}{\partial P_E}-\sum_{j=1}^{n}\sum_{\mathrm{x}=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_x}}\frac{1}{\varepsilon_x}\right)\frac{\mathrm{d}P_E}{\mathrm{d}\varepsilon_i} \\ = \left(-\frac{P_E}{{\varepsilon_i}^2}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_i}}-E_i\right)\end{aligned} \tag{1.45}$$

Writing the system formed by the equations (1.43), (1.44), and (1.45) in matrix form, yields:

$$(\mathrm{T}^t\Gamma\mathrm{T}+\Phi\Upsilon^{-1})\Upsilon\vec{p} = \frac{P_E}{{\varepsilon_i}^2}\vec{b}-\vec{k}E_i \tag{1.46}$$

where $\Gamma = \begin{bmatrix} \frac{\partial Y}{\partial P_Y} & \frac{\partial Y}{\partial P_{Q_1}} & \cdots & \frac{\partial Y}{\partial P_{Q_m}} & \frac{\partial Y}{\partial P_{S_1}} & \cdots & \frac{\partial Y}{\partial P_{S_n}} \\ -\frac{\partial Q_1}{\partial P_Y} & -\frac{\partial Q_1}{\partial P_{Q_1}} & & -\frac{\partial Q_1}{\partial P_{Q_m}} & -\frac{\partial Q_1}{\partial P_{S_1}} & \cdots & -\frac{\partial Q_1}{\partial P_{S_n}} \\ \vdots & & \ddots & & \vdots & & \\ -\frac{\partial Q_m}{\partial P_Y} & -\frac{\partial Q_m}{\partial P_{Q_1}} & & -\frac{\partial Q_m}{\partial P_{Q_m}} & -\frac{\partial Q_m}{\partial P_{S_1}} & \cdots & -\frac{\partial Q_m}{\partial P_{S_n}} \\ -\frac{\partial S_1}{\partial P_Y} & -\frac{\partial S_1}{\partial P_{Q_1}} & \cdots & -\frac{\partial S_1}{\partial P_{Q_m}} & -\frac{\partial S_1}{\partial P_{S_1}} & \cdots & -\frac{\partial S_1}{\partial P_{S_n}} \\ \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial S_n}{\partial P_Y} & -\frac{\partial S_n}{\partial P_{Q_1}} & & -\frac{\partial S_n}{\partial P_{Q_m}} & -\frac{\partial S_n}{\partial P_{S_1}} & \cdots & -\frac{\partial S_n}{\partial P_{S_m}} \end{bmatrix}$ is the HMS of order $(1+m+n)$ from the production function $Y = f(\vec{Q}, \vec{S})$ ($\vec{Q}$ represents the vector $[Q_1, \dots, Q_m]$ and $\vec{S}$ represents the vector $[S_1, \dots, S_n]$), T is a rectangular block matrix of order $(1+m+n) \times (2+m)$ equal to $\begin{bmatrix} \mathrm{I} & \vec{0} \\ 0 & \vec{\varepsilon} \end{bmatrix}$, I is an identity matrix of order $(1+m)$, 0 is a zero matrix of order $n \times (1+m)$, $\vec{0}$ is a zero column vector of order $(1+m)$, $\vec{\varepsilon}$ is a column vector of order n equal to $\vec{\varepsilon}^t = \begin{bmatrix} \frac{\varepsilon_i}{\varepsilon_1} & \frac{\varepsilon_i}{\varepsilon_2} & \cdots & \frac{\varepsilon_i}{\varepsilon_{(n-1)}} & \frac{\varepsilon_i}{\varepsilon_n} \end{bmatrix}$, Φ is a diagonal matrix of order $(2+m)$, where the main diagonal is equal to $\begin{bmatrix} -\frac{\partial Y^D}{\partial P_Y} & \tau_1 \frac{\partial q_1{}^S}{\partial P_{q_1}} & \cdots & \tau_m \frac{\partial q_m{}^S}{\partial P_{q_m}} & \varepsilon_i \frac{\partial E^S}{\partial P_E} \end{bmatrix}$, and $\vec{b}$ and $\vec{k}$ are column vectors of order $(2+m)$, where $\vec{b}^t = \begin{bmatrix} \frac{\partial Y}{\partial P_{S_i}} & -\frac{\partial Q_1}{\partial P_{S_i}} & \cdots & -\frac{\partial Q_m}{\partial P_{S_i}} & -\sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{S_i}} \end{bmatrix}$ and $\vec{k}^t = [0 \quad 0 \quad \cdots \quad 0 \quad 1]$.

Denoting $\mathrm{A} = (\mathrm{T}^t \Gamma \mathrm{T} + \Phi \Upsilon^{-1})$ and assuming that the inverse matrix of A exists, then equation (1.46) yields:

$$\vec{p} = \Upsilon^{-1} \mathrm{A}^{-1} \left[\frac{P_E}{\varepsilon_i{}^2} \vec{b} - \vec{k} E_i \right] \tag{1.47}$$

Note that Υ is invertible (since $\tau_h, \varepsilon_i > 0 \; \forall h, i$) and symmetric. Substituting equation (1.47) in equation (1.42), we have:

$$R_{\varepsilon_i} E = (\vec{g})^t \mathrm{A}^{-1} \left[\frac{P_E}{\varepsilon_i{}^2} \vec{b} - \vec{k} E_i \right] - \frac{P_E}{\varepsilon_i{}^2} \sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{S_i}} \tag{1.48}$$

Using the classic adjunct matrix (transpose of its cofactor matrix), the inverse matrix of A can be determined: $\mathrm{A}^{-1} = \frac{adj(\mathrm{A})}{\det(\mathrm{A})}$, where $adj(\mathrm{A})$ is the classic adjunct matrix of A and

$\det(\mathrm{A})$ is the determinant of the matrix A. Using the transposed matrix property and substituting the inverse matrix A^{-1} in equation (1.48), we obtain:

$$R_{\varepsilon_i}E = \frac{(adj(\mathrm{A}^t)\vec{g})^t}{\det(\mathrm{A})}\left[\frac{P_E}{\varepsilon_i{}^2}\vec{b} - \vec{k}E_i\right] - \frac{P_E}{\varepsilon_i{}^2}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_i}} \quad \textbf{(1.49)}$$

Note that $adj(\mathrm{A}^t)(\vec{g}) = \begin{bmatrix} \det\left({}^1_g\mathrm{A}\right) \\ \det\left({}^2_g\mathrm{A}\right) \\ \vdots \\ \det\left({}^{(1+m)}_{g}\mathrm{A}\right) \\ \det\left({}^{(2+m)}_{g}\mathrm{A}\right) \end{bmatrix}$, where ${}^l_g\mathrm{A}$ is the matrix obtained from A replacing l-th row of A by the vector $\vec{g}$. In order to find the determinants of ${}^l_g\mathrm{A}$, we will use the Laplace expansion in $(2+m)$-th row of ${}^l_g\mathrm{A}$. For any ${}^l_g\mathrm{A}$ with $l \neq (2+m)$, the determinant is equal to:

$$\begin{aligned}\det\left({}^l_g\mathrm{A}\right) = -\Bigg\{&(-1)^{(2+m)+1}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_Y}\det\left({}^l_g\mathrm{A}_{(2+m)\times 1}\right) \\ &+ (-1)^{(2+m)+2}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{Q_1}}\det\left({}^l_g\mathrm{A}_{(2+m)\times 2}\right) + \cdots \\ &+ (-1)^{2(2+m)-1}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{Q_m}}\det\left({}^l_g\mathrm{A}_{(2+m)\times(1+m)}\right) \\ &+ (-1)^{2(2+m)}\sum_{j=1}^{n}\sum_{x=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_x}}\frac{\varepsilon_i}{\varepsilon_x}\det\left({}^l_g\mathrm{A}_{(2+m)\times(2+m)}\right)\Bigg\} \\ &+ (-1)^{2(2+m)}\varepsilon_i{}^2\frac{\partial E^S}{\partial P_E}\det\left({}^l_g\mathrm{A}_{(2+m)\times(2+m)}\right)\end{aligned} \quad \textbf{(1.50)}$$

where ${}^l_g\mathrm{A}_{(2+m)\times k}$ is the submatrix of ${}^l_g\mathrm{A}$ of order $(1+m)$ obtained by suppressing its $(2+m)$-th row and its k-th column. Note that $(-1)^{2(2+m)} = 1$ and that the sum of the elements inside the braces in equation (1.50) is equal to the determinant of a matrix with two equal rows (l-th row is equal to $(2+m)$-th row), then elements inside the braces are equal to zero. Therefore, the equation (1.50) comes down to:

$$\det\left({}_{g}^{l}\mathrm{A}\right) = \varepsilon_i{}^2 \frac{\partial E^S}{\partial P_E} \det\left({}_{g}^{l}\mathrm{A}_{(2+m)\times(2+m)}\right) \quad \textbf{(1.51)}$$

We can do the same procedures for $l = (2+m)$, but now we will add and subtract the right side of the equation by $(-1)^{2(2+m)} \varepsilon_i{}^2 \frac{\partial E^S}{\partial P_E} \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times(2+m)}\right)$. Thus:

$$\begin{aligned}
\det\left({}_{g}^{(2+m)}\mathrm{A}\right) = \Bigg\{ & (-1)^{(2+m)+1} \sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_Y} \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times 1}\right) \\
& + (-1)^{(2+m)+2} \sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{Q_1}} \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times 2}\right) + \cdots \\
& + (-1)^{2(2+m)-1} \sum_{j=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{Q_m}} \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times(1+m)}\right) \\
& + (-1)^{2(2+m)} \left(\sum_{j=1}^{n} \sum_{x=1}^{n} \frac{\varepsilon_i}{\varepsilon_j} \frac{\partial S_j}{\partial P_{S_x}} \frac{\varepsilon_i}{\varepsilon_x} - \varepsilon_i \frac{\partial E^S}{\partial P_E} \right) \\
& \times \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times(2+m)}\right) \Bigg\} \\
& + (-1)^{2(2+m)} \varepsilon_i{}^2 \frac{\partial E^S}{\partial P_E} \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times(2+m)}\right)
\end{aligned} \quad \textbf{(1.52)}$$

Note that the sum of the elements inside the braces in equation (1.52) is equal to minus the determinant of matrix A. Then, equation (1.52) comes down to:

$$\det\left({}_{g}^{(2+m)}\mathrm{A}\right) = \varepsilon_i{}^2 \frac{\partial E^S}{\partial P_E} \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times(2+m)}\right) - \det(\mathrm{A}) \quad \textbf{(1.53)}$$

Using equations (1.51) and (1.53) it is possible to obtain the result of $adj(\mathrm{A}^t)\vec{g}$:

$$adj(\mathrm{A}^t)\vec{g} = \varepsilon_i{}^2 \frac{\partial E^S}{\partial P_E} \vec{d} - \det(\mathrm{A})\,\vec{k} \quad \textbf{(1.54)}$$

where $\vec{d}$ is a column vector of order $(2+m)$: $\vec{d}^t = \left[\det\left({}_{g}^{1}\mathrm{A}_{(2+m)\times(2+m)}\right) \;\; \ldots \;\; \det\left({}_{g}^{(2+m)}\mathrm{A}_{(2+m)\times(2+m)}\right)\right]$.

Before proceeding, it is necessary to determine the values of $\det\left({}_{g}^{l}\mathrm{A}_{(2+m)\times(2+m)}\right)$. Since the vector $\vec{g}$ is equal to minus the $(2+m)$-th row of A, it is possible to see that $\det\left({}_{g}^{l}\mathrm{A}_{(2+m)\times(2+m)}\right)$ is equal to the $\det\left(\mathrm{A}_{l\times(2+m)}\right)$, but with a row being multiplied by minus one and with $[(1+m)-l]$ row swaps. So, using the properties of the determinant and knowing that $(-1)^{(2+m)-l} = (-1)^{(2+m)+l}$, then:

$$\det\left(A^{l}_{(2+m)\times(2+m)}\right) = (-1)^{(2+m)+l}\det\left(A_{l\times(2+m)}\right) \quad \textbf{(1.55)}$$

Therefore, by equation (1.55) we have: $\vec{d} = \begin{bmatrix} (-1)^{(2+m)+1}\det\left(\mathrm{A}_{1\times(2+m)}\right) \\ (-1)^{(2+m)+2}\det\left(\mathrm{A}_{2\times(2+m)}\right) \\ \vdots \\ (-1)^{2(2+m)}\det\left(\mathrm{A}_{(2+m)\times(2+m)}\right) \end{bmatrix}$.

Substituting equation (1.54) into equation (1.49) yields:

$$R_{\varepsilon_i}E = \frac{\left({\varepsilon_i}^2 \frac{\partial E^S}{\partial P_E}\vec{d} - \det(\mathrm{A})\,\vec{k}\right)^t}{\det(\mathrm{A})}\left[\frac{P_E}{{\varepsilon_i}^2}\vec{b} - \vec{k}E_i\right] - \frac{P_E}{{\varepsilon_i}^2}\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_i}} \quad \textbf{(1.56)}$$

Applying the distributive property and multiplying both sides of equation (1.56) by $\frac{1}{E}$ and using the equilibrium relation $E^S = E$, yields:

$$\begin{aligned} R_{\varepsilon_i} = \frac{1}{\det(\mathrm{A})}\Bigg[&\eta_{P_E}(E^S)\left(\vec{d}\right)^t\vec{b} - \sigma_i\eta_{P_E}(E^S)\left(\frac{S_i}{\sigma_i P_{S_i}}\right)\left(\vec{d}\right)^t\vec{k} \\ &- \frac{P_E}{E{\varepsilon_i}^2}\det(\mathrm{A})\left(\vec{k}\right)^t\vec{b} + \sigma_i\det(\mathrm{A})\left(\vec{k}\right)^t\vec{k}\Bigg] - \sum_{j=1}^{n}\sigma_j\eta_{P_{S_i}}(S_j) \end{aligned} \quad \textbf{(1.57)}$$

Note that $\left(\vec{d}\right)^t\vec{k} = (-1)^{2(2+m)}\det\left(\mathrm{A}_{(2+m)\times(2+m)}\right)$, $\left(\vec{k}\right)^t\vec{b} = -\sum_{j=1}^{n}\frac{\varepsilon_i}{\varepsilon_j}\frac{\partial S_j}{\partial P_{S_i}}$ and $\left(\vec{k}\right)^t\vec{k} = 1$. Using the Laplace expansion, it is also possible to identify that $\left(\vec{d}\right)^t\vec{b}$ is equal to the determinant of the matrix ${}_{(2+m)}^{\;\;b}\mathrm{A}$, where ${}_{(2+m)}^{\;\;b}\mathrm{A}$ is the matrix obtained from A replacing $(2+m)$-th column by the vector $\vec{b}$, that is, $\left(\vec{d}\right)^t\vec{b} = \det\left({}_{(2+m)}^{\;\;b}\mathrm{A}\right)$. Note that ${}_{(2+m)}^{\;\;b}\mathrm{A} = \left(\mathrm{T}^t\Gamma\widehat{\mathrm{T}} + \widehat{\Phi\Upsilon^{-1}}\right)$, where $\widehat{\Phi\Upsilon^{-1}}$ is a matrix obtained from $\Phi\Upsilon^{-1}$ by replacing its element $a_{(2+m)\times(2+m)}$ by zero and $\widehat{\mathrm{T}}$ is a matrix obtained from T replacing all elements of the vector $\vec{\varepsilon}$ by zero, except for the element $a_{i\times 1}$, $\vec{\varepsilon}^t = \begin{bmatrix} 0 & \cdots & \frac{\varepsilon_i}{\varepsilon_i} & \cdots & 0 \end{bmatrix}$. Substituting these equalities in equation (1.57) yields:

$$R_{\varepsilon_i} = \left[\frac{\eta_{P_E}(E^S) \det\left({}_{(2+m)}^{\ b}\mathrm{A}\right)}{\det(\mathrm{A})} - \frac{\sigma_i \eta_{P_E}(E^S) \left(\frac{S_i}{\sigma_i P_{S_i}}\right) (-1)^{2(2+m)} \det\left(\mathrm{A}_{(2+m)\times(2+m)}\right)}{\det(\mathrm{A})} + \sum_{j=1}^{n} \sigma_j \eta_{P_{S_i}}(S_j) + \sigma_i \right] - \sum_{j=1}^{n} \sigma_j \eta_{P_{S_i}}(S_j) \quad \textbf{(1.58)}$$

Note that the elements of the matrices A and ${}_{(2+m)}^{\ b}\mathrm{A}$ are not elasticities, they are derivatives. However, it is possible to use elasticity passing matrices (Ψ and Ω) and the equilibrium relation ($Y^D = Y$, $E^S = E$ and ${q_h}^S = q_h \; \forall \; h = 1,2 \ldots, m$) to transform these matrix elements into elasticities. Where Ψ and Ω are diagonal matrices of order $(2 + m)$, where the main diagonals are equal, respectively, to $\begin{bmatrix} P_Y & P_{Q_1} & \cdots & P_{Q_m} & P_{S_i} \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{Y} & \frac{1}{Q_1} & \cdots & \frac{1}{Q_m} & \frac{\sigma_i}{S_i} \end{bmatrix}$. Thus, dividing and multiplying the first term of equation (1.58) by $\det(\Omega)\det(\Psi)$ and applying determinant property, we obtain:

$$R_{\varepsilon_i} = \left[\frac{\eta_{P_E}(E^S) \det\left(\Omega_{(2+m)}^{\ b}\mathrm{A}\Psi\right)}{\det(\Omega\mathrm{A}\Psi)} - \frac{\sigma_i \eta_{P_E}(E^S) (-1)^{2(2+m)} \det\left((\Omega\mathrm{A}\Psi)_{(2+m)\times(2+m)}\right)}{\det(\Omega\mathrm{A}\Psi)} + \sum_{j=1}^{n} \sigma_j \eta_{P_{S_i}}(S_j) + \sigma_i \right] - \sum_{j=1}^{n} \sigma_j \eta_{P_{S_i}}(S_j) \quad \textbf{(1.59)}$$

Note that, in order to find the term $\det\left((\Omega\mathrm{A}\Psi)_{(2+m)\times(2+m)}\right)$, it is first necessary to using Laplace expansion in $(2+m)$-th columns of $\det(\Omega)$ and $\det(\Psi)$. That is, as Ψ and Ω are diagonals matrices, then $\det(\Omega)\det(\Psi) = \frac{\sigma_i P_{S_i}}{S_i} \det\left(\Omega_{(2+m)\times(2+m)}\right) \det\left(\Psi_{(2+m)\times(2+m)}\right)$. From equation (1.59), it is possible to separate the different reallocation effects. The terms in brackets correspond to the sum of the energy price, input price, and output price effects, while the last term is the sum of the direct effect ($R_{\varepsilon_i}(S_i) = -\sigma_i \eta_{P_{S_i}}(S_i)$) and the cross-price effect ($R_{\varepsilon_i}(S_j) = -\sum_{j \neq i} \sigma_j \eta_{P_{S_i}}(S_j)$). In order to disaggregate the price effects, it is necessary to use the Laplace expansion on the $(2+m)$-th row of $\Omega_{(2+m)}^{\ b}\mathrm{A}\Psi$. Since the only difference between $\Omega_{(2+m)}^{\ b}\mathrm{A}\Psi$ and $\Omega\mathrm{A}\Psi$ is found in $(2+m)$-th column, then $(\Omega\mathrm{A}\Psi)_{(2+m)\times(2+m)} = \left(\Omega_{(2+m)}^{\ b}\mathrm{A}\Psi\right)_{(2+m)\times(2+m)}$. Denoting $\Lambda = \Omega\mathrm{A}\Psi$ and ${}_{(2+m)}^{\ b}\Lambda = \Omega_{(2+m)}^{\ b}\mathrm{A}\Psi$, then the output price effect, input price effect, and energy price effect are equal, respectively, to:

$$R_{\varepsilon_i}(P_Y) = \frac{\eta_{P_E}(E^S)}{\det(\Lambda)} \eta_{P_{S_i}}(Y)(-1)^{(2+m)+1} \det\left({}_{(2+m)}^{\ b}\Lambda_{1\times(2+m)}\right) \quad \textbf{(1.60)}$$

$$R_{\varepsilon_i}(P_{q_h}) = -\frac{\eta_{P_E}(E^S)}{\det(\Lambda)} \sum_{h=1}^{m} \eta_{P_{S_i}}(Q_h)(-1)^{(2+m)+(1+h)} \det\left({}_{(2+m)}^{\ b}\Lambda_{(1+h)\times(2+m)}\right) \quad \textbf{(1.61)}$$

$$\begin{aligned} R_{\varepsilon_i}\left(P_{E_j}\right) = {} & \frac{\eta_{P_E}(E^S)}{\det(\Lambda)}(-1)^{2(2+m)} \det\left({}_{(2+m)}^{\ b}\Lambda_{(2+m)\times(2+m)}\right)\left[R_{\varepsilon_i}(S_i) + R_{\varepsilon_i}(S_j) - \sigma_i\right] \\ & - \left[R_{\varepsilon_i}(S_i) + R_{\varepsilon_i}(S_j) - \sigma_i\right] \end{aligned} \quad \textbf{(1.62)}$$

It is noteworthy that the energy price effect in equation (1.62) incorporates all other rebound effects, weighting them by the price elasticity of the energy supply. In order to see this, just added and subtracted in equation (1.62) the term $\frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)}{\det(\Lambda)}$, where H is a square matrix of order $(2+m)$ with all elements equal to zero, except the element $a_{(2+m)\times(2+m)}$ which is equal to 1. Thus, equation (1.62) yields:

$$\begin{aligned} R_{\varepsilon_i}\left(P_{E_j}\right) = {} & \sigma_i - \left[R_{\varepsilon_i}(S_i) + R_{\varepsilon_i}(S_j) + R_{\varepsilon_i}(P_{q_h}) + R_{\varepsilon_i}(P_Y)\right] \\ & + \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)}{\det(\Lambda)} \end{aligned} \quad \textbf{(1.63)}$$

Equation (1.59) can be rewritten as:

$$\begin{aligned} R_{\varepsilon_i} = \sigma_i + {} & \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ b}\Lambda\right)}{\det(\Lambda)} \\ & - \frac{\sigma_i \eta_{P_E}(E^S)(-1)^{2(2+m)}\det\left({}_{(2+m)}^{\ b}\Lambda_{(2+m)\times(2+m)}\right)}{\det(\Lambda)} \end{aligned} \quad \textbf{(1.64)}$$

Equation (1.64) can be manipulated in two equivalent ways. First, note that $\det\left({}_{(2+m)}^{\ b}\Lambda\right) - \sigma_i(-1)^{2(2+m)}\det\left({}_{(2+m)}^{\ b}\Lambda_{(2+m)\times(2+m)}\right) = \det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)$. Second, if we multiply and divide the first element of equation (1.64), σ_i, by $\det(\Lambda)$ it is possible to rearrange the terms and get that $\det(\Lambda) - \eta_{P_E}(E^S)(-1)^{2(2+m)}\det\left({}_{(2+m)}^{\ b}\Lambda_{(2+m)\times(2+m)}\right) = \det(\widehat{\Lambda})$, where $\widehat{\Lambda} = \Omega\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right)\Psi$. That is, the only difference between $\widehat{\Lambda}$ and Λ is the absence of the term $\eta_{P_E}(E^S)$ in the element $a_{(2+m)\times(2+m)}$ of $\widehat{\Lambda}$. Therefore, we can, equivalently, rewrite the equation (1.64) as:

$$R_{\varepsilon_i} = \sigma_i + \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ \ b}\Lambda - \sigma_i \mathrm{H}\right)}{\det(\Lambda)} \tag{1.65}$$

$$R_{\varepsilon_i} = \frac{\sigma_i \det(\widehat{\Lambda})}{\det(\Lambda)} + \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ \ b}\Lambda\right)}{\det(\Lambda)} \tag{1.66}$$

If there is only a single energy service, then $\sigma_i = 1$ and ${}_{(2+m)}^{\ \ b}\Lambda = \widehat{\Lambda}$, since $\mathrm{T} = \widehat{\mathrm{T}} = \mathrm{I}$. In this case, equations (1.65) and (1.66), respectively, yields:

$$R_{\varepsilon_i} = 1 + \frac{\eta_{P_E}(E^S)\det\left(\widehat{\Lambda} - \mathrm{H}\right)}{\det(\Lambda)} \tag{1.67}$$

$$R_{\varepsilon_i} = \frac{\left[1 + \eta_{P_E}(E^S)\right]\det(\widehat{\Lambda})}{\det(\Lambda)} \tag{1.68}$$

The short-run reallocation effect with endogenous prices can be obtained analogously. Note that equation (1.36) can be rewritten as $R^r_{\varepsilon_i}\frac{1}{\sigma^r_i} = -\eta_{P_{S_i}}({S_i}^r) + \eta_{P_{S_i}}({S_i}^r)\eta_{\varepsilon_i}(P_E) + \eta_{P_Y}({S_i}^r)\eta_{\varepsilon_i}(P_Y)$. That is, in order to find the short-run effect, we can use the same procedures as used above, but assuming that the model only incorporates a single energy service (S_i) and that there are no other inputs in the model ($m = 0$). In other words, the short-run model is equivalent to the long-run model with a single energy service and without non-energy inputs. Thus, by equation (1.68), $R^r_{\varepsilon_i}\frac{1}{\sigma^r_i} = \frac{\left[1+\eta_{P_E}\left(E^{S^r}\right)\right]\det(\widehat{\Lambda}^r)}{\det(\Lambda^r)}$, where Λ^r and $\widehat{\Lambda}^r$ represent, respectively, the matrix Λ and $\widehat{\Lambda}$ assuming that there are no other inputs ($m = 0$) and where all its elements come from short-run functions.[26] Therefore, the short-run rebound effect with endogenous prices is equal to:

$$R^r_{\varepsilon_i} = \frac{\left[1 + \eta_{P_E}\left(E^{S^r}\right)\right]\sigma^r_i \det(\widehat{\Lambda}^r)}{\det(\Lambda^r)} \tag{1.69}$$

[26] In fact, the system expressed in equation (1.46) will incorporate the non-energy inputs. However, as these inputs will be fixed, matrix A can be written as a diagonal block matrix of order $(2 + m)$. This diagonal block matrix will be composed of a diagonal matrix of order m, where the elements of the main diagonal correspond to the

Appendix 1.D Super-conservation and energy service numbers

We will show that super-conservation is not allowed when the rebound model includes only a single energy service. That is, when there is a single energy service in the model, the reallocation effect is always greater than or equal to zero ($R_{\varepsilon_i} \geq 0$). In other words, when there is only a single energy service, the different rebound effects can be positive or negative (except the direct effect which is always positive), but their sum is always greater than or equal to zero. We will demonstrate this fact using the long-run rebound effect. Nevertheless, as the short-run model is equivalent to the long-run model with only a single energy service and without non-energy inputs, then the short-run rebound effect cannot be negative either.

Proof: To prove that the super-conservation is not allowed when the model has a single energy service, just show that equation (1.68) will always be greater than or equal to zero: $R_{\varepsilon_i} = \frac{[1+\eta_{P_E}(E^S)]\det(\widehat{\Lambda})}{\det(\Lambda)} \geq 0$. Note that $[1+\eta_{P_E}(E^S)] > 0$. Thus, just show that $\frac{\det(\widehat{\Lambda})}{\det(\Lambda)} \geq 0$. However, $\det(\Lambda) = \det(\Omega)\det(\mathrm{T}^t\Gamma\mathrm{T} + \Phi\Upsilon^{-1})\det(\Psi)$ and $\det(\widehat{\Lambda}) = \det(\Omega)\det\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right)\det(\Psi)$. In addition, when there is a single energy service, then $\mathrm{T} = \mathrm{I}$.[27] Thus, just prove that $\frac{\det(\Gamma+\widehat{\Phi\Upsilon^{-1}})}{\det(\Gamma+\Phi\Upsilon^{-1})} \geq 0$. That is, it is enough to prove that the matrices $(\Gamma + \Phi\Upsilon^{-1})$ and $\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)$ are positive semidefinite. Note that this implies that $\det(\Gamma + \Phi\Upsilon^{-1}), \det\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right) \geq 0$.

As Γ is the Hessian matrix of second-order partials derivatives of the profit function, then it must be symmetric by Young's theorem and positive semidefinite by convexity of the profit function (JEHLE; RENY, 2011). $\Phi\Upsilon^{-1}$ is a diagonal matrix of order $(2+m)$, where the

derivatives of the supply function of the other inputs ($\frac{\partial {q_h}^S}{\partial P_{q_h}}$), and a square matrix of order 2, that have the same element as A when we assume $m = 0$. The matrix ${}_{(2+m)}^{\,b}\mathrm{A}$ can also be written in the same way, that is, as a diagonal block matrix. Thus, if we apply the property of the determinants of diagonal block matrix, we can in equation (1.64) eliminate all the elements of the matrix that corresponds to the m non-energy inputs. Therefore, remaining only the determinate of square matrices of order 2 that have the same elements as the matrices when we assume $m = 0$.

[27] Appendix 1.C shows that T is a rectangular block matrix of order $(1+m+n) \times (2+m)$ equal to $\begin{bmatrix} \mathrm{I} & \vec{0} \\ 0 & \vec{\varepsilon} \end{bmatrix}$, I is an identity matrix of order $(1+m)$, 0 is a zero matrix of order $n \times (1+m)$, $\vec{0}$ is a zero column vector of order $(1+m)$, $\vec{\varepsilon}$ is a column vector of order n equal to $\vec{\varepsilon}^t = \begin{bmatrix} \frac{\varepsilon_i}{\varepsilon_1} & \frac{\varepsilon_i}{\varepsilon_2} & \cdots & \frac{\varepsilon_i}{\varepsilon_{(n-1)}} & \frac{\varepsilon_i}{\varepsilon_n} \end{bmatrix}$. When there is a single energy service, then $\vec{\varepsilon}^t = 1$ and 0 is a zero matrix of order $1 \times (1+m)$. In this way, $\mathrm{T} = \mathrm{I}$ of order $(2+m)$.

main diagonal elements are $\left[-\frac{\partial Y^D}{\partial P_Y} \quad {\tau_1}^2 \frac{\partial {q_1}^S}{\partial P_{q_1}} \quad \cdots \quad {\tau_m}^2 \frac{\partial {q_m}^S}{\partial P_{q_m}} \quad {\varepsilon_i}^2 \frac{\partial E^S}{\partial P_E}\right]$[28]. $\widehat{\Phi\Upsilon^{-1}}$ is a diagonal matrix of order $(2+m)$, where the elements of its main diagonal are $\left[-\frac{\partial Y^D}{\partial P_Y} \quad {\tau_1}^2 \frac{\partial {q_1}^S}{\partial P_{q_1}} \quad \cdots \quad {\tau_m}^2 \frac{\partial {q_m}^S}{\partial P_{q_m}} \quad 0\right]$.[29] As $\Phi\Upsilon^{-1}$ and $\widehat{\Phi\Upsilon^{-1}}$ are diagonal matrices, then their eigenvalues are the diagonal elements themselves. In addition, a matrix is positive semidefinite if and only if all of its eigenvalues are nonnegative. Therefore, it is enough that the diagonal elements of the matrix $\Phi\Upsilon^{-1}$ and $\widehat{\Phi\Upsilon^{-1}}$ are non-negative for the matrices to be positive semidefinite. As $\tau_h, \varepsilon_i > 0\ \forall h, i$ and as Production theory requires that any supply function will be increasing in output price $\frac{\partial {q_h}^S}{\partial P_{q_h}}, \frac{\partial E^S}{\partial P_E} \geq 0\ \forall h$, then, in order to the matrix $\Phi\Upsilon^{-1}$ and $\widehat{\Phi\Upsilon^{-1}}$ to be positive semidefinite, it is necessary that $\frac{\partial Y^D}{\partial P_Y} \leq 0$, that is, that the output Y is not a Giffen good.

Therefore, in well-behaved situations ($\frac{\partial Y^D}{\partial P_Y} \leq 0$), Γ, $\Phi\Upsilon^{-1}$ and $\widehat{\Phi\Upsilon^{-1}}$ are positive semidefinite matrices. As shown in Horn and Johnson (2012, p. 430), the sum of two positive semidefinite matrices is also a positive semidefinite matrix. Thus, the matrices $(\Gamma + \Phi\Upsilon^{-1})$ and $\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)$ are positive semidefinite[30] and so $\det(\Gamma + \Phi\Upsilon^{-1}), \det\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right) \geq 0$. ■

Two observations are worth making. First, in order to the system in equation (1.46) to have a single solution (i.e., in order to the inverse matrix of $\mathrm{A} = (\Gamma + \Phi\Upsilon^{-1})$ to exist), it is necessary that $(\Gamma + \Phi\Upsilon^{-1})$ is positive definite. This means that Γ or $\Phi\Upsilon^{-1}$ must be a positive definite matrix and so $\det(\Gamma + \Phi\Upsilon^{-1}) > 0$. Second, although $\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)$ is positive semidefinite and so $\det\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right) \geq 0$, the signs of the determinants of submatrices $\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)_{l\times k}\ \forall\, l \neq k$ will be indeterminate. Thus, as $\widehat{\Lambda} = \Omega\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)\Psi$, the signs of, input price, output price, and energy price effects expressed, respectively, in equations (1.60), (1.61), and (1.63), are indeterminate, regardless of the number of energy services in the model.

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[28] Appendix 1.C shows that Υ is a diagonal matrix of order $(2+m)$, where the main diagonal is equal to $\left[1 \quad \frac{1}{\tau_1} \quad \cdots \quad \frac{1}{\tau_m} \quad \frac{1}{\varepsilon_i}\right]$ and Φ is a diagonal matrix of order $(2+m)$, where the main diagonal is equal to $\left[-\frac{\partial Y^D}{\partial P_Y} \quad \tau_1 \frac{\partial {q_1}^S}{\partial P_{q_1}} \quad \cdots \quad \tau_m \frac{\partial {q_m}^S}{\partial P_{q_m}} \quad \varepsilon_i \frac{\partial E^S}{\partial P_E}\right]$.

[29] That is, $\widehat{\Phi\Upsilon^{-1}}$ is equal to $\Phi\Upsilon^{-1}$, except for its last element of the diagonal $a_{(2+m)\times(2+m)}$.

[30] Proof: Let $\vec{x} \neq 0$ nonzero vector. As Γ, $\Phi\Upsilon^{-1}$ and $\widehat{\Phi\Upsilon^{-1}}$ are positive semidefinite matrices, by definition we have: $\vec{x}^t\Gamma\vec{x} \geq 0$, $\vec{x}^t(\Phi\Upsilon^{-1})\vec{x} \geq 0$ and $\vec{x}^t\left(\widehat{\Phi\Upsilon^{-1}}\right)\vec{x} \geq 0$. Therefore, $\vec{x}^t(\Gamma + \Phi\Upsilon^{-1})\vec{x} = \vec{x}^t\Gamma\vec{x} + \vec{x}^t(\Phi\Upsilon^{-1})\vec{x} \geq 0$ and $\vec{x}^t\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)\vec{x} = \vec{x}^t\Gamma\vec{x} + \vec{x}^t\left(\widehat{\Phi\Upsilon^{-1}}\right)\vec{x} \geq 0$. This means that $(\Gamma + \Phi\Upsilon^{-1})$ and $\left(\Gamma + \widehat{\Phi\Upsilon^{-1}}\right)$ are positive semidefinite.

In addition, it is noteworthy that when the model has more than one energy service, equation (1.66) require that $\det\left(\mathrm{T}^t\Gamma\widehat{\mathrm{T}} + \widehat{\Phi\Upsilon^{-1}}\right) < -\frac{\sigma_i}{\eta_{P_E}(E^S)}\det\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right)$ in order to the super-conservation to occur ($R_{\varepsilon_i} < 0$). As $\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right)$ is positive semidefinite[31] and so $\det\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right) \geq 0$. Also, $\sigma_i, \eta_{P_E}(E^S) \geq 0$. Therefore, a necessary (but not sufficient) condition for a rebound effect to be negative is that $\det\left(\mathrm{T}^t\Gamma\widehat{\mathrm{T}} + \widehat{\Phi\Upsilon^{-1}}\right) < 0$. Nothing prevents this from happening, since $\left(\mathrm{T}^t\Gamma\widehat{\mathrm{T}} + \widehat{\Phi\Upsilon^{-1}}\right)$ is not a symmetric matrix (i.e., $\mathrm{T} \neq \widehat{\mathrm{T}}$). That is, we cannot use the properties of positive semidefinite of Γ (and $\widehat{\Phi\Upsilon^{-1}}$) to get the sign of the determinant: let $\vec{x} \neq 0$ nonzero vector, then $\vec{x}^t\left(\mathrm{T}^t\Gamma\widehat{\mathrm{T}} + \widehat{\Phi\Upsilon^{-1}}\right)\vec{x} = \vec{w}^t\Gamma\vec{z} + \vec{x}^t\left(\widehat{\Phi\Upsilon^{-1}}\right)\vec{x}$, where $\vec{z} = \widehat{\mathrm{T}}\vec{x} \neq \mathrm{T}\vec{x} = \vec{w}$. The example shown in section 1.5.3 illustrates this fact.

Appendix 1.E The importance of energy supply

We will show that when the energy price energy is endogenous (i.e., when the energy supply function is introduced), the magnitude of the AES decreases. That is, we will show that $\left|\dot{\eta}_{\varepsilon_i}(E)'\right| \geq \left|\dot{\eta}_{\varepsilon_i}(E)\right|$, where $\dot{\eta}_{\varepsilon_i}(E)'$ is the elasticity of energy consumption with respect to energy efficiency when the energy price is exogenous.

Proof: Using equations (1.30) and (1.65), it is possible to see that $\dot{\eta}_{\varepsilon_i}(E) = \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\;\;b}\Lambda - \sigma_i\mathrm{H}\right)}{\det(\Lambda)}$. Since $\det(\Lambda) = \det\left(\widehat{\Lambda}\right) + \eta_{P_E}(E^S)(-1)^{2(2+m)}\det\left(\Lambda_{(2+m)\times(2+m)}\right)$, then $\dot{\eta}_{\varepsilon_i}(E) = \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\;\;b}\Lambda - \sigma_i\mathrm{H}\right)}{\det(\widehat{\Lambda}) + \eta_{P_E}(E^S)(-1)^{2(2+m)}\det\left(\Lambda_{(2+m)\times(2+m)}\right)}$. When energy price is exogenous, this means $\eta_{P_E}(E^S) \to \infty$. Thus, $\dot{\eta}_{\varepsilon_i}(E)' = \lim_{\eta_{P_E}(E^S)\to\infty}\dot{\eta}_{\varepsilon_i}(E) = \lim_{\eta_{P_E}(E^S)\to\infty}\frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\;\;b}\Lambda - \sigma_i\mathrm{H}\right)}{\det(\widehat{\Lambda}) + \eta_{P_E}(E^S)(-1)^{2(2+m)}\det\left(\Lambda_{(2+m)\times(2+m)}\right)}$. Applying the L´hopital rule[32], it is easy to see that $\dot{\eta}_{\varepsilon_i}(E)' = \frac{\det\left({}_{(2+m)}^{\;\;b}\Lambda - \sigma_i\mathrm{H}\right)}{\det\left(\Lambda_{(2+m)\times(2+m)}\right)}$. Note that, as already shown in Appendix 1.D, $\det\left(\widehat{\Lambda}\right)$, $\det(\Lambda)$, $\det\left(\Lambda_{(2+m)\times(2+m)}\right)$ and $\eta_{P_E}(E^S)$ are non-negative numbers, then $\dot{\eta}_{\varepsilon_i}(E)'$ and $\dot{\eta}_{\varepsilon_i}(E)$

[31] Proof: Let $\vec{x} \neq 0$ nonzero vector, then $\vec{x}^t\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right)\vec{x} = \vec{x}^t\mathrm{T}^t\Gamma\mathrm{T}\vec{x} + \vec{x}^t\left(\widehat{\Phi\Upsilon^{-1}}\right)\vec{x} = (\mathrm{T}\vec{x})^t\Gamma(\mathrm{T}\vec{x}) + \vec{x}^t\left(\widehat{\Phi\Upsilon^{-1}}\right)\vec{x} = \vec{w}^t\Gamma\vec{w} + \vec{x}^t\left(\widehat{\Phi\Upsilon^{-1}}\right)\vec{x}$, where $\vec{w} = \mathrm{T}\vec{x}$. Note that T has full column rank and therefore T is nonsingular (i.e., $\mathrm{T}\vec{x} = 0$ if and only if $\vec{x} = 0$). As $\vec{x} \neq 0$, then $\vec{w} = \mathrm{T}\vec{x} \neq 0$. In this way, $\mathrm{T}^t\Gamma\mathrm{T}$ is also positive semidefinite (i.e., $\vec{x}^t(\mathrm{T}^t\Gamma\mathrm{T})\vec{x} = (\mathrm{T}\vec{x})^t\Gamma(\mathrm{T}\vec{x}) = \vec{w}^t\Gamma\vec{w} \geq 0$, since Γ is positive semidefinite – see Horn and Johnson (2012, p. 431) for more details). As already seen, $\widehat{\Phi\Upsilon^{-1}}$ is also positive semidefinite. Therefore, $\left(\mathrm{T}^t\Gamma\mathrm{T} + \widehat{\Phi\Upsilon^{-1}}\right)$ is positive semidefinite.

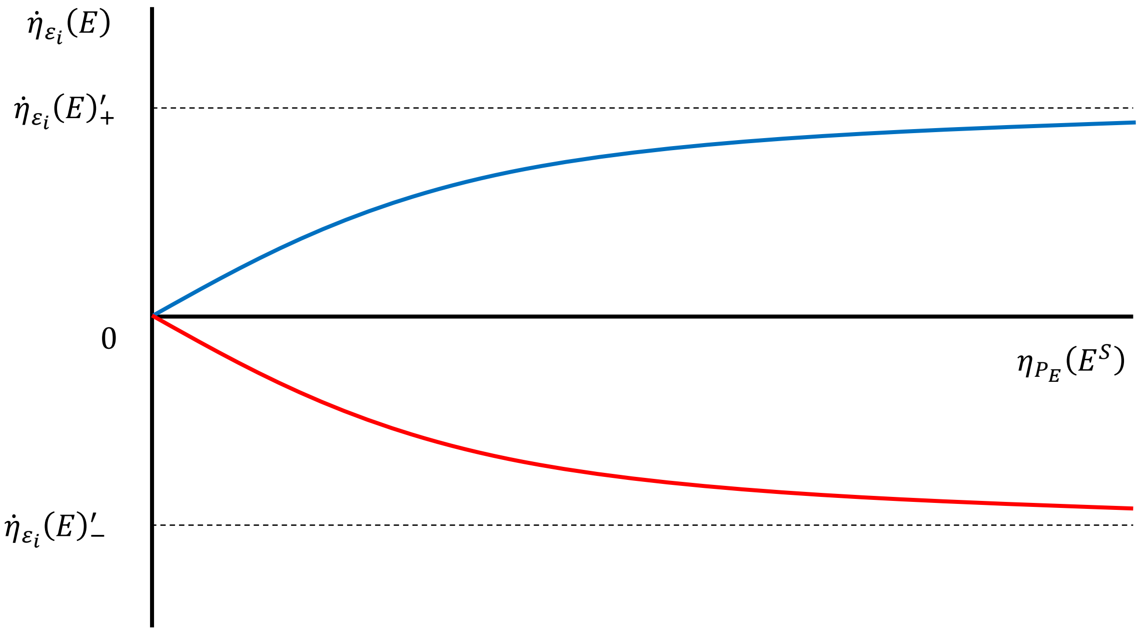
[32] Note that $\eta_{P_E}(E^S)$ is not an element of any of these three matrices, that is, $\widehat{\Lambda}$, $\Lambda_{(2+m)\times(2+m)}$, and ${}_{(2+m)}^{\;\;b}\Lambda - \sigma_i\mathrm{H}$. Therefore, the respective determinants can be treated as a scalar.

will either be positive or negative, depending on the sign of $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)$. It is possible to see that $\dot{\eta}_{\varepsilon_i}(E)' - \dot{\eta}_{\varepsilon_i}(E) = \frac{\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)\det(\widehat{\Lambda})}{\det\left(\Lambda_{(2+m)\times(2+m)}\right)\det(\Lambda)}$. Thus, when $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right) \gtrless 0$, then $\dot{\eta}_{\varepsilon_i}(E)' \gtrless \dot{\eta}_{\varepsilon_i}(E)$ and, since both have the same sign , then $\left|\dot{\eta}_{\varepsilon_i}(E)'\right| \geq \left|\dot{\eta}_{\varepsilon_i}(E)\right|$, with equality being satisfied when $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right) = 0$ or when $\det\left(\widehat{\Lambda}\right) = 0$. ■

In addition, we will also show that the more price-inelastic the energy supply is, the smaller the magnitude of the AES (i.e., $\dot{\eta}_{\varepsilon_i}(E)$). In the extreme case, when the energy supply is fixed ($\eta_{P_E}(E^S) = 0$), the energy consumption does not respond to shocks in energy efficiency ($\dot{\eta}_{\varepsilon_i}(E) = 0$).

Proof: Note that $\frac{\partial \dot{\eta}_{\varepsilon_i}(E)}{\partial \eta_{P_E}(E^S)} = \frac{\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)\det(\widehat{\Lambda})}{\det(\Lambda)^2}$ and $\frac{\partial \dot{\eta}_{\varepsilon_i}(E)}{\partial^2 \eta_{P_E}(E^S)} = \frac{-2\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)\det(\widehat{\Lambda})\det\left(\Lambda_{(2+m)\times(2+m)}\right)}{\det(\Lambda)^3}$. Thus, the sign of both derivatives depends only on the sign of $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)$. When $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right) \gtrless 0$, then $\frac{\partial \dot{\eta}_{\varepsilon_i}(E)}{\partial \eta_{P_E}(E^S)} \gtrless 0$ and $\frac{\partial \dot{\eta}_{\varepsilon_i}(E)}{\partial^2 \eta_{P_E}(E^S)} \lessgtr 0$ (assuming $\det\left(\widehat{\Lambda}\right) > 0$). That is, when $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right) > 0$, $\dot{\eta}_{\varepsilon_i}(E)$ is an increasing and concave function (with an upper limit) in $\eta_{P_E}(E^S)$ and, when $\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right) < 0$, it is a decreasing and convex function (with a lower limit). Furthermore, since $\dot{\eta}_{\varepsilon_i}(E) = \frac{\eta_{P_E}(E^S)\det\left({}_{(2+m)}^{\ b}\Lambda - \sigma_i \mathrm{H}\right)}{\det(\Lambda)}$ and $\det(\Lambda) > 0$ (see Appendix 1.D), hence $\dot{\eta}_{\varepsilon_i}(E) = 0$, when $\eta_{P_E}(E^S) = 0$. ■

We can graphically illustrate the relationship between price elasticity of energy supply and elasticity of energy consumption with respect to energy efficiency (see Figure 1.2).



Note: $\dot{\eta}_{\varepsilon_i}(E)'_+$ represents the term $\dot{\eta}_{\varepsilon_i}(E)'$ when $\det\left(\frac{b}{(2+m)}\Lambda - \sigma_i \mathrm{H}\right) > 0$ and $\dot{\eta}_{\varepsilon_i}(E)'_-$ represents the term $\dot{\eta}_{\varepsilon_i}(E)'$ when $\det\left(\frac{b}{(2+m)}\Lambda - \sigma_i \mathrm{H}\right) < 0$.

Figure 1.2 – Relationship between price elasticity of energy supply and elasticity of energy consumption with respect to energy efficiency

Source: Authors' own elaboration.

2 AN ECONOMIC MODEL OF ENERGY SECURITY: A PROPOSAL TO UNIFY THE CONCEPT*

2.1 Introduction

In the 20th century, energy security has been a political concern since the outbreak of the World Wars and the need to supply oil to armies (CHERP; JEWELL, 2011; YERGIN, 2011). However, energy security became an object of academic reflection only in the 1960s and its first definition emerged in 1976 (AZZUNI; BREYER, 2018; CHERP; JEWELL, 2014).[33] While the 1980s and 1990s were characterized by a low interest in the topic of energy security, in the 21st century it has been widely studied and intensely debated in scientific circles (ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018). Nevertheless, despite six decades of academic debate on energy security, there are still two major gaps in the literature that must be addressed. First, there is no consensus on its definition and whether it would be possible to define energy security universally since it is highly context-dependent. Second, there is no rigorous methodological framework that explains reasonably well the theoretical foundations of energy security, that is, the logical structure that explains how its dimensions interact with each other and consequently how they affect energy security has not yet been properly developed.

Therefore, the purpose of this article goes far beyond just proposing a definition for energy security and compiling a long list of its dimensions. The goal is to develop a simplified model that provides the theoretical foundations of energy security. We start from the definition of the concept of security proposed by Baldwin (1997, p. 13): "*low probability of damage to acquired values*". This definition is converted into microeconomic concepts and random variables are inserted, which represent the energy supply disruptions. Thus, this simplified model combines economic theory and the concept of security in a probabilistic framework. Of course, no theoretical model can capture all aspects of the complex reality of the energy security problem. Nevertheless, this simplified model is only a first step in the attempt to rigorously

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[33] According to Azzuni and Breyer (2018), the first record of an energy security definition is presented in Willrich (1976, p. 747): "*assurance of sufficient energy supplies to permit the national economy to function in a politically acceptable manner*". Willrich (1976, p. 747) also presents two other ways to define energy security: 1) "*guarantee of sufficient energy supplies to permit a country to function during war*"; and 2) "*assurance of adequate energy supplies to maintain the national economy at a normal level*".

develop an integrative methodological framework. The model can and should be further developed to become more realistic.

The paper is structured as follows. Section 2.2 performs a literature review on energy security. Section 2.3 develops the energy security concept through the definition of the security concept proposed by Baldwin (1997). Section 2.4 presents the subjectivity of energy security. Section 2.5 develops the energy security model in detail. Section 2.6 shows that there is a necessary condition for an economy to be in an energy security situation. Section 2.7 defines the concept of energy independence. Section 2.8 develops the concept of preferable energy security strategy. Section 2.9 presents the relationship between energy price and energy security. Section 2.10 concludes. Furthermore, three appendices provide a detailed description of some aspects of the model.

2.2 Literature review

Recently, many literature reviews have been conducted (ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018; CHESTER, 2010; KOULOURI; MOURAVIEV, 2019; PARAVANTIS et al., 2019; SOVACOOL, 2010; SOVACOOL; BROWN, 2010; WINZER, 2012) and they all came to the same conclusion: there are numerous energy security definitions. For example, Azzuni and Breyer (2018), when reviewing 104 studies, found 66 different definitions. Some authors argue that several definitions exist because the concept of energy security is polysemic in nature, being inherently slippery and capable of holding multiple dimensions (ANG; CHOONG; NG, 2015a; CHESTER, 2010; CIUTĂ, 2010; VALENTINE, 2010; YERGIN, 2011). In this view, energy security would have many possible meanings and would be highly context-dependent, in such a way that it would be impractical to define it universally. As highlighted by Ciutã (2010, p. 127), "*energy security clearly means many different things to different authors and actors, and even at times to the same author or actor*".

On the other hand, Cherp and Jewell (2014) argue that the presence of different meanings of energy security does not necessarily mean the existence of different concepts of energy security. Thus, a universal definition would be possible. However, according to Cherp and Jewell (2014), Chester's (2010) statement that energy security is a slippery and multidimensional concept has become a self-fulfilling prophecy, in such a way that it discouraged the search for a rigorous concept of energy security and gave green light to each author proposes his own multidimensional definitions. Thereby, there has been a low concern to define it rigorously. Ang, Choong, and Ng (2015a), when reviewing 104 studies, identified

21 where energy security was not clearly defined. Moreover, as Valdés (2018) points out, only a few studies include a rigorous definition of energy security, most of them build their definition on the enumeration of energy security dimensions. For example, Sovacool and Brown (2010, p. 81) only state that "*energy security should be based on the interconnected factors of availability, affordability, efficiency, and environmental stewardship*", without presenting a formal definition.

Many authors have sought to explain the concept of energy security by drawing up lists of energy security concerns (i.e., dimensions). Nevertheless, the lack of rigor in its definition is reflected in the process and criteria for choosing the energy security dimensions. As highlighted by Cherp and Jewell (2011), there is no methodological framework that rigorously justifies the inclusion or omission of energy security dimensions. Simply put, three methods of choosing dimensions can be identified: 1) choices based on meta-analysis of previous studies (ABDULLAH et al., 2020; ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018; ERAHMAN et al., 2016; RAGHOO et al., 2018; REN; SOVACOOL, 2014; SOVACOOL; BROWN, 2010); 2) choices based on interviews with experts (SOVACOOL, 2011, 2016; SOVACOOL; MUKHERJEE, 2011); and 3) arbitrary choices (APERC, 2007; HUGHES, 2009; LI; SHI; YAO, 2016; VIVODA, 2010; VON HIPPEL et al., 2011). Meta-analysis and interviews are relatively systematic methods, but their values are diminished by the fact that the underlying studies and experts can be based on the method of arbitrary choice (CHERP, 2012; CHERP; JEWELL, 2011).

Thus, in order to explain the energy security concept, each author has included the dimensions that they believed to be relevant. Some authors use a small number of dimensions, such as the four 'A's: Availability, Affordability, Acceptability, Accessibility (APERC, 2007; HUGHES; SHUPE, 2010; KRUYT et al., 2009; REN; SOVACOOL, 2014; SHIN; SHIN; LEE, 2013). Other authors use as many dimensions as possible, such as land use, health, culture, energy literacy, military, and cyber security issues (AZZUNI; BREYER, 2018; SOVACOOL, 2011; VIVODA, 2010). Furthermore, some authors argue that, for energy security to be defined comprehensively, the concept of sustainability and energy justice (or energy poverty) has to be taken into consideration (AZZUNI; BREYER, 2018; GOLDTHAU; SOVACOOL, 2012), while other authors argue that these concepts should not be included to better understand energy security (CHERP; JEWELL, 2014; WINZER, 2012).

In addition, albeit the dimensions usually have understandable names that appeal to common sense, the lack of rigor is also found in their definitions. For example, despite the term Affordability is commonly used to indicate the energy price dimension, there is no consensus on what Affordability means (ANG; CHOONG; NG, 2015a; CHERP; JEWELL, 2014; KOULOURI; MOURAVIEV, 2019). Affordability can mean absolute price level (i.e., low price) (HUGHES, 2012; KRUYT et al., 2009; RAGHOO et al., 2018), price stability, a price level that guarantees equitable energy consumption (SOVACOOL, 2010), a price level that guarantees the profitability of energy investments (APERC, 2007), or market liquidity (REN; SOVACOOL, 2014). Moreover, one of the most frequently quoted definitions is the "*availability of sufficient supplies at affordable prices*" suggested by Yergin (2006, p. 70–71). However, if affordable prices mean absolute price level, the energy demand function will associate the affordable price level with an amount of energy consumption (i.e., sufficient supplies). On the other hand, the inverse function of energy demand will associate sufficient supplies with a price level (i.e., affordable price). Therefore, in this case, using the terms sufficient supplies and affordable prices in the definition is redundant.

Despite the clear utility of the method of drawing up lists of energy security dimensions, this categorization remains only as a technical exercise in taxonomy and it does not provide a theoretical basis for the concept of energy security. As Valentine (2010) points out, adding more dimensions to a long and disconnected list does not help to better understand energy security; instead, it only helps to enhance intellectual discord and widen the lack of consensus. This is because the method of drawing up lists does not result in an integrative framework (CHERP; JEWELL, 2011). Moreover, it does not develop a logical structure that explains the relationship between the energy security dimensions. Notwithstanding, as emphasized by Sovacool and Brown (2010) and Sovacool and Saunders (2014), dimensions can be competing or allied, that is, changes in one dimension can shrink or enlarge other dimensions.

Furthermore, as highlighted by Axon, Darton, and Winzer (2013), defining energy security and measuring it are two sides of the same coin, since the measurement only has meaning if it quantifies a clearly defined entity. Nevertheless, the lack of rigor in the energy security definition results in a highly inconsistent measurement (GASSER, 2020; VALDÉS, 2018). Depending on the definition and subsequent choice of dimensions, energy security measurements can present the most diverse results (BÖHRINGER; BORTOLAMEDI, 2015; CHERP; JEWELL, 2010; NARULA; REDDY, 2015; VALENTINE, 2010; WINZER, 2012). Thus, the ability of energy security studies to inform energy policy has so far been limited, since it is only possible to improve energy security if policy-makers know what it really means. Moreover, in the absence of a clear definition, energy security has become an umbrella term for many different policy goals (WINZER, 2012).

All these points stress the need for a rigorous theoretical foundation for energy security. It is not because energy security depends on personal judgment that it is just a matter of opinion. On the contrary, there is such a thing as consistency in reasoning from premises to conclusions and so to prescriptions. Therefore, energy security means much more than compiling long lists of various disparate concerns. However, its multidimensionality makes the development of a unified concept a non-trivial task.

Nevertheless, the literature imposes three essential prerequisites for the conceptualization of energy security. First, the concept of energy security should be based on the concept of security in general (BOSWORTH; GHEORGHE, 2011; CHERP; JEWELL, 2014; CIUTĂ, 2010; JOHANSSON, 2013; KARLSSON-VINKHUYZEN; JOLLANDS, 2013; PARAVANTIS et al., 2019; VON HIPPEL et al., 2011). Second, energy security is concerned with risks (AZZUNI; BREYER, 2018; CHERP; JEWELL, 2014; CHESTER, 2010; KUCHARSKI; UNESAKI, 2015; WINZER, 2012) and therefore energy security is closely related to the concept of probability. Third, many of the concepts used in the energy security literature are economic concepts (e.g., welfare, externalities, price, consumption, production, imports, supply, demand, investments). Thus, the concept of energy security should present a rigorous microeconomic foundation. As Böhringer and Bortolamedi (2015) points out, without a rigorous microeconomic foundation, energy security remains a vague catchword rather than an operational concept. However, economic theory has been neglected in the conceptualization of energy security; it is used only to explain some issues related to the energy price and without integration with the concept of security (e.g., Bohi and Toman (1996), Greene (2010), Markandya and Pemberton (2010), and Zhang et al. (2018)).

2.3 From the security concept to the energy security concept

Likewise Cherp and Jewell (2014), we will develop the energy security model starting from the definition of the security concept proposed by Baldwin (1997, p. 13): "*low probability of damage to acquired values*". Baldwin (1997) argues that seven questions need to be answered for this definition to be broadly specified:

1. Security for whom?
2. Security for which values?
3. In what time period?
4. How much security?
5. From what threats?

6. By what means?
7. At what cost?

However, our simplified model will not be able to answer the last question.[34]

The first two questions are paramount for a clear specification of the security concept (BALDWIN, 1997). That is, it is necessary to specify which are the acquired values and to whom those values belong. We will resort to the economic concepts of utility and social welfare. The social welfare function, $W(u_1, \dots, u_I)$, aggregates the utilities u_i of the I agents of the economy. We will define *acquired values* as being a minimum level of social welfare ($\overline{W}$) associated with the minimum levels of utility ($\overline{u_i}$) chosen by each agent of the economy: $\overline{W} = W(\overline{u_1}, \dots, \overline{u_I})$. In other words, the acquired values correspond to a lifestyle desired by a group of individuals. The answer to "Security for whom?" is straightforward: for the group of the I agents, that is, for the economy (e.g., country, region, or world). Furthermore, as a utility function necessarily incorporates some time aggregation, the third question is also answered. That is, depending on the time aggregation embedded in the utility function, the security concept can be assessed in the short or long term.

It is also necessary to specify what means *damage* in the security definition. The *damage to acquired values* occurs when the difference between the equilibrium level of social welfare, $W^* = W(u_1^*, \dots, u_I^*)$, and its minimum desired level is negative: $W^* - \overline{W} < 0$. That is, the damage occurs when the economy does not experience the desired lifestyle for some reason. Therefore, $P(W(u_1^*, \dots, u_I^*) - W(\overline{u_1}, \dots, \overline{u_I}) < 0)$ is the *probability of damage to acquired values*, that is, it is the insecurity probability of the economy. Finally, *low probability* means that there is a maximum level for the insecurity probability of the economy, P^{max}, that the I agents require in order not to feel insecure. Each agent has its own maximum level, P_i^{max}, for the insecurity probability of the economy, which are aggregated through a social probability function, $f_P(.)$. That is, $P^{max} = f_P(P_1^{max}, \dots, P_I^{max})$. Thereby, Baldwin's (1997) definition can be expressed as:

$$P(W(u_1^*, \dots, u_I^*) - W(\overline{u_1}, \dots, \overline{u_I}) < 0) \leq f_P(P_1^{max}, \dots, P_I^{max}) \qquad \textbf{(2.1)}$$

Equation (2.1) shows that the concept of security can be defined by two approaches: the perspective of insecurity and the perspective of security. It is possible to identify that Baldwin's

[34] According to Baldwin (1997), the last question (At what cost?) must be answered by specifying the price that an economy (i.e., its population) is willing to pay in the search for security, since all security policies involve some kind of cost, that is, the sacrifice of other goals that could have been pursued with the resources devoted to security.

(1997) definition approaches the security concept through the perspective of insecurity. That is, Baldwin's (1997) definition means that insecurity must be low. On the other hand, equation (2.1) can be rewritten through the complementary event, that is, through the perspective of security:

$$P(W(u_1^*, \ldots, u_I^*) - W(\overline{u_1}, \ldots, \overline{u_I}) \geq 0) \geq P^{min} \quad \textbf{(2.2)}$$

where $P^{min} = 1 - P^{max}$ is the minimum level for the security probability of the economy, $P(W^* - \overline{W} \geq 0)$, that the I agents require to feel secure.

Thereby, equation (2.2) shows that Baldwin's (1997) definition also means "*high probability of non-damage to acquired values*", that is, it means that security must be high. Thus, the fourth question is answered: How much security? at least $P^{min} = 1 - P^{max}$. The value $P^{min} = 0$ means that the agents do not desire security, whereas the value $P^{min} = 1$ means that the agents desire total security. Thus, P^{min} can be understood as a measure of risk aversion of the economy as a whole.

The relationship between the concepts of security and energy security is due to the extreme importance of energy to the modern lifestyle. As highlighted in Sovacool and Brown (2010), energy is not a commodity like any other, rather it is a prerequisite for the production of all other commodities, a basic factor equal to air, water, and earth. All modern technology uses large amounts of energy when producing goods and services for human well-being. Nevertheless, economic agents do not demand energy per se, but rather the services generated by this energy, called energy services.[35] For example, it is the consumption of transportation services that will require some kind of energy to be generated (e.g., gasoline or electricity). Thereby, human well-being depends largely on the amount of energy services consumed. At the same time, energy contributes heavily to several of the most important environmental problems, especially climate change. That is, energy consumption is a strong generator of negative externalities, which decreases human well-being.

Therefore, the concept of energy security emerges when we assume that social welfare depends only on the consumption of energy services and (negatively) on energy consumption. Thus, we will assume that $u_i(S_i, E_{-i})$, where S_i is the energy service consumption of the agent i and $E_{-i} = [E_1, \ldots, E_{i-1}, E_{i+1}, \ldots, E_I]$ is the vector of energy consumption of all other agents,

[35] In the literature, there is no consensus on energy services definition (FELL, 2017). However, the definition proposed by Fell (2017, p. 137) will be followed: "*Energy services are those functions performed using energy which are means to obtain or facilitate desired end services or states*". This definition encompasses examples of energy services that are commonly used (e.g., lighting, cooking, heating, cooling, transport, etc.).

that is, it is the negative environmental externalities. It is noteworthy that when the agent chooses the amount of energy service to be consumed, the agent implicitly decides on his energy consumption. The energy consumption of each agent is determined by the relationship: $E_i = \frac{S_i}{\varepsilon_i}$, where ε_i is the energy efficiency of the equipment used by agent i; and so $S_i = E_i \varepsilon_i$ (ROCHA; ALMEIDA, 2021; SORRELL; DIMITROPOULOS, 2008).

Therefore, the choice of the minimum level of utility ($\overline{u_i}$) is associated with a choice of a minimum amount of energy service consumption (and energy). Thus, this choice is very similar to the "affirmative principle" of energy justice developed by Sovacool, Sidortsov, and Jones (2014, p. 46), which states: "*if any of the basic goods to which every person is justly entitled can only be secured by means of energy services, then in that case there is also a derivative right to the energy service*". However, the minimum level of utility (or energy service consumption) does not necessarily refer to the minimum level necessary to sustain life. The agents choose the minimum level that they deem necessary for their well-being.

Using equation (2.2), energy security can be defined by the following equation:

$$P\big(W\big(u_1(S_1^*, E_{-1}^*), \dots, u_I(S_I^*, E_{-I}^*)\big) - W(\overline{u_1}, \dots, \overline{u_I}) \geq 0\big) \geq P^{min} \qquad \textbf{(2.3)}$$

The term $P^{ES} = P(W^* - \overline{W} \geq 0)$ will be called degree of energy security. On the other hand, the complementary event $P^{EI} = 1 - P^{ES} = P(W^* - \overline{W} < 0)$, will be called degree of energy insecurity.

The economy is said to be in a condition (or situation) of energy security when the inequality expressed in equation (2.3) is true, that is, when $P^{ES} \geq P^{min}$ or $P^{EI} \leq P^{max}$. On the other hand, it is said that the economy is in a condition of energy insecurity when the opposite inequality expressed in equation (2.3) is verified, that is, when $P^{ES} < P^{min}$ or $P^{EI} > P^{max}$. Therefore, energy security is a condition experienced by the economy at a point in time. It is not an action or policy.[36]

Therefore, the economy is either in a condition of energy security or energy insecurity. There is no middle ground (or partial security condition). Nevertheless, the economy may be in a more or less secure condition, that is, the degree of energy security may be greater or lesser, in such a way that the damage to the acquired values may be more or less likely. Thus, energy

[36] It is noteworthy that some authors have defined energy security as an action, such as the four 'R's (review, reduce, replace and restrict) proposed by Hughes (2009). The understanding of energy security as a policy (or action) clearly contributes to the lack of consensus on its definition, since there may be numerous policies to achieve an energy security condition.

security has a relative aspect (a greater or lesser degree of energy security) and an absolute aspect (a condition of energy security or insecurity).

2.4 The subjectivity of energy security

Equation (2.3) exposes the subjectivity of energy security. The choice of the social welfare function is effectively a choice between alternative sets of ethical values. What is the method used to aggregate the individual utilities? The aggregation can be done using only the ordinal meaning of the utilities, such as the criteria of majority voting, oligarchy, and dictatorship, or the aggregation can be done using the cardinal meaning, such as utilitarian, generalized utilitarian, rawlsian or constant elasticity forms (MAS-COLELL; WHINSTON; GREEN, 1995).[37]

Thus, the choice of the social welfare function is closely related to the concept of energy justice developed by Sovacool, Sidortsov, and Jones (2014), that is, energy security includes an energy justice dimension. Energy justice is, in part, about the distribution of energy services as a social good, but it is also about how the harms of energy production and use are allocated (SOVACOOL; SIDORTSOV; JONES, 2014). For example, if the preferences are aggregated through the dictatorial criteria and if the dictator does not suffer from losses of well-being due to environmental damages, this economy will not include the environmental dimension of energy security, even if all other agents suffer great losses of well-being due to environmental damages. Also, the dictatorial criteria imply that the energy poverty dimension is not included, since what matters, in this case, is the dictator's energy service consumption. Therefore, energy security may not correspond to equity between individuals; this will depend on the ethical values of society.

Furthermore, the social probability function presents the same concern and personal judgments as the social welfare function. How are the agents' maximum values for insecurity probability (P_i^{max}) aggregated? That is, who defines what is secure or insecure (i.e., P^{min})? In addition, the acquired values of the economy ($\overline{W}$) also depend on the personal judgment of each agent on their minimum level of well-being, as well as how these levels are aggregated. On this score, then, matters of opinion really are involved in the energy security definition. They rightfully belong in the very first stage of the energy security concept. This means that, although

[37] The cardinal meaning of utility, that is, the idea that intensity of preference can be compared in a coherent way across individuals is controversial at best.

it is a universal concept, energy security is highly context-dependent and can have different meanings.

The subjectivity implies that energy security is an ordinal measure rather than a cardinal one. Thus, there is some difficulty in comparing different degrees of energy security. Since each economy has its own forms of social welfare functions and the utility is essentially an ordinal measure, comparing the degrees of energy security for different economies at a point in time does not make much sense. Nevertheless, the comparison of the energy security over time can be performed. Notwithstanding, the comparison between distant periods of time can present some difficulties, since the social welfare and utility functions can change over time. This would explain the results found in Narula and Reddy (2015) that the ranking of energy security between countries varies depending on its measurement, although its measurement over time is fairly consistent.

2.5 The meaning of energy security definition

In order to understand the meaning of energy security in detail, we will make some simplifying assumptions. We will use the hypothesis of small open economy in a partial equilibrium model. We will also assume that there is a single representative consumer and a single representative firm that produces energy. Thus, we will put aside the relationship between energy justice and energy security.

2.5.1 Social welfare

To introduce the negative externalities in our simplified model, we will make the following assumption: the consumer is ignorant of the negative effects that energy consumption has on well-being. This means that the utility function depends on energy service consumption, but the social welfare function, in addition to depending on it, also depends negatively on energy consumption. Therefore, $W(S, E) = u(S) - g(E)$, where $g(.)$ is the environmental externality function. As $S = \varepsilon E$, social welfare function can be rewritten as $W(\varepsilon E, E) = u(\varepsilon E) - g(E)$.

We also assume that $u(0) = 0$, $u'(S) > 0$, $u''(S) < 0$, $g(0) = 0$, $g'(E) > 0$ and $g''(E) > 0$. Given these assumptions, it is easy to see that the social welfare function has a maximum ($W^{max} = W(\varepsilon E^{max}, E^{max}) > 0$), which is obtained when $\varepsilon u'(\varepsilon E) = g'(E)$. This maximum value can be understood as a sort of Pareto optimal allocation. However, as the consumer chooses the level of consumption taking into account only its utility, just by chance the energy consumption will occur at the optimum point. Thereby, we will assume that the consumer chooses a positive minimum level of social welfare, but different from the maximum level: $0 < \overline{W} < W^{max}$. That is, the consumer chooses a minimum level of social welfare in which the utility derived from the energy consumption more than compensates for the loss of welfare due to environmental damages.

2.5.2 Energy demand and energy supply

As we are using a partial equilibrium model, let's assume that the consumer's demand function for energy services is equal to $S\left(\frac{p_E}{\varepsilon}, Z_d\right)$, where p_E is the energy price and Z_d represents another variable that affects this function (e.g., income, exogenous prices of non-energy goods, or an energy conservation parameter).[38] Therefore, the energy demand function is equal to: $E_d = \frac{S\left(\frac{p_E}{\varepsilon}, Z_d\right)}{\varepsilon}$.[39] Regarding the energy supply function, let's assume that it is given by $E_s(p_E, Z_s)$, where Z_s represents another variable that affects this function (e.g., exogenous prices of inputs, technical change, level of reserves, or an extraction difficulty parameter).[40]

2.5.3 Energy market equilibrium in the small open economy

The small open economy assumption implies that changes in a country's demand for imports or its supply of exports have negligible effects on the world market, so the international energy price is not affected by the small country. That is, the international energy price is exogenous ($\overline{p_E}$) and the economy is price-taker. As the majority of the energy security studies focus on large energy importing countries (ANG; CHOONG; NG, 2015a), we will assume that the international energy price is always less than or equal to the equilibrium energy price in a closed economy. Thus, the economy will be an energy importer. Therefore, in the equilibrium of the small open economy, energy consumption (E_d^*), domestic production (E_s^*) and imports (M^*) are given, respectively, by: $E_d^* = \frac{S_d^*}{\varepsilon}$, where $S_d^* = S\left(\frac{\overline{p_E}}{\varepsilon}, Z_d\right)$, $E_s^* = E_s(\overline{p_E}, Z_s)$, and $M^* = E_d^* - E_s^*$ (and so $E_d^* = E_s^* + M^*$).[41] Moreover, social welfare in the equilibrium is equal to $W^* = u(\varepsilon E_d^*) - g(E_d^*)$.

[38] It is noteworthy that $p_S = \frac{p_E}{\varepsilon}$ is the implicit price of the energy service (SORRELL; DIMITROPOULOS, 2008).

[39] For more details on the microeconomic foundations of energy demand functions with the same structure see Chan and Gillingham (2015) and Sorrell and Dimitropoulos (2008).

[40] It is noteworthy that Z_d and Z_s represent, respectively, changes in consumption and domestic production of energy that are not induced by the energy price.

[41] It is noteworthy that the international energy price is in the same currency unit as the domestic energy prices. That is, the exchange rate that transforms the international currency unit (e.g., dollar) into domestic currency units

2.5.4 Threats, energy supply disruption, and random variables

Following Martišauskas, Augutis, and Krikštolaitis (2018), a threat is defined as any potential danger that exists within or outside the energy system and that has the potential to result in a disruption in that system. Baldwin (1997) argues that his fifth question of security concept (From what threats?) must be answered by specifying the threats to acquired values. Nevertheless, as the energy system is highly complex, the number of threats to the energy supply chain is huge. As Ciută (2010) points out, in conceptual terms, there is an infinite number of targets that are subject to an infinite number of vulnerabilities; and therefore energy security means the security of everything, everywhere, against everything. For example, solar storms can produce a massive blackout, such as the power blackout in Québec, Canada, in March 1989 (BOTELER, 2019; BOTELER; PIRJOLA; NEVANLINNA, 1998). Thereby, in conceptual terms, it is impossible to specify all threats to acquired values.[42] Nevertheless, threats can be divided into three broad categories: technical, natural, and socio-political (MARTIŠAUSKAS; AUGUTIS; KRIKŠTOLAITIS, 2018). Technical threats are endogenous to the energy system, such as failures in its components (e.g., transmission lines, power plants, or transformers) or accidents caused by unintentional human error. Natural threats are those caused by the Environment, such as natural disasters. Socio-political threats are related to geopolitical issues and can be malicious (e.g., sabotage, terrorism, or export embargoes) or non-malicious (e.g., political instability or wars in energy-exporting countries).

The idea of avoiding sudden changes in the availability of energy supplies in relation to demand, that is, avoiding energy supply disruptions, is an aspect shared by many energy security studies (WINZER, 2012). Similar to Beccue and Huntington (2005), we will define energy supply disruption (ESD) as a physical constraint (measured as a percentage) imposed on the current energy supply chain (i.e., domestic production and import) of an economy that occurs when a threat is materialized.[43] It is noteworthy that when the physical constraint ends,

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is implicitly embedded in the international energy price. Thus, an exchange rate devaluation results in an increase in the international energy price accounted for in domestic currency. This fact demonstrates the importance of external accounts (balance of payments) and the countries' financial import capacity for energy security, something that has already been highlighted by Radovanović, Filipović, and Golušin (2018) and Aslanturk and Kıprızlı (2020).

[42] Of course, when the goal is to implement energy security policies, the identification of the main threats is paramount.

[43] Some examples of ESD are: the loss of 5.6 million barrels of crude oil per day due to the 1978-79 Iranian Revolution, the disruption of 1.5 million barrels of crude oil per day due to Hurricane Katrina in 2005 and the gas supply disruption of about 7 billion cubic meters that occurred in Europe in 2009 due to Russia's decision to suspend gas supply to Ukraine (IEA, 2014).

domestic energy production and energy imports can recover the initial level and so ESD is not a displacement in the energy supply.[44]

As we are using the small open economy hypothesis, we will assume that ESD imposed on the energy import and the domestic energy production do not affect the international energy price. Thus, to introduce ESD in our simplified model, we will assume that there are two independent and continuous random variables that represent the ESD in a given period of time[45]: D_{E_s} is the ESD in the internal energy supply chain (i.e., domestic production) and D_M is the ESD in the external energy supply chain (i.e., import). ESD cannot be a negative value (energy is not earned for free) and cannot be a value greater than the unit (energy that is not produced is not disrupt). Also, we will assume that each probability density function depends conditionally on a parameter that represents the level of threats on the energy supply chain: where t_{E_s} is the level of threats on the domestic energy production and t_M is the level of threats on the energy imports. For simplicity, we will standardize the level of threats between 0 and 1, that is, $t_i \in (0,1)$, where $t_i \to 1$ means an imminent potential danger and $t_i \to 0$ means a remote potential danger.

Therefore, the probability density functions are equal to $f_{D_i}(x_i|t_i) \geq 0$ if $x_i \in [0,1]$, and $f_{D_i}(x_i|t_i) = 0$ if $x_i \notin [0,1]$. Thus, the cumulative distribution function is represented by: $F_{D_i}(y_i|t_i) = \begin{cases} 1 & y_i > 1 \\ \int_0^{y_i} f_{D_i}(x_i|t_i)dx_i & y_i \in [0,1] \\ 0 & y_i < 0 \end{cases}$. Furthermore, the definition of threat implies that a lower level of threats is associated with a lower chance of ESD. To represent this phenomenon, we will assume that if $t_i^* < t_i^{**}$, then $F_{D_i}(y_i|t_i^*) \geq F_{D_i}(y_i|t_i^{**})\ \forall y_i$. That is, the distribution $F_{D_i}(y_i|t_i^*)$ first-order stochastically dominates $F_{D_i}(y_i|t_i^{**})$ whenever $t_i^* < t_i^{**}$. In addition, in this case, it is said that the energy supply chain associated with t_i^* is more secure. Thus, if there is a lower level of threats, the chances of energy supply disruption are small and then the energy supply chain is more secure.

The values $D_{E_s} = x_{E_s}$ and $D_M = x_M$ mean that the amount of energy that is disrupted is equal to $E_d^D = E_s^* x_{E_s} + M^* x_M$, in such a way that the amount of energy actually supplied (or consumed) is $E_d^A = E_d^* - E_d^D = E_s^*(1 - x_{E_s}) + M^*(1 - x_M)$. In addition, the ESD is equal to

[44] For example, low energy production caused by a lack of investment in the energy sector is not an ESD, and neither is a lack of investment a threat. Nevertheless, this does not mean that the lack of investments is not a concern for energy security. Investments to expand energy resources are extremely important for energy security, as they affect the energy supply function.

[45] The time aggregation embedded in the utility function.

$\frac{E_d^D}{E_d^*} = x_{E_s}\sigma + x_M(1-\sigma)$, where $\sigma = \frac{E_s^*}{E_d^*}$ is the self-sufficiency ratio and $(1-\sigma) = \frac{M^*}{E_d^*}$ is the dependence ratio. As illustrated in Figure 2.1, ESD by constraining domestic production and imports, generate a new market equilibrium (change from point A to point B), where E_d^A is the maximum amount of energy that can be supplied and, therefore, it will be the amount of energy consumed in the equilibrium and the price of energy will be equal to $p_{E_d^A} = \varepsilon S^{-1}(\varepsilon E_d^A, Z_d)$. Note that $S_d^A = \varepsilon E_d^A$ is the amount of energy service actually consumed.

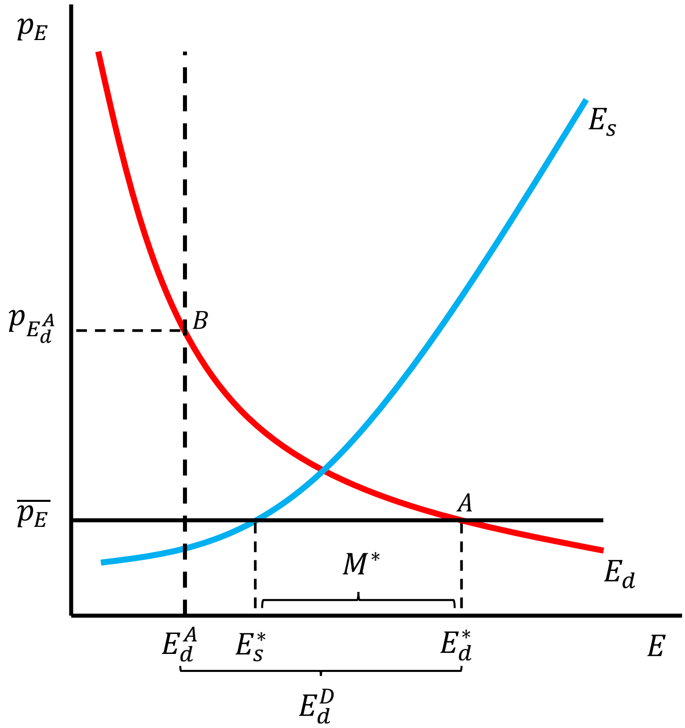

Figure 2.1 – Market equilibrium and energy supply disruption
Source: Author's own elaboration.

We will also assume that the ESD does not affect the environmental externality function. This is justified because the first stages of the energy supply chain generate a lot of environmental damage (e.g., burning of coal to generate electricity) and ESD can occur in later stages of the supply chain (e.g., transmission or distribution). This means that to reduce environmental harms, the reduction in energy consumption must be permanent, rather than a temporary reduction caused by a momentary constrain in the energy supply chain. Thereby, the social welfare function in the new market equilibrium is equal to $W(\varepsilon E, E_d^*) = u(\varepsilon E) - g(E_d^*)$ and its new equilibrium value is given by $W^A = u(\varepsilon E_d^A) - g(E_d^*)$.

2.5.5 Sufficient social welfare, adequate energy consumption, and affordable energy prices

We will refer to a level of social welfare W as sufficient when it is at least equal to the desired minimum level, that is, when $W \geq \bar{W}$. In addition, we will call the amount of energy consumption that produces the minimum level of social welfare and that compensates for the loss of social welfare in equilibrium due to environmental harms as security energy consumption (E^q). That is, $E^q = \frac{u^{-1}\left(\bar{W}+g(E_d^*)\right)}{\varepsilon}$ and so $\bar{W} = u(\varepsilon E^q) - g(E_d^*)$. Thus, we will refer to an amount of energy consumption E as adequate when it is at least equal to the security energy consumption, that is, when $E \geq E^q$. Similarly, we will call $S^q = u^{-1}\left(\bar{W} + g(E_d^*)\right)$ as security energy service consumption and therefore an amount of energy service consumption S is adequate when $S \geq S^q$. From another point of view, since energy consumption depends on the energy price through demand function, $E_d = \frac{S\left(\frac{p_E}{\varepsilon}, Z_d\right)}{\varepsilon}$, we will refer to an energy price p_E as affordable when it allows energy consumption to be adequate: $p_E \leq p_{E^q} = \varepsilon S^{-1}(\varepsilon E^q, Z_d)$.

2.5.6 Energy security definitions

Winzer (2012) argues that energy security definition can be expressed through three different lenses, that is, through energy consumption, energy service consumption, or social welfare. Nevertheless, equation (2.3) shows that energy security can be expressed in several equal ways, in such a way that the three lenses mentioned by Winzer (2012) are all faces of the same story. Given the assumptions, we can rewrite equation (2.3) as:

$$P(W^A \geq \bar{W}) \geq P^{min} \quad \textbf{(2.4)}$$

Thereby, energy security can be defined as a condition in which economic agents perceive a high probability that the social welfare will be sufficient. Nevertheless, equation (2.4) can be rewritten as $P(S_d^A \geq S^q) \geq P^{min}$. Thus, energy security can also be defined as a condition in which economic agents perceive a high probability that the energy services consumption will be adequate. Moreover, equation (2.4) can also be rewritten as $P(E_d^A \geq E^q) \geq P^{min}$. Therefore, a third way of defining it would be as a condition in which economic agents perceive a high probability that energy consumption will be adequate. Additionally, energy security can also be defined as a condition in which economic agents perceive a high probability that the energy price will be affordable, that is, $P\left(p_{E_d^A} \leq p_{E^q}\right) \geq P^{min}$. Therefore, the terms "sufficient social welfare", "adequate energy service consumption", "adequate energy consumption" and "affordable energy prices" are redundant in the energy security definition.

There is also a fifth way of defining energy security. Since $E_d^A = E_s^*\left(1 - x_{E_s}\right) + M^*(1 - x_M)$, equation (2.4) can be rewritten as:

$$P\left(D_{E_s}\sigma + D_M(1-\sigma) \le e\right) \ge P^{min} \qquad \textbf{(2.5)}$$

where $e = \frac{E_d^* - E^q}{E_d^*}$ is the tolerance capacity ratio. The tolerance capacity ratio represents the maximum percentage of energy consumption that an economy can tolerate losing in order to keep a sufficient level of social welfare. Thus, an ESD is tolerable when it is less than (or equal to) the tolerance capacity ratio, that is, when $\frac{E_d^D}{E_d^*} = x_{E_s}\sigma + x_M(1-\sigma) \le e$. Therefore, energy security can be defined as a condition in which economic agents perceive a high probability that ESD will be tolerable.

Furthermore, it is always possible to define energy security through the perspective of insecurity, that is, $P(W^A < \bar{W}) = P(S_d^A < S^q) = P(E_d^A < E^q) = P\left(p_{E_d^A} > p_{E^q}\right) = P\left(D_{E_s}\sigma + D_M(1-\sigma) > e\right) \le P^{max}$. This means replacing "high probability" for "low probability" and "will be" for "will not be" in the energy security definitions presented above.

2.6 The necessary condition for energy security

There is a necessary (but not sufficient) condition for the economy to be in an energy security situation, that is, equilibrium energy consumption must be adequate: $E_d^* \ge E^q$. Equivalently, this condition also means that, in the equilibrium, the social welfare must be sufficient ($W^* \ge \bar{W}$), the energy services consumption must be adequate ($S^* \ge S^q$) and the energy price must be affordable ($\overline{p_E} \le p_{E^q}$). Figure 2.2 illustrates this fact. As $E_d^A \le E_d^*$, since $x_i \in [0,1]$ ($i = E_s, M$), then $W^A \le W^*$. Thus, if $E_d^* = E_{d_2}^*$ is not adequate (i.e., $E_d^* \notin [E_1, E_2]$) then $W^* = W_2^* < \bar{W}$. In this case, $W^A \le W_2^* < \bar{W}$ and therefore $P(W^A \ge \bar{W}) = 0$. Thus, if the agents desire some energy security, that is, if $P^{min} > 0$, the economy will always be in an energy insecurity situation when equilibrium energy consumption is not adequate. In other words, if the economy is not able to generate the desired minimum level of social welfare without ESD, it will not be able to generate it with ESD. On the other hand, if $E_d^* = E_{d_1}^*$ is adequate (i.e., $E_d^* \in [E_1, E_2]$) then $W^* = W_1^* > \bar{W}$. Thus, depending on the value of ESD, W^A

can be greater or less than the desired minimum level. That is, if $E_d^A \in \left[E_1^q\ , E_{d_1}^*\ \right]$, then $W^A \geq \overline{W}$, whereas, if $E_d^A \in \left[0, E_1^q\right)$, then $W^A < \overline{W}$.

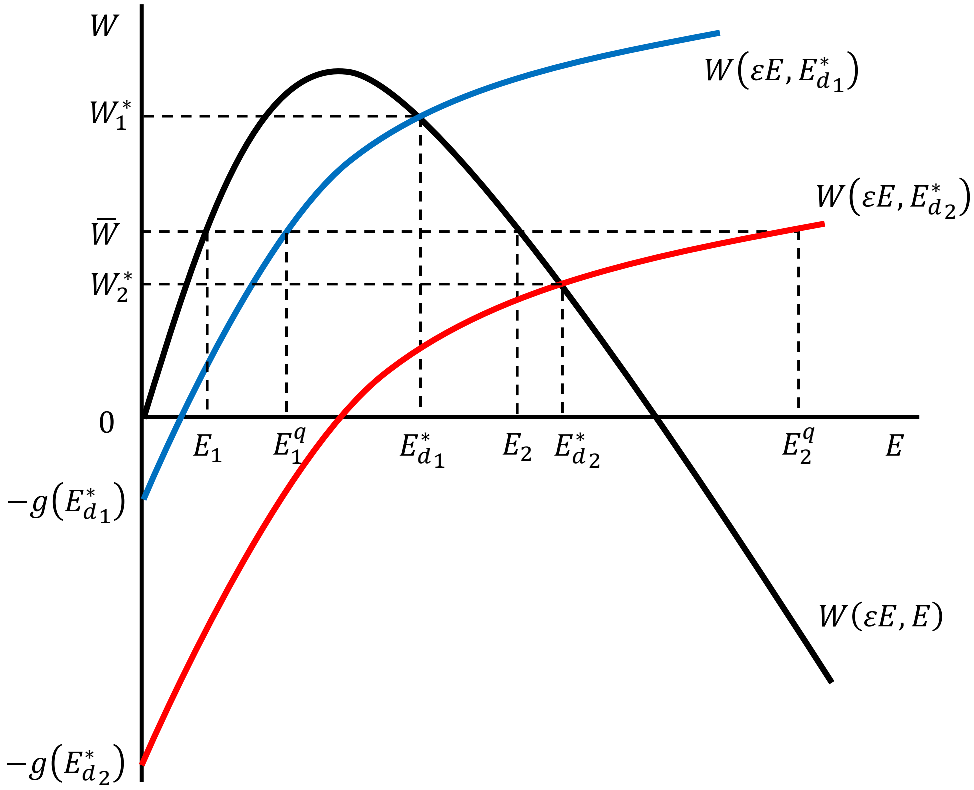

Figure 2.2 – The necessary condition for energy security and adequate energy consumption
Source: Author's own elaboration.

Figure 2.2 also shows that the necessary condition for energy security does not mean that the amount of energy consumed in the equilibrium must be large. Indeed, this condition requires that $E_d^* \in [E_1, E_2\ ]$, where E_1 and E_2 are the two roots that solve $u(\varepsilon E) - g(E) - \overline{W} = 0$. That is, the equilibrium energy consumption cannot be too small, to the point of not producing the minimum level of social welfare, nor too great, in such a way that the environmental harms generate an excessive loss of social welfare. Thus, the condition that the energy price must be affordable does not mean that energy must be cheap, but that $\overline{p_E} \in \left[p_{E_2}, p_{E_1}\ \right]$, where $p_{E_1} = \varepsilon S^{-1}(\varepsilon E_1, Z_d)$ and $p_{E_2} = \varepsilon S^{-1}(\varepsilon E_2, Z_d)$. However, if the environmental dimension is omitted (i.e., $g(E) = 0\ \forall E$), the necessary condition will mean that a large amount of energy must be consumed ($E_d^* \geq E^q = \frac{u^{-1}(\overline{W})}{\varepsilon}$) and that the energy must be cheap ($\overline{p_E} \leq p_{E^q} = \varepsilon S^{-1}(u^{-1}(\overline{W}), Z_d)$).

The necessary condition for energy security can also be demonstrated through equation (2.5). Since $\sigma \in [0,1]$ and $x_i \in [0,1]$ $(i = E_s, M)$, hence $x_{E_s}\sigma + x_M(1-\sigma) \in [0,1]$. Thus, if

$E_d^* < E^q$, which implies $e < 0$, therefore $P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq e\right) = 0$. That is, the necessary condition for energy security also means that the tolerance capacity ratio must be a positive value. In this way, the definition expressed in equation (2.5) does not only mean "being protected from threats that cause ESD". It does not matter how much the energy supply chain is protected from ESD, that is, it does not matter how large is $P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq z\right)$ for any $z \in [0,1]$, if the energy consumption is not adequate (i.e., if $e < 0$). In this case, the economy will always be in an energy insecurity situation, no matter how protected it is from threats. Thus, the meaning behind energy security definition is to be protected from threats that cause ESD while having adequate equilibrium energy consumption.[46]

2.7 Self-sufficiency, energy independence and energy security

We will distinguish the concepts of self-sufficiency and energy independence from the concept of energy security. Self-sufficiency is a condition in which the energy consumption is supplied only by domestic energy production ($E_s^* = E_d^* \Leftrightarrow \sigma = 1$), that is, energy import is null ($M^* = 0 \Leftrightarrow 1 - \sigma = 0$). Energy independence is a condition in which economic agents perceive a high probability that domestic energy production will generate a sufficient level of social welfare. In other words, energy independence is a scenario in which energy security is assessed in the context of a complete disruption on energy imports. That is, if an ESD ceased all imported energy, would the economy still be in a condition of energy security? In other words, how much is energy security independent of energy imports?

Mathematically, energy independence is defined as $P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq e \middle| D_M = 1\right) \geq P^{min}$. Therefore, energy independence is a particular case of energy security. That is, if an economy is in a condition of energy independence, then it is also in a condition of energy security.[47] However, the reverse is not true. It is noteworthy that self-sufficiency does not imply energy independence and vice versa. When an economy is self-sufficiency ($\sigma = 1$), it just means that its energy independence is equal to energy security ($P\left(D_{E_s} \leq e\right) \geq P^{min}$). Furthermore, similar to energy security, energy independence can also be defined in several ways. In addition, there is also a necessary condition for energy

[46] This statement demonstrates that energy security can always be defined using synonyms. This means that, in order to the energy security definition to be rigorous, it cannot be defined using just words. That is, the energy security definition must be measurable.
[47] It is easy to see that $P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq e\right) \geq P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq e \middle| D_M = 1\right)$ (see Appendix 2.A).

independence, that is, domestic energy production must be adequate: $E_S^* \geq E^q$ (if and only if $e \geq (1-\sigma)$).[48]

2.8 Preferable energy security strategies

The degree of energy security in equation (2.5) can be expressed as:

$$P\big(D_{E_S}\sigma + D_M(1-\sigma) \leq e\big) = \iint\limits_{A_{(\sigma,e)}} f_{D_{E_S}}\big(x_{E_S}\big|t_{E_S}\big) f_{D_M}(x_M|t_M)\, dx_{E_S} dx_M \qquad \textbf{(2.6)}$$

where $A_{(\sigma,e)} = \{(x_{E_S}, x_M) | x_{E_S}\sigma + x_M(1-\sigma) \leq e, 0 \leq x_{E_S} \leq 1, 0 \leq x_M \leq 1\}$. We will denote $P^{ES}\big(A_{(\sigma,e)}, t_{E_S}, t_M\big) = \iint_{A_{(\sigma,e)}} f_{D_{E_S}}\big(x_{E_S}\big|t_{E_S}\big) f_{D_M}(x_M|t_M)\, dx_{E_S} dx_M$.

An energy security strategy is a vector $(\sigma, e, t_{E_S}, t_M)$ that generates a certain value for $P^{ES}\big(A_{(\sigma,e)}, t_{E_S}, t_M\big)$. In other words, an energy security strategy is the featured (or measure) of its dimensions that generate a certain degree of energy security. We will define $(\sigma_i, e_i, t_{E_S}^i, t_M^i)$ as an energy security strategy preferable to $(\sigma_j, e_j, t_{E_S}^j, t_M^j)$, when $P^{ES}\big(A_{(\sigma_i,e_i)}, t_{E_S}^i, t_M^i\big) \geq P^{ES}\left(A_{(\sigma_j,e_j)}, t_{E_S}^j, t_M^j\right)$, and we will denote it as $(\sigma_i, e_i, t_{E_S}^i, t_M^i) \succsim (\sigma_j, e_j, t_{E_S}^j, t_M^j)$. This means that the strategy $(\sigma_i, e_i, t_{E_S}^i, t_M^i)$ is at least as good for energy security as the strategy $(\sigma_j, e_j, t_{E_S}^j, t_M^j)$. Similarly, we will denote $(\sigma_i, e_i, t_{E_S}^i, t_M^i) \precsim (\sigma_j, e_j, t_{E_S}^j, t_M^j)$, when $P^{ES}\big(A_{(\sigma_i,e_i)}, t_{E_S}^i, t_M^i\big) \leq P^{ES}\left(A_{(\sigma_j,e_j)}, t_{E_S}^j, t_M^j\right)$; and, $(\sigma_i, e_i, t_{E_S}^i, t_M^i) \backsim (\sigma_j, e_j, t_{E_S}^j, t_M^j)$, when $P^{ES}\big(A_{(\sigma_i,e_i)}, t_{E_S}^i, t_M^i\big) = P^{ES}\left(A_{(\sigma_j,e_j)}, t_{E_S}^j, t_M^j\right)$, that is, when strategies are equally preferable.

The preferable energy security strategies answer Baldwin's (1997) sixth question. That is, this shows by what means the degree of energy security can be improved. When the necessary condition for energy security is met ($E_d^* \geq E^q$)[49], it is possible to identify two situations where one strategy will always be preferable to another. First, for a given self-

[48] This is because $P\big(D_{E_S}\sigma + D_M(1-\sigma) \leq e \big| D_M = 1\big) = P\left(D_{E_S} \leq \frac{e-(1-\sigma)}{\sigma}\right)$, where $\frac{e-(1-\sigma)}{\sigma} = \frac{E_S^* - E^q}{E_S^*}$. Therefore, if $E_S^* < E^q$, then $P\big(D_{E_S}\sigma + D_M(1-\sigma) \leq e \big| D_M = 1\big) = 0$.

[49] In this case, we have $e \in [0,1)$. Note that $E_d^* \geq E^q$ implies $e \geq 0$. On the other hand, we have $e < 1$ by definition. This is because $e \geq 1$ if and only if $E^q \leq 0$. However, given the assumptions on the utility function (i.e., $u(0) = 0$ and $u'(S) > 0$), for $E^q = \frac{u^{-1}\left(\overline{W} + g(E_d^*)\right)}{\varepsilon} \leq 1$ it would be necessary that $\overline{W} + g(E_d^*) \leq 0$. Notwithstanding, the assumptions on the minimum level of social welfare (i.e., $0 < \overline{W} < W^{max}$) and on the environmental externality function (i.e., $g(0) = 0$ and $g'(E) > 0$) guarantee $\overline{W} > 0$ and $g(E_d^*) \geq 0$ and, therefore, $\overline{W} + g(E_d^*) > 0$. Thus, $E^q > 0$ and $e < 1$.

sufficiency ratio and tolerance capacity ratio, a reduction in the level of threats implies a more preferable strategy. That is, as shown in Appendix 2.A, when $t_M^i < t_M^j$, then $(\sigma, e, t_{E_s}, t_M^i) \gtrsim (\sigma, e, t_{E_s}, t_M^j)$; and when $t_{E_s}^i < t_{E_s}^j$, then $(\sigma, e, t_{E_s}^i, t_M) \gtrsim (\sigma, e, t_{E_s}^j, t_M)$. Second, as illustrated in the Figure 2.3, for a given level of threats, when $\frac{e_i}{\sigma_i} \geq \frac{e_j}{\sigma_j}$ and $\frac{e_i}{(1-\sigma_i)} \geq \frac{e_j}{(1-\sigma_j)}$, then $A_{(\sigma_i,e_i)} \supseteq A_{(\sigma_j,e_j)}$ and therefore $(\sigma_i, e_i, t_{E_s}, t_M) \gtrsim (\sigma_j, e_j, t_{E_s}, t_M)$. Similarly, when $\frac{e_i}{\sigma_i} \leq \frac{e_j}{\sigma_j}$ and $\frac{e_i}{(1-\sigma_i)} \leq \frac{e_j}{(1-\sigma_j)}$, then $A_{(\sigma_i,e_i)} \subseteq A_{(\sigma_j,e_j)}$ and therefore $(\sigma_i, e_i, t_{E_s}, t_M) \lesssim (\sigma_j, e_j, t_{E_s}, t_M)$. In addition, when $\frac{e_i}{\sigma_i} = \frac{e_j}{\sigma_j}$ and $\frac{e_i}{(1-\sigma_i)} = \frac{e_j}{(1-\sigma_j)}$, then $A_{(\sigma_i,e_i)} = A_{(\sigma_j,e_j)}$ and therefore $(\sigma_i, e_i, t_{E_s}, t_M) \backsim (\sigma_j, e_j, t_{E_s}, t_M)$. This means that when the derivatives of $\frac{e}{(1-\sigma)}$ and $\frac{e}{\sigma}$ with respect to some exogenous variable of the demand or supply functions are positive, that is, when $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial y} \geq 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial y} \geq 0$ where $y = \overline{p_E}, \varepsilon, Z_d, Z_s$, then an increase in this variable result in a more preferable strategy. Otherwise, when $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial y} \leq 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial y} \leq 0$, an increase in this variable result in a less preferable strategy. Also, when $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial y} = 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial y} = 0$, changes in this variable result in equally preferable strategy.

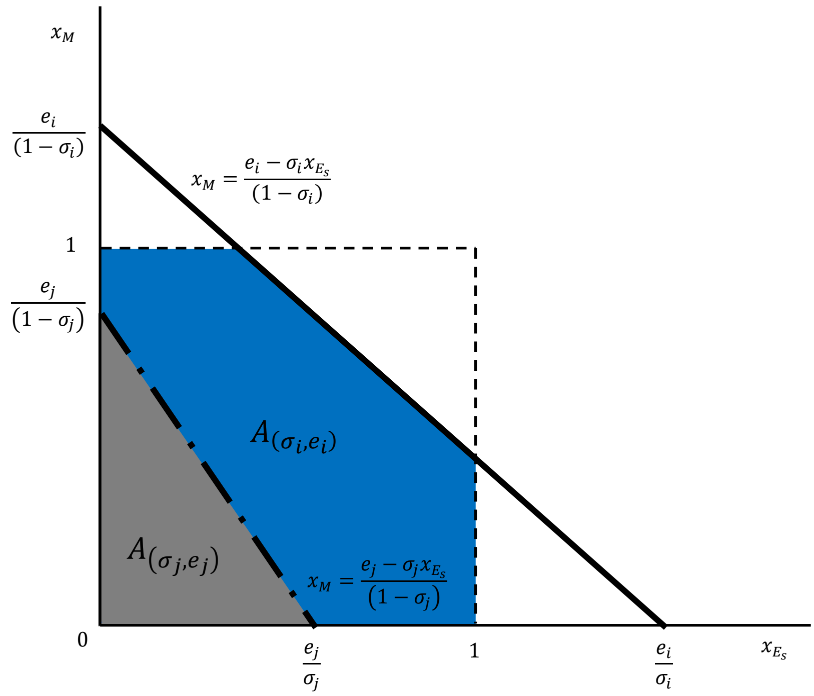

Figure 2.3 – Preferable energy security strategies and the integration area for the probability of the degree of energy security

Source: Authors' own elaboration.

2.9 Energy price and Energy Security

As IEA (2014) and Bohi and Toman (1996) point out, since physical constraints in the energy supply chain are generally limited to extreme events, energy security concerns are primarily related to extreme price spikes. To understand the relationship between energy price and energy security, it is necessary to identify whether a change in price results in a more (or less) preferable strategy. For this, we will assume that the necessary condition for energy security is met ($E_d^* \geq E^q$). Thus, as energy prices do not affect the level of threats[50], it is necessary to identify if $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} \geq 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} \geq 0$, or if $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} \leq 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} \leq 0$. The following equations show, respectively, the derivatives $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}}$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}}$ (see Appendix 2.B):

$$\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial \overline{p_E}}}{\sigma E_d^*}\left[\frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}} - 1 - \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)e\right] \tag{2.7}$$

$$\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial \overline{p_E}}}{(1-\sigma)E_d^*}\left\{\frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}} - 1 + \left[1 + \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)\right]\frac{e}{(1-\sigma)}\right\} \tag{2.8}$$

Since $\frac{\partial E_s^*}{\partial \overline{p_E}} > 0$ and $\frac{\partial E_d^*}{\partial \overline{p_E}} < 0$, equations (2.8) and (2.7) show that if $\frac{\partial g(E_d^*)}{\partial E_d^*} \geq \frac{\partial u(\varepsilon E^q)}{\partial E^q}\left[1 + \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)e\right]$, then $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} > 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} \geq 0$.[51] This situation occurs when: 1) the equilibrium energy consumption is excessive, that is, when a reduction in it results in gains in social welfare: $E^{max} < E_d^*$ and so $\frac{\partial g(E_d^*)}{\partial E_d^*} > \frac{\partial u(\varepsilon E_d^*)}{\partial E_d^*}$; 2) the minimum desired level of social welfare is close to its equilibrium level, which implies that the security energy consumption is

[50] Although the level of threats is exogenous in our model, Huntington (2011) points out that the revenue received by oil-exporting countries may finance terrorism or belligerent dictators controlling oil resources. In this context, the international energy price would influence the level of threats on energy imports, since it impacts on the revenues received by energy-exporting countries.

[51] This is because $\frac{\partial g(E_d^*)}{\partial E_d^*} > \frac{\partial u(\varepsilon E^q)}{\partial E^q}\left[1 + \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)e\right] > \frac{\partial u(\varepsilon E^q)}{\partial E^q} > \frac{\partial u(\varepsilon E^q)}{\partial E^q}\left\{1 - \left[1 + \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)\right]\frac{e}{(1-\sigma)}\right\}$.

close to the equilibrium energy consumption: $E^{max} < E^q \leq E_d^*$ and then $\frac{\partial g(E_d^*)}{\partial E_d^*} > \frac{\partial u(\varepsilon E^q)}{\partial E^q} \geq \frac{\partial u(\varepsilon E_d^*)}{\partial E_d^*}$; and 3) the energy consumed has a high environmental impact: $g'(E)$ is a large enough value for all E. Under these conditions, increases (decreases) in the international energy price improve (worsen) the degree of energy security.

On the other hand, equations (2.8) and (2.7) show that if $\frac{\partial g(E_d^*)}{\partial E_d^*} \leq \frac{\partial u(\varepsilon E^q)}{\partial E^q}\left\{1 - \left[1 + \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)\right]\frac{e}{(1-\sigma)}\right\}$, then $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} \leq 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} < 0$. In this case, increases (decreases) in the international energy price worsen (improve) the degree of energy security. This situation occurs when the opposite of conditions 1 and 3 presented above is observed. That is, when energy consumption is scarce ($E^{max} > E_d^* \geq E^q$ and consequently $\frac{\partial g(E_d^*)}{\partial E_d^*} < \frac{\partial u(\varepsilon E_d^*)}{\partial E_d^*} \leq \frac{\partial u(\varepsilon E^q)}{\partial E^q}$) and when it has a low environmental impact ($g'(E)$ is a small enough value for all E).

Also, when $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} > 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} < 0$ or when $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} < 0$ and $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} > 0$, it is not possible to identify at first whether increases in the international energy price improve or worsen the degree of energy security. Furthermore, price changes can affect other variables that are not being accounted for here. For example, price changes can affect energy efficiency and investments to expand energy resources. Therefore, the relationship between energy prices and energy security cannot be determined at first.

However, there is a situation where increases in the energy price generally worsen the degree of energy security. This situation occurs when the domestic energy supply is fixed (i.e., $E_s(p_E, Z_s) = \overline{E_s}$ and then $\frac{\partial E_s^*}{\partial \overline{p_E}} = 0$) and the environmental harms are not perceived by the agents (i.e.,$g(E) = 0\ \forall E$ and then $\frac{\partial g(E_d^*)}{\partial E_d^*} = 0$). This situation can be understood as short-term energy security. In this case, equations (2.7) and (2.8) can be rewritten, respectively, as:

$$\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} = \frac{\frac{\partial E_d^*}{\partial \overline{p_E}}}{\sigma E_d^*} \qquad \textbf{(2.9)}$$

$$\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial\overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial\overline{p_E}}}{(1-\sigma)E_d^*}\left[\frac{e}{(1-\sigma)} - 1\right] \quad \mathbf{(2.10)}$$

Since $\frac{\partial E_d^*}{\partial\overline{p_E}} < 0$, equation (2.9) yields: $\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial\overline{p_E}} < 0$. Equation (2.10) shows that: if $\frac{e}{(1-\sigma)} \geq 1$, then $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial\overline{p_E}} \geq 0$, otherwise, if $\frac{e}{(1-\sigma)} < 1$, then $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial\overline{p_E}} < 0$. However, $\frac{e}{(1-\sigma)} \geq 1$ occurs if and only if $\overline{E_s} \geq E^q = \frac{u^{-1}(\overline{W})}{\varepsilon}$. This means that necessary condition for energy independence is meet, that is, even if all imported energy is disrupted, domestic energy production can maintain a sufficient level of social welfare, that is, $\overline{E_s} \geq E^q$ implies that $u\left(\varepsilon\overline{E_s}\right) \geq \overline{W}$. This is a somewhat unrealistic hypothesis for countries that depend excessively on energy imports. As the large importing countries are, in general, the focus of the literature (ANG; CHOONG; NG, 2015a), as a rule, we have that $\overline{E_s} < E^q$ and consequently $\frac{e}{(1-\sigma)} < 1$, in such a way that $\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial\overline{p_E}} < 0$. Therefore, in general, increases (decreases) in the international energy price worsen (improve) the degree of energy security in the short term.

2.10 Conclusion

This article developed a simplified energy security model that combines economic theory and the concept of security in a probabilistic framework. The article seeks to contribute to the broadening of the theoretical foundation of energy security, by clarifying some points of the two main gaps in the literature, namely: the lack of consensus in its definition and the lack of a rigorous methodological framework.

Our simplified model shows that energy security is a universal concept, but it has several meanings. The agents' choices about the minimum desired level of utility and the maximum values for insecurity probability, as well as the forms of aggregation of utilities and probabilities, make energy security a highly subjective concept. For example, depending on the social welfare function, the concept of energy security may include the environmental and energy poverty dimensions in some cases, while these dimensions may be omitted in others. This means that personal judgments are an integral part of the energy security definition. Thus, energy security is an ordinal measure rather than a cardinal one. However, energy security is not just a matter of opinion; there is consistency in its reasoning, ranging from premises to conclusions and then to prescriptions. Our model also shows that the meaning of energy security can be expressed through different lenses, regardless of its subjectivity. That is, energy security can be defined using social welfare, energy service consumption, energy consumption, energy price, or energy supply disruptions, as well as the perspective of security or insecurity. Furthermore, the importance of each dimension of energy security varies for different economies. This is because each economy will present different values for each parameter of the model. For example, energy efficiency can be high and energy import dependency can be small in some countries, while the opposite may occur in another. All these facts would explain the existence of several energy security definitions in the literature. Of course, the lack of concern to define energy security rigorously in the literature also contribute to this.

In addition, our simplified model incorporates the multidimensionality of energy security in a rigorous methodological framework, in such a way that it allows an integration of different dimensions of energy security. Although the model does not include all dimensions, as they are numerous, it presents a logical mechanism that determines how the different dimensions interact with each other and consequently how they affect energy security. That is, albeit our simplified model imposes that the level of threats is exogenous and it does not include all parameters that affect the energy demand and supply functions, once a change in one of these variables is identified, the model determines rationally how energy security will be affected. This is illustrated by the relationship between energy price and energy security. Despite this relationship cannot be determined in general, it is possible to determine, via the concept of preferable energy security strategy, in which situations energy price increases improve or worsen the degree of energy security.

As Cherp and Jewell (2011) points out, energy security challenges have their roots in different disciplines. Our simplified model is just a first attempt to integrate them and, when used in conjunction with these disciplines, it is a strong tool to explain the energy security concept. That is, international relations and political science would explain socio-political threats, while engineering and natural science would explain technical and natural threats. That is, together these disciplines would explain the level of threats and so the likelihood of ESD. On the other hand, economic theory would explain the issues related to energy demand and supply, which in our model are synthesized in the variables Z_d and Z_s respectively. Therefore, the operationalization of the model can guide energy policies to improve energy security.

It is noteworthy that our simplified model is insufficient to capture many aspects of the energy security problem, as it is based on several fairly strict assumptions. Among other things, it uses the hypotheses of perfect competition and small open economy in a partial equilibrium model, as well as aggregating all energy sources (e.g., oil, gas, coal, solar, wind, etc.) in a common energy unit.[52] However, our model is flexible enough to incorporate new dimensions of energy security. For example, Appendix 2.C includes the resilience dimension (i.e., emergency stockpiles) in the model. Moreover, it is possible to further expand the model to make it more realistic. For this, it is necessary to develop it using a general equilibrium model in an open economy. Thus, it would be possible to include the different energy sources and various energy supply chains (i.e., several random variables of ESD and several levels of threats).

Appendix 2.A Threat reduction and more preferable strategy

We will show that when $t_M^i < t_M^j$, then $P^{ES}\left(A_{(\sigma,e)}, t_{E_S}, t_M^i\right) \geq P^{ES}\left(A_{(\sigma,e)}, t_{E_S}, t_M^j\right)$ and therefore $\left(\sigma, e, t_{E_S}, t_M^i\right) \gtrsim \left(\sigma, e, t_{E_S}, t_M^j\right)$.

Figure 2.4 shows that $A_{(\sigma,e)} = A_1 \cup A_2$ and $A_1 \cap A_2 = \emptyset$, where $A_1 = \left\{\left(x_{E_S}, x_M\right) \middle| x_{E_S}\sigma + (1-\sigma) \leq e, 0 \leq x_{E_S} \leq 1, 0 \leq x_M \leq 1\right\}$ and $A_2 = \left\{\left(x_{E_S}, x_M\right) \middle| e - (1-\sigma) < x_{E_S}\sigma \leq e - x_M(1-\sigma), 0 \leq x_{E_S} \leq 1, 0 \leq x_M \leq 1\right\}$. In this way, equation (2.6) can be rewritten as:

$$P^{ES}\left(A_{(\sigma,e)}, t_{E_S}, t_M\right) = P^{ES}\left(A_1, t_{E_S}\right) + P^{ES}\left(A_2, t_{E_S}, t_M\right) \tag{2.11}$$

where $P^{ES}\left(A_1, t_{E_S}\right) = F_{D_{E_S}}\left(\frac{e-(1-\sigma)}{\sigma} \middle| t_{E_S}\right)$ and $P^{ES}\left(A_2, t_{E_S}, t_M\right) = \int_{max\left\{0,\frac{e-(1-\sigma)}{\sigma}\right\}}^{min\left\{1,\frac{e}{\sigma}\right\}} f_{D_{E_S}}\left(x_{E_S} \middle| t_{E_S}\right) F_{D_M}\left(\frac{e-\sigma x_{E_S}}{(1-\sigma)} \middle| t_M\right) dx_{E_S}$. It is noteworthy that $P^{ES}\left(A_1, t_{E_S}\right) = P\left(D_{E_S}\sigma + D_M(1-\sigma) \leq e \middle| D_M = 1\right)$ is the degree of energy independence. As $P^{ES}\left(A_2, t_{E_S}, t_M\right) \geq 0$, then $P^{ES}\left(A_{(\sigma,e)}, t_{E_S}, t_M\right) \geq P^{ES}\left(A_1, t_{E_S}\right)$. That is, if an economy is in a condition of energy independence, then it is also in a condition of energy security.

Given the hypothesis of first-order stochastic dominance, when threats to energy imports are reduced, that is, when $t_M^i < t_M^j$, then $F_{D_M}\left(\frac{e-\sigma x_{E_S}}{(1-\sigma)} \middle| t_M^i\right) \geq F_{D_M}\left(\frac{e-\sigma x_{E_S}}{(1-\sigma)} \middle| t_M^j\right)$ and, therefore, $P^{ES}\left(A_2, t_{E_S}, t_M^i\right) \geq P^{ES}\left(A_2, t_{E_S}, t_M^j\right)$. Thus, $P^{ES}\left(A_{(\sigma,e)}, t_{E_S}, t_M^i\right) \geq P^{ES}\left(A_{(\sigma,e)}, t_{E_S}, t_M^j\right)$, which implies that $\left(\sigma, e, t_{E_S}, t_M^i\right) \gtrsim \left(\sigma, e, t_{E_S}, t_M^j\right)$.

[52] Another limitation of our simplified model is that it is static, in such a way that it ignores the dynamics of possible adjustment processes to energy supply disruptions. Furthermore, the model includes only the energy consumption of the end users, not including the energy consumption of firms.

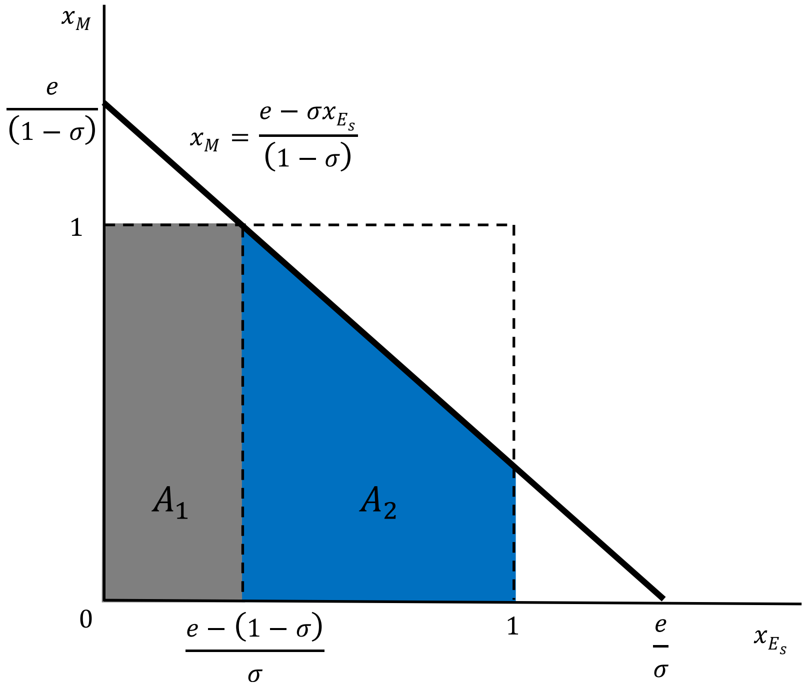

Figure 2.4 – Integration area for the probability of the degree of energy security
Source: Author's own elaboration.

Note that the analogous is true for t_{E_s}, that is, when $t^i_{E_s} < t^j_{E_s}$, then $P^{ES}\left(A_{(\sigma,e)}, t^i_{E_s}, t_M\right) \geq P^{ES}\left(A_{(\sigma,e)}, t^j_{E_s}, t_M\right)$ and consequently $\left(\sigma, e, t^i_{E_s}, t_M\right) \gtrsim \left(\sigma, e, t^j_{E_s}, t_M\right)$. In order to show this, just analyze $P^{ES}\left(A_{(\sigma,e)}, t_{E_s}, t_M\right)$ using the inverse function $x_{E_s} = \frac{e-(1-\sigma)x_M}{\sigma}$.

Appendix 2.B Energy price and energy security

The derivative of $\frac{e}{(1-\sigma)}$ with respect to international energy price ($\overline{p_E}$) is equal to:

$$\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial\overline{p_E}} = \frac{\frac{\partial e}{\partial\overline{p_E}} - \frac{\partial(1-\sigma)}{\partial\overline{p_E}}\frac{e}{(1-\sigma)}}{(1-\sigma)} \quad \textbf{(2.12)}$$

Since $\frac{\partial(1-\sigma)}{\partial\overline{p_E}} = -\frac{\partial\sigma}{\partial\overline{p_E}} = -\frac{\left(\frac{\partial E^*_s}{\partial\overline{p_E}}E^*_d - \frac{\partial E^*_d}{\partial\overline{p_E}}E^*_s\right)}{\left(E^*_d\right)^2}$ and $\frac{\partial e}{\partial\overline{p_E}} = \frac{\left(\frac{\partial E^*_d}{\partial\overline{p_E}}E^q - \frac{\partial E^q}{\partial\overline{p_E}}E^*_d\right)}{\left(E^*_d\right)^2}$, we can rewrite equation (2.12) as:

$$\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial\overline{p_E}} = \frac{-\frac{\partial E^*_d}{\partial\overline{p_E}}\left(\frac{\frac{\partial E^q}{\partial\overline{p_E}}}{\frac{\partial E^*_d}{\partial\overline{p_E}}}E^*_d - E^q\right) + \left(\frac{\partial E^*_s}{\partial\overline{p_E}}E^*_d - \frac{\partial E^*_d}{\partial\overline{p_E}}E^*_s\right)\frac{e}{(1-\sigma)}}{(1-\sigma)(E^*_d)^2} \quad \textbf{(2.13)}$$

By definition, we have that $\overline{W} = u(\varepsilon E^q) - g(E_d^*)$ and then $\frac{\partial \overline{W}}{\partial \overline{p_E}} = \frac{\partial u(\varepsilon E^q)}{\partial E^q}\frac{\partial E^q}{\partial \overline{p_E}} - \frac{\partial g(E_d^*)}{\partial E_d^*}\frac{\partial E_d^*}{\partial \overline{p_E}} = 0$. Thus, $\frac{\frac{\partial E^q}{\partial \overline{p_E}}}{\frac{\partial E_d^*}{\partial \overline{p_E}}} = \frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}}$. Therefore, equation (2.13) yields:

$$\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial \overline{p_E}}\left[\frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}}E_d^* - E^q\right] + \left(\frac{\partial E_s^*}{\partial \overline{p_E}}E_d^* - \frac{\partial E_d^*}{\partial \overline{p_E}}E_s^*\right)\frac{e}{(1-\sigma)}}{(1-\sigma)(E_d^*)^2} \quad \textbf{(2.14)}$$

After some algebraic manipulation, it is possible to rewrite equation (2.14) as:

$$\frac{\partial\left(\frac{e}{(1-\sigma)}\right)}{\partial \overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial \overline{p_E}}}{(1-\sigma)E_d^*}\left\{\frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}} - 1 + \left[1 + \left(\frac{\frac{\partial E_s^*}{\partial \overline{p_E}}}{-\frac{\partial E_d^*}{\partial \overline{p_E}}}\right)\right]\frac{e}{(1-\sigma)}\right\}$$

The derivative of $\frac{e}{\sigma}$ with respect to international energy price ($\overline{p_E}$) is equal to:

$$\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} = \frac{\frac{\partial e}{\partial \overline{p_E}} - \frac{\partial \sigma}{\partial \overline{p_E}}\frac{e}{\sigma}}{\sigma} \quad \textbf{(2.15)}$$

Since $\frac{\partial \sigma}{\partial \overline{p_E}} = \frac{\left(\frac{\partial E_s^*}{\partial \overline{p_E}}E_d^* - \frac{\partial E_d^*}{\partial \overline{p_E}}E_s^*\right)}{\left(E_d^*\right)^2}$ and $\frac{\partial e}{\partial \overline{p_E}} = \frac{\left(\frac{\partial E_d^*}{\partial \overline{p_E}}E^q - \frac{\partial E^q}{\partial \overline{p_E}}E_d^*\right)}{\left(E_d^*\right)^2}$, we can rewrite equation (2.15) as:

$$\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial \overline{p_E}}\left(\frac{\frac{\partial E^q}{\partial \overline{p_E}}}{\frac{\partial E_d^*}{\partial \overline{p_E}}}E_d^* - E^q\right) - \left(\frac{\partial E_s^*}{\partial \overline{p_E}}E_d^* - \frac{\partial E_d^*}{\partial \overline{p_E}}E_s^*\right)\frac{e}{\sigma}}{\sigma(E_d^*)^2} \quad \textbf{(2.16)}$$

Since $\frac{\frac{\partial E^q}{\partial \overline{p_E}}}{\frac{\partial E_d^*}{\partial \overline{p_E}}} = \frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}}$, equation (2.16) can be rewritten as:

$$\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial \overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial \overline{p_E}}\left(\frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}}E_d^* - E^q\right) - \left(\frac{\partial E_s^*}{\partial \overline{p_E}}E_d^* - \frac{\partial E_d^*}{\partial \overline{p_E}}E_s^*\right)\frac{e}{\sigma}}{\sigma(E_d^*)^2} \quad \textbf{(2.17)}$$

Therefore, after some algebraic manipulation, equation (2.17) yields:

$$\frac{\partial\left(\frac{e}{\sigma}\right)}{\partial\overline{p_E}} = \frac{-\frac{\partial E_d^*}{\partial\overline{p_E}}}{\sigma E_d^*}\left[\frac{\frac{\partial g(E_d^*)}{\partial E_d^*}}{\frac{\partial u(\varepsilon E^q)}{\partial E^q}} - 1 - \left(\frac{\frac{\partial E_s^*}{\partial\overline{p_E}}}{-\frac{\partial E_d^*}{\partial\overline{p_E}}}\right)e\right]$$

Appendix 2.C Resilience and energy security

Resilience is an important dimension of energy security and emergency energy stockpiles are a fundamental aspect of the resilience of the energy system (AZZUNI; BREYER, 2018; JEWELL, 2011; KUCHARSKI; UNESAKI, 2015; SOVACOOL, 2011; YERGIN, 2006). According to IEA (2014), emergency energy stockpiles are the most effective first line of defense for providing additional energy to an undersupplied market.[53] In other words, emergency stockpiles increase the capacity of the energy system to absorb disturbance and retain its essential function and structure, in such a way that its release can fully or partially cover an ESD.

In order to show how emergency stockpile (i.e., resilience) fits into our simplified model, we will assume for simplicity that the stored energy is always available when an ESD occurs (i.e., there is no chance of disruption in the release of the emergency stockpile) and that whenever there is an ESD, emergency stocks are released until the ESD is remedied or until the stock runs out. Given these hypotheses, the inclusion of emergency stock in the model results in a new amount of energy that is disrupted: $E_d^{D'} = E_d^D - E_\Delta$, where $E_\Delta = \begin{cases} \Delta & \Delta < E_d^D \\ E_d^D & \Delta \geq E_d^D \end{cases}$ is the amount of energy that is released from the emergency stockpile and Δ is the amount of energy stored in it. Thus, the new ESD is equal to $\frac{E_d^{D'}}{E_d^*} = \frac{E_d^D}{E_d^*} - \frac{E_\Delta}{E_d^*}$ and so $\max_{E_\Delta}\left(\frac{E_d^{D'}}{E_d^*}\right) = \frac{E_d^D}{E_d^*} - \frac{\Delta}{E_d^*}$. Therefore, in order to the ESD to be tolerable, it is necessary that $x_{E_s}\sigma + x_M(1-\sigma) \leq e_\Delta$, where $e_\Delta = e + \frac{\Delta}{E_d^*}$ is the new tolerance capacity ratio. Thereby, equation (2.6) can be rewritten as:

[53] It is no wonder that IEA member countries are obliged to hold oil stock levels equivalent to at least 90 days of their net imports (IEA, 2014). IEA members have already acted in coordination three times to release emergency stockpiles (Iraqi invasion of Kuwait in 1990-91, Hurricanes Katrina and Rita in the United States in 2005, and the Libyan civil war in 2011), bringing additional oil to the market and helping the market adjust to the disruption (IEA, 2019b). Furthermore, IEA members have already released emergency stocks in domestic crises several times (IEA, 2019b).

$$P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq e_\Delta\right) = \iint\limits_{A_{(\sigma, e_\Delta)}} f_{D_{E_s}}\left(x_{E_s} \middle| t_{E_s}\right) f_{D_M}(x_M|t_M)\, dx_{E_s} dx_M$$

If equilibrium energy consumption is adequate ($E_d^* \geq E^q$), then increases in the emergency stockpile (Δ) result in a more preferable strategy, since $\frac{\partial\left(\frac{e_\Delta}{(1-\sigma)}\right)}{\partial\Delta} = \frac{1}{(1-\sigma)E_d^*} > 0$ and $\frac{\partial\left(\frac{e_\Delta}{\sigma}\right)}{\partial\Delta}\frac{1}{\sigma E_d^*} > 0$.[54] Furthermore, if the emergency stockpile is sufficient to ensure that the energy consumption will be at least equal to the security energy consumption ($\Delta \geq E^q$), then the economy will experience a total energy security condition. This is because when $\Delta \geq E^q$, hence $e_\Delta \geq 1$ and therefore $P\left(D_{E_s}\sigma + D_M(1-\sigma) \leq e_\Delta\right) = 1$.

[54] Note that we are assuming that the size of the emergency energy stockpile (Δ) does not affect the level of threats. However, the mere existence of a large enough emergency stockpile has the ability to reduce threats, by deterring the use of energy as a political weapon by energy-exporting countries (AHN, 2007; BALAS, 1981). This is because the imposition of an energy embargo generates a cost for the energy-exporting country, such as the loss of revenue from energy exports or geopolitical costs. Thus, the existence of a large enough emergency stockpile in the importing country, with the ability to withstand the embargo for a long period of time, means that the imposition of the embargo only will result in costs for the energy-exporting country and its goal of inflicting damage on the importing country will not be achieved. In this way, the energy-exporting country would rationally choose not to impose the embargo.

3 ANALYZING THE RELATIONSHIP BETWEEN ENERGY SECURITY AND SUPPLIER DIVERSITY OF ENERGY IMPORTS USING AN ECONOMIC MODEL OF ENERGY SECURITY*

3.1 Introduction

The debate on the importance of diversity for energy security emerged in the years just preceding the First World War when Britain converted its battleships from coal to oil (YERGIN, 2011). Although oil made ships in the Royal Navy faster and more flexible than those in the German Navy, Britain had no such resources. This meant that the Royal Navy would not depend on Wales' coal, which was safe within its borders, but on the insecure oil supply from Persia, now Iran. In this way, Winston Churchill claimed that the security of Britain's oil supply would "*lie in variety and variety alone*" (YERGIN, 2011).

Since then, many authors have defended the supplier diversity (e.g. regions, countries) of energy imports in favor of energy security (ANG; CHOONG; NG, 2015a; APERC, 2007; AZZUNI; BREYER, 2018; COHEN; JOUTZ; LOUNGANI, 2011; FRONDEL; SCHMIDT, 2008; GE; FAN, 2013; GENG; JI, 2014; GUPTA, 2008; JEWELL, 2011; KISEL et al., 2016; LE COQ; PALTSEVA, 2009; LESBIREL, 2004; MOHSIN et al., 2018; NOVIKAU, 2019; PAVLOVIĆ; BANOVAC; VIŠTICA, 2018; REN; SOVACOOL, 2014; SOVACOOL, 2010, 2011; SOVACOOL; BROWN, 2010; SOVACOOL; MUKHERJEE, 2011; STOKES, 2007; SUTRISNO; NOMALER; ALKEMADE, 2021; VAN MOERKERK; CRIJNS-GRAUS, 2016; VIVODA, 2009, 2010, 2014, 2019; VIVODA; MANICOM, 2011; VON HIPPEL et al., 2011; WABIRI; AMUSA, 2010; WU et al., 2007; YANG et al., 2014; YERGIN, 2006; ZHANG; JI; FAN, 2013). In addition, some authors argue that the greater the energy import dependency, the greater the relevance of the supplier diversity for energy security (VAN MOERKERK; CRIJNS-GRAUS, 2016; VIVODA, 2009; YANG et al., 2014). Therefore, there is a strong consensus that, *ceteris paribus*, the greater the supplier diversity of energy imports, the greater the energy security, that is, supplier diversity is beneficial for energy security.[55]

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[*] Paper yet to be submitted.

[55] It is noteworthy that some authors emphasize that diversity, albeit prominent, is just one among several other dimensions of energy security (DHARFIZI; GHANI; ISLAM, 2020; KUCHARSKI; UNESAKI, 2015; RANJAN; HUGHES, 2014; RUBIO-VARAS; MUÑOZ-DELGADO, 2019; STIRLING, 2010a; VIVODA, 2009). Thus, an energy system considered diverse need not be secure and a secure energy system need not be considered diverse (RANJAN; HUGHES, 2014), since other dimensions could have an adverse effect on energy security. In addition, other authors recognize that promoting diversity is costly, that is, it is not a free lunch (SKEA, 2010; STIRLING, 2010a; WEITZMAN, 1992).

However, in the literature, there is no theoretical basis to support the existence of a positive relationship between supplier diversity and energy security. As Valdés (2018) points out, most studies build their definition of energy security on the enumeration of dimensions. Notwithstanding, as highlighted by Cherp and Jewell (2011), these methodological frameworks do not rigorously justify the inclusion or omission of dimensions, as well as their effects on energy security. Simply put, three methods of choosing dimensions can be identified: 1) choices based on meta-analysis of previous studies (ABDULLAH et al., 2020; ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018; ERAHMAN et al., 2016; RAGHOO et al., 2018; REN; SOVACOOL, 2014; SOVACOOL; BROWN, 2010); 2) choices based on interviews with experts (SOVACOOL, 2011, 2016; SOVACOOL; MUKHERJEE, 2011); and 3) arbitrary choices (APERC, 2007; HUGHES, 2009; LI; SHI; YAO, 2016; VIVODA, 2010; VON HIPPEL et al., 2011). Although meta-analysis and interviews are relatively systematic approaches, their value is lessened because the underlying research and experts can choose the dimensions through the arbitrary method (CHERP, 2012; CHERP; JEWELL, 2011).

The method of enumerating dimensions implies that energy security (or at least part of it) is defined as being diversity itself. That is, the methodological frameworks based on the enumeration of dimensions assume in advance that, *ceteris paribus*, greater supplier diversity is equivalent, by definition, to greater energy security. This means that the positive relationship between supplier diversity and energy security is not a result, or a conclusion, of these methodological frameworks, but rather an assumption.

Thereby, the purpose of this article is to identify whether there is any theoretical basis that justifies the existence of a positive relationship between supplier diversity of energy imports and energy security. Also, this article examines whether this positive relationship is more relevant when the import dependency is greater. To do this, it is necessary to rigorous define what "energy security" and "diversity" mean. In this way, we will use the definition of diversity proposed by Stirling (2007, 2010a) and the definition of energy security proposed in the second essay of this Thesis. We expanded the energy security model presented in the second essay to include two suppliers for energy imports. Thus, a simulation of this model is performed, identifying how the Hirschman-Herfindahl diversity index relates to the degree of energy security

The paper is structured as follows. Section 3.2 performs a literature review on the relationship between diversity and energy security. Section 3.3 shows the concept of energy security. Section 3.4 presents the concept of diversity. Section 3.5 examines the relationship between energy security and supplier diversity of energy imports. Section 3.6 concludes.

Furthermore, one appendix provides a detailed description of the assumptions on energy security.

3.2 Literature review on energy security and diversity

In the literature, when there is any justification for the assumption of a positive relationship between energy security and diversity, it is, in general, justified by the popular proverb: “Don’t put all your eggs in one basket” (AWERBUCH et al., 2006; AZZUNI; BREYER, 2018; CHERP; JEWELL, 2010; CHUANG; MA, 2013; GRUBB; BUTLER; TWOMEY, 2006; KAMSAMRONG; SORAPIPATANA, 2014; LESBIREL, 2004, 2013; STIRLING, 1994, 2010a; VAN MOERKERK; CRIJNS-GRAUS, 2016; WU et al., 2007; WU; LIU; WEI, 2009; YANG et al., 2014). The logic behind this proverb is that it is better to be exposed to various risks with limited consequences than to one risk with unbearable consequences. The analogy is simple. If we concentrate all the eggs in one basket and if that basket is lost, we will be without eggs; but if we divide the eggs evenly between the baskets, even if we lose some baskets, we still have enough eggs. In the context of energy security, just change the words “eggs” for “energy” and “baskets” for “energy options” (e.g., supplier, source, transport route, and transport mode). Thus, according to this proverb, diversification would reduce vulnerability and so increase energy security.

In this way, since energy security is often measured using indicators (ANG; CHOONG; NG, 2015a; ERAHMAN et al., 2016; GASSER, 2020; KRUYT et al., 2009; VALDÉS, 2018), empirical studies include the diversity dimension through diversity indices. The most used are the Shannon index and the Hirschman-Herfindahl index (CHALVATZIS; RUBEL, 2015; CHUANG; MA, 2013; COOKE; KEPPO; WOLF, 2013; KRUYT et al., 2009; MÅNSSON; JOHANSSON; NILSSON, 2014). These empirical studies assume in advance that, *ceteris paribus*, the higher the value of the diversity index, the greater the energy security.[56] While some authors use these indices alone as a *proxy* for energy security (DELGADO et al., 2013; DHARFIZI; GHANI; ISLAM, 2020; GRUBB; BUTLER; TWOMEY, 2006; HICKEY; LON CARLSON; LOOMIS, 2010; IOANNIDIS et al., 2019; RUBIO-VARAS; MUÑOZ-DELGADO, 2019; VIVODA, 2014, 2019; VIVODA; MANICOM, 2011), other use these indices in combination with other energy security indicators, since diversity is only one of several dimensions of energy security (ABDULLAH et al., 2020; ANG; CHOONG; NG,

[56] It is noteworthy that, in relation to the Hirschman-Herfindahl index, the assumption is that the lower the value of this indicator, the greater the energy security. This is because Hirschman-Herfindahl index measures concentration or similarity, rather than diversity (see section 3.4.1).

2015b; BHATTACHARYYA, 2009; CHALVATZIS; IOANNIDIS, 2017; CHALVATZIS; RUBEL, 2015; COSTANTINI et al., 2007; ERAHMAN et al., 2016; JEWELL, 2011; JEWELL; CHERP; RIAHI, 2014; LI; SHI; YAO, 2016; PAVLOVIĆ; BANOVAC; VIŠTICA, 2018).

Although the proverb for eggs and baskets is intuitive, it fails to robustly justify the existence of a positive relationship between energy security and diversity. First, as Grubb, Butler, and Twomey (2006) points out, it does not clarify what the diverse set of ''baskets'' should be. The diversity dimension covers several energy options, such as supplier (e.g., regions, countries, companies), source (e.g., coal, oil, gas, wind, solar, biomass), transport route, and transport mode (e.g., pipeline, rail, ship, grid interconnects). Often, these types of diversity can conflict (COOKE; KEPPO; WOLF, 2013). For example, a country can consume a small number of energy sources, but they can be supplied by a large number of suppliers or vice versa. That is, it is possible to have a low source diversity while having a high supplier diversity, and vice versa.

Second, the proverb considers that the "probability of losing the basket" is equal for all baskets. If we assume that basket refers to an energy supplier, this means assuming that the probability of an energy supply disruption is equal for all suppliers. For example, we are assuming that the oil supply from Canada and Norway is as secure as the oil supply from Iran and Iraq. Clearly, the probability of an energy supply disruption is different depending on the supplier, the source, the transport route, the transport mode, etc. Furthermore, if we knew that the probability of losing a certain basket is zero, why should we divide the eggs evenly among the baskets, since if we concentrate all the eggs in that basket, we will lose none? For example, if a country's oil imports were concentrated in Norway, why should that country diversify its imports towards less secure suppliers, such as Iraq or Iran? Or, if a country concentrates its energy consumption on domestic supply, why should that country diversify its energy consumption towards foreign suppliers? What would be the gains in energy security? Therefore, the relationship between diversity and energy security is not straightforward. It depends on the probability of the energy supply disruption of each energy option.

Although the "probability of losing the basket" may differ between baskets, Stirling argues that, in the energy security context, these probabilities are unknown (STIRLING, 1994, 1999, 2010a, 2010b). That is, Stirling argues that energy security is dominated by ignorance. Ignorance means that there is no knowledge about the probability of the occurrence of threats (e.g., failures in the equipment of the energy system, natural disasters, terrorism, export embargoes, political instability, and wars) and about how the occurrence of these threats will affect the energy supply. Therefore, no basis exists to assign probabilities to energy supply disruptions. Thus, diversity would provide a hedge against ignorance and so increase energy security.

However, the argument of ignorance in favor of diversity is not a theoretical justification for the positive relationship between energy security and diversity. On the contrary, it is an empirical argument that uses a precautionary recommendation due to the lack of knowledge. Back to the proverb of eggs and baskets, as much as the probabilities are unknown, if the probability of losing a certain basket is zero, from a theoretical point of view, dividing the eggs evenly between the baskets is a worse strategy than concentrating the eggs in that basket that has no chance of being lost. Nevertheless, from a practical point of view, in the state of ignorance, we do not know what this basket is and we do not even know if there is a basket with such property. Notwithstanding, consider the state of ignorance seems overly restrictive since it ignores any additional information available (AWERBUCH et al., 2006). That is, the traditional diversity indices do not consider additional information available that can provide some reliable guide to the probabilities of energy supply disruptions.

To take advantage of the available information that can serve as a guide to estimate these probabilities, some authors have modified the diversity indices. This modification occurs through the inclusion of correction factors, such as political risk, import dependency, correlation of energy prices, depletion of energy resources, distance between the importing country and energy suppliers, reserve-production ratio, fungibility of the energy supply, supply capacity of energy, and size of the energy importing country (CHUANG; MA, 2013; COHEN; JOUTZ; LOUNGANI, 2011; FRONDEL; SCHMIDT, 2008; GENG; JI, 2014; GUPTA, 2008; IEA, 2007; JANSEN; VAN ARKEL; BOOTS, 2004; KAMSAMRONG; SORAPIPATANA, 2014; LE COQ; PALTSEVA, 2009; LEFÈVRE, 2010; MATSUMOTO; DOUMPOS; ANDRIOSOPOULOS, 2018; MATSUMOTO; SHIRAKI, 2018; MOHSIN et al., 2018; VAN MOERKERK; CRIJNS-GRAUS, 2016; VAN VLIET et al., 2012; WANG et al., 2018; YANG et al., 2014; ZHANG; JI; FAN, 2013).

As Månsson, Johansson, and Nilsson (2014) points out, a typical modification of the Hirschman-Herfindahl index can be formulated as: $\sum_{i=1}^{n} {\omega_i}^2 a_i$, where ω_i is the proportion of elements assigned to energy option i in a sample of n options and a_i is the correction factor.[57]

[57] Similarly, a typical modification of the Shannon index can be formulated as: $-\sum_{i=1}^{n} \omega_i \ln(\omega_i)\left(\frac{1}{a_i}\right)$ or $-\sum_{i=1}^{n} \omega_i \ln(\omega_i)\,(1 - a_i)$. Note that the a_i must be used inversely in the Shannon index – see Chuang and Ma (2013). This is because the Shannon index measures diversity, while the Hirschman-Herfindahl index measures concentration or similarity (see section 3.4.1).

The main disadvantage of using this modified index is that it does not represent the diversity concept. That is, albeit it is based on a diversity index, the modified index is no longer a diversity index, since it does not satisfy the scaling of variety, monotonicity of variety, and balance properties (STIRLING, 2007).

The modified index puts more weight on energy options that have a low correction factor value. If a certain a_i is equal to zero, then the lowest value of the modified Hirschman-Herfindahl index (i.e., greater energy security) will occur when we have the total concentration in option i (i.e., zero diversity). Thus, the application of the modified diversity indices generally results in the conclusion that, in order to improve energy security, one should concentrate on energy options where the correction factor value is low (COHEN; JOUTZ; LOUNGANI, 2011; FRONDEL; SCHMIDT, 2008; LE COQ; PALTSEVA, 2009; MOHSIN et al., 2018; VAN MOERKERK; CRIJNS-GRAUS, 2016; YANG et al., 2014; ZHANG; JI; FAN, 2013). In other words, one should not diversify in a general way, but one should diversify away from insecure energy options, that is, replacing less secure options with more secure ones. Therefore, the use of modified diversity indices implies that the positive relationship between energy security and diversity advocated in theoretical conceptualization no longer exists. On the contrary, if diversification implies a greater share of less secure options (i.e., options with high correction factors), it will result in a worsening of energy security.

A third reason why the proverb of eggs and baskets fails to justify the positive relationship between energy security and diversity is that it does not divide risks into systematic and specific ones. According to Lesbirel (2004), systematic risk is the risk that affects all energy options together. In general, fluctuations in the international energy price are taken as systematic risk, since they affect the international energy market as a whole (GE; FAN, 2013; LESBIREL, 2004; WABIRI; AMUSA, 2010; WU et al., 2007). Hence, diversification does not affect systematic risks, which is why they are also called non-diversifiable risks. On the other hand, the specific risks are those that are unique to energy options, that is, they affect each energy option individually. Some examples are political instability, accident, and natural disaster that affects a certain energy exporting country. Thus, the specific risk is affected by diversification, which is why they are also called diversifiable risks.

In the analogy of eggs and baskets, to separate these two types of risks, we need to imagine that all baskets are inside a box, where the systematic risk refers to “losing the box” and specific risk refers to “losing a certain basket”. It does not matter the degree of diversification of the eggs between the baskets if the box with all the baskets inside is lost. In the context of energy security, it does not matter the degree of diversification of oil imports, if there is a shock in the international oil price that will be spread by all exporting countries. This demonstrates that diversity can have a limited effect on energy security, depending on the severity of the systematic risk.

Some authors have used the Portfolio theory to develop different indices for the systematic and specific risks (GE; FAN, 2013; LESBIREL, 2004; NEFF, 1997; WABIRI; AMUSA, 2010; WU et al., 2007). However, these indices are essentially modified diversity indices[58], which, as already mentioned, result in the conclusion that less secure energy options should be replaced with more secure ones. Moreover, the Portfolio theory does not rigorously define energy security. It sees energy security only as variations in the energy imports price (GE; FAN, 2013; LESBIREL, 2004; WABIRI; AMUSA, 2010; WU et al., 2007). As noted by Awerbuch et al. (2006) and Hickey, Lon Carlson, and Loomis (2010), the Portfolio theory neglects several important aspects of energy security, such as social, environmental, and geopolitical issues.

This brings us to the fourth and most important reason why the proverb of eggs and baskets is not a robust justification for diversification, namely: it does not define what energy security means. In other words, what is the purpose of dividing eggs evenly between baskets? Is the goal to keep a high average number of eggs? Or is the goal to keep at least a minimum number of eggs? If the goal is to keep a high average number of eggs, the application of the diversity indices, and their modified versions, will not be appropriate. In this case, the appropriate index would be that which represents the expected value of the eggs that will be kept (i.e., that will not be lost); that is, an average of the probabilities of not losing the baskets weighted by the number of eggs in each basket. In the case of energy security, such an indicator was proposed by Bompard et al. (2017). Simply put, this indicator is represented as $E \times \sum_{i=1}^{n} \omega_i (1 - a_i)$, where $E = \sum_{i=1}^{n} E_i$ is the total amount of energy supplied, E_i is the energy supplied by option i, $\omega_i = \frac{E_i}{E}$ is the share of energy supplied by option i, and $0 \leq a_i \leq 1$ is the correction factor that is a *proxy* for the expected value of energy supply disruption of option i. The maximum value of this indicator is obtained when the entire energy supply is concentrated

[58] The systematic risk index is essentially a modified Hirschman-Herfindahl index, where the correction factors are the variance of the international energy price and a parameter that represents the impact that this variation has on the energy import price (GE; FAN, 2013; LESBIREL, 2004; WABIRI; AMUSA, 2010; WU et al., 2007). The specific risk index is also a modified Hirschman-Herfindahl index, but the correction factors are the variance of the energy import price that is not explained by the international price (i.e. variance of the error) and the political risk (LESBIREL, 2004; WABIRI; AMUSA, 2010; WU et al., 2007). It should be noted that the specific risk index developed by Ge and Fan (2013) is essentially a Stirling index (STIRLING, 2007), where the disparity includes the political risk, correlation of oil import prices from different regions and standard deviation of the error.

on the option that has the least correction factor value. Therefore, in this case, there is no positive relationship between energy security and diversity.

Thus, the relationship between energy security and diversity depends intrinsically on the definition of energy security. However, as already mentioned, in the literature, energy security (or at least part of it) is equivalent, by definition, to greater diversity. Nevertheless, diversity is only a property of any energy system (STIRLING, 2010a) and so diversity indices, and their modified versions, say nothing about the theoretical foundations of energy security. As Skea (2010) points out, an indicator is just an indicator and nothing more, as it does not explain fundamental physical or economic properties.

Furthermore, as demonstrated by Cooke, Keppo, and Wolf (2013), there are several diversity indices with different properties and each index tells a particular version of the "diversity story" (i.e., they can present different results, which are often conflicting). Thereby, diversity is never directly measured, instead a specific index is used to measure a particular view of diversity. In this way, which diversity index (or its modified version) is adequate to measure energy security? Moreover, what is the value of this index that results in a satisfactory or unsatisfactory level of energy security? As several authors point out, the values of the diversity indices do not provide a guide on the absolute level of energy security, that is, they do not provide a cardinal measure of energy security (CHALVATZIS; IOANNIDIS, 2017; CHALVATZIS; RUBEL, 2015; GRUBB; BUTLER; TWOMEY, 2006; HICKEY; LON CARLSON; LOOMIS, 2010).[59] Its use provides only an imprecise guide, given the lack of theoretical justification, on the ordinal level (e.g., evolution over time) of energy security.

Therefore, in order to identify whether there is any relationship between energy security and supplier diversity of energy imports, it is necessary to strictly define what "energy security" and "diversity" mean.

3.3 What does energy security mean?

Many literature reviews have been conducted and all conclude that there are numerous energy security definitions (ANG; CHOONG; NG, 2015a; AZZUNI; BREYER, 2018; CHESTER, 2010; KOULOURI; MOURAVIEV, 2019; PARAVANTIS et al., 2019; SOVACOOL, 2010; SOVACOOL; BROWN, 2010). For example, Azzuni and Breyer (2018),

[59] In order to express different levels of energy security (e.g., satisfactory and unsatisfactory) through the diversity indices, some authors have defined arbitrary ranges for the values of the diversity indices (GRUBB; BUTLER; TWOMEY, 2006; JEWELL, 2011), while others have used the ranges stipulated by the United States Department of Justice and the Federal Trade Commission for the Hirschman-Herfindahl index (DHARFIZI; GHANI; ISLAM, 2020). However, this does not explain what energy security means.

found 66 different definitions in 104 studies. According to Cherp and Jewell (2014), this lack of consensus is reflected in the absence of a rigorous definition of energy security. Most studies build their definition on the enumeration of energy security dimensions (VALDÉS, 2018).

Despite the obvious utility of the method of drawing up lists of energy security dimensions, it is just a technical exercise in taxonomy and it does not give a theoretical basis for the concept of energy security. As highlighted by Valentine (2010), adding more dimensions to a long and disconnected list does not help to better understand energy security; rather, it exacerbates intellectual discord and amplifies the lack of consensus. Furthermore, it does not develop a methodological framework that explains the relationship between the energy security dimensions. That is, the approach of enumerating dimensions does not produce an integrative framework (CHERP; JEWELL, 2011). As Sovacool and Brown (2010) and Sovacool and Saunders (2014) point out, dimensions can be competing or allied, meaning that changes in one dimension can shrink or enlarge other dimensions.

To explain energy security, we will use the energy security model developed in the second essay of this Thesis. This choice is justified for four reasons. First, this model combines economic theory and the concept of security in a probabilistic framework. Thus, it fulfills the three prerequisites for a rigorous conceptualization of energy security. That is, the concept of energy security should be: 1) based on the concept of security in general (BOSWORTH; GHEORGHE, 2011; CHERP; JEWELL, 2014; CIUTĂ, 2010; JOHANSSON, 2013; KARLSSON-VINKHUYZEN; JOLLANDS, 2013; PARAVANTIS et al., 2019; VON HIPPEL et al., 2011); 2) related to the concept of risk and therefore should be related to the concept of probability (AZZUNI; BREYER, 2018; CHERP; JEWELL, 2014; CHESTER, 2010; KUCHARSKI; UNESAKI, 2015; WINZER, 2012); and 3) based on rigorous microeconomic fundamentals (BÖHRINGER; BORTOLAMEDI, 2015).

Second, the model includes personal judgments (i.e., context-dependency) in the definition of energy security, while presenting consistency in its reasoning. That is, energy security is a universal concept, but its definition can have different meanings (i.e., include different dimensions) that can be expressed through different lenses. Third, the model does not assume in advance how the dimensions affect energy security; this is a result of the model. Fourth, although the model does not include all dimensions, it is capable of integrating different dimensions of energy security in a rigorous methodological framework. That is, the model explains how the different dimensions interact with each other and how they affect energy security. Of course, no theoretical model can capture all aspects of the complex reality of the energy security problem, and therefore this model is a simplified representation of energy security.

3.3.1 Energy security definition

In the second essay (see section 2.5.6), energy security is defined as:

$$P\big(D_{E_S}\sigma + D_M(1-\sigma) \leq e\big) \geq P^{min} \tag{3.1}$$

Where σ is the self-sufficiency ratio, $(1-\sigma)$ is the dependence ratio, e is the tolerance capacity ratio, D_{E_S} is the random variable that represents energy supply disruptions in the internal energy supply chain (i.e. domestic energy production) and D_M is the random variable that represents energy supply disruptions in the external energy supply chain (i.e., energy import), $P\big(D_{E_S}\sigma + D_M(1-\sigma) \leq e\big)$ is the degree of energy security, and P^{min} is the minimum level for the degree of energy security that the economy agents require to feel secure.

The economy is said to be in a condition (or situation) of energy security when the inequality expressed in equation (3.1) is true. Otherwise, the economy is in a condition of energy insecurity. Thus, energy security has a relative aspect (a greater or lesser degree of energy security) and an absolute aspect (condition of energy security or insecurity).

3.3.2 Assumptions on energy security

We will expand this energy security model to include two suppliers for energy imports (e.g., two geographic regions). That is, will assume that $M^* = M_1^* + M_2^*$, where M^* is the equilibrium energy import, M_1^* is the energy import from supplier 1 and M_2^* is the energy import from supplier 2. Energy supply disruptions for each supplier can be represented by a random variable: D_{M_1} is the random variable that represents energy supply disruptions in the energy supply chain of supplier 1 and D_{M_2} is the random variable that represents energy supply disruptions in the energy supply chain of supplier 2. We will assume that D_{E_S}, D_{M_1} and D_{M_2} are independents.

Therefore, we have $D_M = D_{M_1}\gamma + D_{M_2}(1-\gamma)$, where $\gamma = \frac{M_1^*}{M^*}$ and $(1-\gamma) = \frac{M^*-M_1^*}{M^*} = \frac{M_2^*}{M^*}$ are the respective shares of imports from suppliers 1 and 2 in total energy imports. Thus, it is possible to rewrite equation (3.1) as $P\big(D_{E_S}\sigma + D_{M_1}\gamma(1-\sigma) + D_{M_2}(1-\gamma)(1-\sigma) \leq e\big) \geq P^{min}$. In order to analyze the relationship between energy security and supplier diversity of energy import, we will focus on the relative aspect of energy security, that is, the degree of energy security:

$$P\left(D_{E_s}\sigma + D_{M_1}\gamma(1-\sigma) + D_{M_2}(1-\gamma)(1-\sigma) \leq e\right) \qquad \textbf{(3.2)}$$

It is noteworthy that equation (3.2) allows the separation of systematic and specific risks. Systematic risk is equivalent to variations in the value of the tolerance capacity ratio (e), since reductions in e will affect the degree of energy security (i.e., equation (3.2)) regardless of the level of supplier diversity (i.e., γ). As highlighted in the second essay of this Thesis, the tolerance capacity ratio is a function of several variables, including the international energy price.[60] On the other hand, the specific risks are represented by the (independent) random variables, D_{i}, of each supplier.

We will assume that D_{M_1} and D_{M_2} have beta distribution, that is, $D_{M_i} \sim Beta(\alpha_i, 1)$, where $\alpha_i = \frac{t_i}{1-t_i}$ and t_i ($i = M_1, M_2$) is the level of threats on the energy supply chain of supplier i. It is noteworthy that $t_i \in (0,1)$, where $t_i \to 1$ means an imminent potential danger and $t_i \to 0$ means a remote potential danger (see section 2.5.4). Besides, to simplify, we will assume that there are no threats on the domestic energy supply chain ($t_{E_s} \to 0$), in such a way that it is completely secure. In other words, there are no disruptions in the internal energy supply: $P\left(D_{E_s} = 0\right) = 1$. This means that producing energy domestically is beneficial to energy security, while importing energy is detrimental. This assumption is widely used in the literature (CHALVATZIS; IOANNIDIS, 2017; CHALVATZIS; RUBEL, 2015; CHUANG; MA, 2013; FRONDEL; SCHMIDT, 2008; LE COQ; PALTSEVA, 2009; MÅNSSON; JOHANSSON; NILSSON, 2014; MATSUMOTO; DOUMPOS; ANDRIOSOPOULOS, 2018; SOVACOOL, 2011; SOVACOOL; BROWN, 2010; VAN MOERKERK; CRIJNS-GRAUS, 2016; VIVODA, 2010; YANG et al., 2014).

Furthermore, as the majority of the energy security studies focus on large energy importing countries (ANG; CHOONG; NG, 2015a), we will assume that $0 \leq \sigma, \gamma \leq 1$. This means that the economy does not export energy. It should be noted that $e < 1$ by definition (see section 2.8).

Given these assumptions, Appendix 3.A shows that equation (3.2) is equal to:

[60] It is noteworthy that the effect of the international energy price ($\overline{p_E}$) on tolerance capacity ratio (e) can be positive in some cases ($\frac{\partial e(\overline{p_E})}{\partial \overline{p_E}} > 0$) and negative in others ($\frac{\partial e(\overline{p_E})}{\partial \overline{p_E}} < 0$) (see section 2.9).

$$\begin{cases} 0 & e < 0 \\ 1 & 1 \geq e \geq (1-\sigma) \geq 0 \\ I_{e_\sigma}(\alpha_1, 1) & 0 \leq e < (1-\sigma) \leq 1; \gamma = 1 \\ I_{e_\sigma}(\alpha_2, 1) & 0 \leq e < (1-\sigma) \leq 1; \gamma = 0 \\ I_{\hat{\varphi}}(\alpha_1, 1) + \mathrm{Z}\hat{\Gamma}\left[I_\psi(\alpha_1, \alpha_2 + 1) - I_\varphi(\alpha_1, \alpha_2 + 1)\right] & 0 \leq e < (1-\sigma) \leq 1; \gamma \in (0,1) \end{cases} \quad \textbf{(3.3)}$$

where $e_\sigma = \frac{e}{(1-\sigma)}$, $\varphi = \max\left(0, \frac{e_\sigma - (1-\gamma)}{e_\sigma}\right) = \frac{\gamma}{e_\sigma}\max\left(0, \frac{e_\sigma - (1-\gamma)}{\gamma}\right)$, $\hat{\varphi} = \varphi\left(\frac{e_\sigma}{\gamma}\right)$, $\psi = \min\left(\frac{\gamma}{e_\sigma}, 1\right)$, $\mathrm{Z} = \left[\frac{(e_\sigma)^{\alpha_1 + \alpha_2}}{(\gamma)^{\alpha_1}(1-\gamma)^{\alpha_2}}\right]$, $\hat{\Gamma} = \left[\frac{\Gamma(\alpha_1+1)\Gamma(\alpha_2+1)}{\Gamma(\alpha_1+\alpha_2+1)}\right]$, $I_x(\alpha, \beta)$ is the cumulative distribution function of the beta distribution and $\Gamma(\alpha)$ is the gamma function.

3.3.2.1 Parameter values

We will choose arbitrary values for the parameters e, σ, t_{M_1} and t_{M_2}. The tolerance capacity ratio (e) tends to be a small value since it represents the maximum percentage of energy consumption that an economy can tolerate losing to keep a sufficient level of social welfare (see section 2.5.6). We assume that $e = 0.05$, that is, the economy is only able to tolerate an energy supply disruption of a maximum of 5% of energy consumption. Any energy supply disruption of greater magnitude will result in damage to acquired values of the economy. Since our analysis focuses on specific risks, we will not do sensitivity analysis for the parameter e, that is, we will not analyze systematic risks.[61]

Regarding the self-sufficiency ratio, it should be noted that when $e \geq (1-\sigma)$, equation (3.3) is equal to 1 regardless of the value of γ. Thus, the supplier diversity of energy imports has no impact on the degree of energy security in this case. As highlighted in the second essay of this Thesis (see section 2.7), assuming that $e \geq (1-\sigma)$ is equivalent to assuming that domestic energy production is able to maintain a sufficient level of social welfare, even if all imported energy is disrupted. However, such an assumption is somewhat unrealistic for countries that depend excessively on energy imports. As large energy importing countries are precisely the focus of the energy security literature (ANG; CHOONG; NG, 2015a), we will assume $e < (1-\sigma)$, that is, we will analyze equation (3.3) for 1900 observations of the self-sufficiency ratio: $\sigma = \left(\frac{101}{2000}, \frac{102}{2000}, \frac{103}{2000}, \dots, \frac{2000}{2000}\right)$.

[61] It is noteworthy that, if $e \geq 0$, equation (3.3) does not depend on the absolute value of e. It depends only on its relative value, that is, $e_\sigma = \frac{e}{(1-\sigma)}$. Therefore, we can always choose a new value for the self-sufficiency ratio to get a new value for e_σ.

Regarding the level of threats, it is necessary to make two observations. First, we will define the supplier i as being more secure than the supplier j when the cumulative distribution function $F_{D_i}(x|t_i)$ first-order stochastically dominates $F_{D_j}(x|t_j)$, that is, when $F_{D_i}(x|t_i) \geq F_{D_j}(x|t_j)\ \forall x$. Thereby, given the assumptions about the distributions of D_{M_1} and D_{M_2}, it is easy to see that if $t_{M_1} < t_{M_2}$, then $F_{D_{M_1}}(x|t_{M_1}) \geq F_{D_{M_2}}(x|t_{M_2})\ \forall x$ (and vice versa).[62] Therefore, if there is a lower level of threats, the chances of energy supply disruption are small and then the energy supplier is more secure. Second, when we assume a value for the level of threats, we are choosing the expected value of energy supply disruptions. That is, $E(D_{M_i}) = t_{M_i}$.[63] Thus, the level of threat is not excessively high, since large energy supply disruptions in the international energy market are sporadic events. In this way, we will assume four relative security scenarios between suppliers 1 and 2, that is, we will set $t_{M_1} = 0.01$ and vary t_{M_2}:

- Scenario 1: supplier 2 is as secure as supplier 1 ($t_{M_2} = t_{M_1} = 0.01$)
- Scenario 2: supplier 2 is slightly less secure than supplier 1 ($t_{M_2} = 0.02$ and $t_{M_1} = 0.01$)
- Scenario 3: supplier 2 is moderately less secure than supplier 1 ($t_{M_2} = 0.05$ and $t_{M_1} = 0.01$)
- Scenario 4: supplier 2 is much less secure than supplier 1 ($t_{M_2} = 0.15$ and $t_{M_1} = 0.01$)

Given these assumptions, the parameters e, σ, t_{M_1} and t_{M_2} are constant in equation (3.3). Therefore, equation (3.3) becomes just a function of the share of imports from suppliers 1 in total energy imports (γ). We will denote it as $\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$, where the superscript S indicates the security scenario and the subscript $(1-\sigma)$ indicates the value of the dependence ratio.[64] Since we have 1900 values for $(1-\sigma)$ and four scenarios (i.e., four values for t_{M_2} and one value for t_{M_1}), we will calculate 7600 versions of $\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$ (i.e., equation (3.3)).

3.4 What does diversity mean?

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[62] Note that $D_{M_i} \sim Beta(\alpha_i, 1)$ implies that $F_{D_{M_i}}(x|t_i) = I_x(\alpha_i, 1) = \begin{cases} 1 & x > 1 \\ x^{\alpha_i} & 0 \leq x \leq 1 \\ 0 & x < 0 \end{cases}$, where $\alpha_i = \frac{t_i}{1-t_i}$.

[63] If $X \sim Beta(\alpha, \beta)$, then $E(X) = \frac{\alpha}{\alpha+\beta}$. As $D_{M_i} \sim Beta(\alpha_i, 1)$ and $\alpha_i = \frac{t_i}{1-t_i}$, hence $E(D_{M_i}) = t_{M_i}$.

[64] It is noteworthy that we are suppressing the notation of the parameter e because we only have a single value for it.

According to Stirling (2010a), diversity is defined as an evenly balanced reliance on a variety of mutually disparate options. As such, diversity is a property of any energy system, since its elements can be divided into categories, called options. In this way, diversity is a combination of three necessary but individually insufficient elements: Variety (the number of options), Balance (the spread across the options), and Disparity (the degree to which the options are different from each other) (STIRLING, 2007, 2010a).

Although the concept of energy diversity is strictly defined, it is, at core, subjective and irreducibly context-specific (COOKE; KEPPO; WOLF, 2013). As Cooke, Keppo, and Wolf (2013) points out, the first issue to be considered in the definition of diversity is how the options should be classified. For example, they can be classified as energy suppliers, energy transport routes, energy transport modes, primary energy sources, and many other classification schemes. The second issue is related to the level of disaggregation that will be used in the classification of options (COOKE; KEPPO; WOLF, 2013). For example, energy suppliers can be classified into regions, which can be broken down into countries, which in turn can still be broken down by companies. Likewise, primary energy sources can be decomposed into renewable and non-renewable energies. Renewable energies can be decomposed into biomass, solar, wind, water, and marine, while marine energy can be further decomposed into wave and tidal.

Once the options are classified and the level of disaggregation is chosen, there is still a third issue: determine how different each option is from each other option (COOKE; KEPPO; WOLF, 2013; STIRLING, 2007). The disparity reflects the ''distance'' between two options in terms of their intrinsic characteristics (STIRLING, 2007, 2010a). Notwithstanding, it is necessary to choose which intrinsic characteristics will be used to measure the disparity. For example, the intrinsic characteristics chosen can be the environmental quality (e.g., CO2 emission), technology class, capital intensity, vulnerability of the energy supplier, and many others (STIRLING, 2010a). Furthermore, in the case where these characteristics are expressed ordinally (e.g., the least subject to a specific type of failure, second least, third least) or as categorical variables (e.g., public property or private property) the disparity scores assign to them are necessarily subjective and context-specific (COOKE; KEPPO; WOLF, 2013).

3.4.1 How diversity is measured?

The choice of classification, disaggregation, and disparity characteristics, shapes and delimits the diversity definition. However, there is still another issue from a practical point of view: the choice of the index to measure diversity. Although there are a huge number of diversity indices with elegant mathematical formalizations (CHUANG; MA, 2013; COOKE;

KEPPO; WOLF, 2013; SKEA, 2010; STIRLING, 2010a), which makes the assumptions clear, as demonstrated by Cooke, Keppo, and Wolf (2013), diversity is never directly measured. Instead, a specific index is used to measure a particular view of diversity. Therefore, we will analyze in detail the three most used diversity indices: Shannon index, Hirschman-Herfindahl index, and Stirling index.[65]

3.4.1.1 Shannon index

Shannon index is defined as (SHANNON; WEAVER, 1949):

$$H = -\sum_{i=1}^{n} \omega_i \ln(\omega_i) \tag{3.4}$$

where n is the number of options, ω_i is the proportion of elements assigned to option i in a sample of n options, $0 < \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$.[66] Shannon index is not bounded, that is, $H \geq 0$. Moreover, H is more sensitive to options that occupy a smaller share of the sample (i.e., rare species in an ecological context). For a given number of options (n), the maximum value for Shannon index is obtained when all options are evenly balanced: if $\omega_i = \frac{1}{n}\ \forall i$, then $H = \ln(n)$.[67]

3.4.1.2 Hirschman-Herfindahl index

Hirschman-Herfindahl index is given by (HIRSCHMAN, 1945, 1964): $HHI = \sum_{i=1}^{n} {\omega_i}^2$, where $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$.[68] Unlike the Shannon index, Hirschman-Herfindahl index is more sensitive to options that occupy a larger share of the sample (i.e., abundant species in an ecological context). It is noteworthy that the Hirschman-Herfindahl index is not a measure of diversity. Rather, it is a measure of concentration or similarity, and

[65] It is noteworthy that the first two indices are special cases of the Hill index and Generalized Entropy index (HILL, 1973; KEYLOCK, 2005), while all three are special cases of the Leinster-Cobbold index (LEINSTER; COBBOLD, 2012).

[66] Shannon index is also known as Shannon–Wiener index and Shannon–Weaver index (SPELLERBERG; FEDOR, 2003).

[67] It is noteworthy that it is possible to normalize Shannon index, in order to get the Shannon Evenness index: $H_E = \frac{H}{\ln(n)}$, where $n > 1$ (STIRLING, 2010a). Shannon Evenness index is bounded, that is, $0 < H_E \leq 1$.

[68] Hirschman–Herfindahl index is also known as Simpson index (ROUSSEAU, 2018), due to Simpson (1949). It is noteworthy that Hirschman initially proposed the Concentration index (C), given by $C = \sqrt{\sum_{i=1}^{n}(100 * \omega_i)^2} = 100\sqrt{HHI}$ (HIRSCHMAN, 1945, p. 159). According to Hirschman (1964), in 1950, Herfindahl proposed the same concentration index, except for the square root, that is, $C^2 = 10.000 * HHI$. Therefore, $0 < C \leq 100$ and $0 < C^2 \leq 10.000$. It is noteworthy that some authors have used Hirschman's original index, but have named it as Hirschman–Herfindahl–Agiobenebo index (AGIOBENEBO, 2000; GE; FAN, 2013; WABIRI; AMUSA, 2010; WU et al., 2007; WU; LIU; WEI, 2009).

therefore the closer HHI is to 1, the more concentrated (or less diverse) is the sample. Nevertheless, it is possible to transform HHI to obtain a diversity index:[69]

$$\lambda = 1 - HHI \tag{3.5}$$

This Hirschman-Herfindahl diversity index is bounded, that is, $0 < HHI \leq 1$ and so $0 \leq \lambda < 1$. For a given number of options (n), the maximum value for Hirschman-Herfindahl diversity index is obtained when all options are evenly balanced: if $\omega_i = \frac{1}{n}\ \forall i$, then $\lambda = \frac{n-1}{n}$.

3.4.1.3 Stirling index

Stirling index is defined as (STIRLING, 2007):

$$\Delta = \sum_{i,j=1}^{n} d_{ij}\omega_i\omega_j \tag{3.6}$$

where d_{ij} is the disparity between option i and j, $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$.[70] As Stirling (2007, 2010a) points out, the Shannon index and Hirschman-Herfindahl index only measure variety and balance, omitting disparity. On the other hand, the Stirling index includes disparity, in addition to variety and balance. However, as shown by Rousseau (2018), the Stirling index does not meet Stirling's third robust property (STIRLING, 2007), that is, the monotonicity of balance property.

Disparity is a non-negative symmetric function, that is, $d_{ij} \geq 0$, $d_{ij} = d_{ji}$, and for all $i = j$, $d_{ij} = 0$.[71] Thus, $\Delta = \sum_{i,j} d_{ij}\omega_i\omega_j = \sum_{i,j\ (i \neq j)} d_{ij}\omega_i\omega_j = 2\sum_{i,j\ (i<j)} d_{ij}\omega_i\omega_j$. By convention, it is possible to normalize the disparity, that is, $0 \leq d_{ij} \leq 1$, in which $d_{ij} = 1$ indicates total disparity (i.e., when options i and j have totally different intrinsic characteristics) e $d_{ij} = 0$ total similarity (i.e., when options i and j have totally equal intrinsic characteristics).

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69 This index is also known as Simpson Diversity index (KEYLOCK, 2005; ROUSSEAU, 2018).

70 Stirling index is also known as Rao-Stirling index (ROUSSEAU, 2018), due to the index proposed by Rao (1982). It should be noted that the Stirling index can also be expressed as $\Delta = \sum_{i,j=1}^{n}(d_{ij})^{\alpha}(\omega_i\omega_j)^{\beta}$, where α and β are allowed to take all possible permutations of the values 0 and 1 (STIRLING, 2007).

71 According to Stirling (2007), distance in a Euclidean m-space offers the most parsimonious and generally applicable framework to measured disparity. Let m be the number of intrinsic characteristics of each option and c_{ik} be the disparity score for option i $(i = 1, \dots, n)$ assigned to the intrinsic characteristic k $(k = 1, \dots, m)$. Thus, disparity can be measured as $d_{ij} = \sqrt{\sum_{k=1}^{m}(c_{ik} - c_{jk})^2}$.

In this way, Stirling index is bounded, that is, $0 \leq \Delta \leq \lambda < 1$, where $\Delta = \lambda$ when $d_{ij} = 1\ \forall i \neq j$.[72]

3.4.2 Assumptions on diversity

Since our model of energy security developed in section 3.3.2 imposes the existence of two suppliers to import energy, we will classify the options as being the energy suppliers and we will disaggregate them into two options ($n = 2$), namely: supplier 1 and supplier 2. It is easy to see that $\omega_1 = \gamma$ and $\omega_2 = (1-\gamma)$. It is noteworthy that, when there are just two options and $d_{12} > 0$, the Stirling index is a linear transformation of the Hirschman-Herfindahl diversity index[73] and so $\rho = 1$, where ρ is the correlation coefficient. Also, in this case, the Hirschman-Herfindahl diversity index and the Shannon index are strongly correlated ($\rho = 0.996$). Thus, the choice of the diversity index will not significantly influence the analysis of the relationship between energy security and supplier diversity of energy imports. In this way, we will use the Hirschman-Herfindahl diversity index. Given the assumptions, we can rewrite equation (3.5) as a function only of the share of imports from suppliers 1 in total energy imports, that is, $\lambda(\gamma)$:

$$\lambda = 2\gamma(1-\gamma) \qquad \textbf{(3.7)}$$

3.5 The relationship between supplier diversity of energy imports and energy security

In order to analyze the relationship between energy security and supplier diversity of energy imports, we will simulate 2001 observations of the share of imports from suppliers 1 in total energy imports, that is, $\gamma = \left(\frac{0}{2000}, \frac{1}{2000}, \frac{2}{2000}, \dots, \frac{2000}{2000}\right)$. From this, we will calculate the respective values of $\lambda(\gamma)$ (i.e., equation (3.7)) and the 7600 versions of $\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$ (i.e., equation (3.3)). With these values, we will:

- calculate the correlation coefficient between $\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$ and $\lambda(\gamma)$;
- compare the values of $\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$ and $\lambda(\gamma)$, given changes in γ, and;
- obtain the optimal value of γ which maximizes $\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$.

[72] Since $0 \leq d_{ij} \leq 1$, then $0 \leq d_{ij}\omega_i\omega_j \leq \omega_i\omega_j$. Also, as $d_{ij} = 0\ \forall i = j$, so $0 \leq \sum_{i,j} d_{ij}\omega_i\omega_j = \sum_{i,j\ (i\neq j)} d_{ij}\omega_i\omega_j \leq \sum_{i,j\ (i\neq j)} \omega_i\omega_j$. Note that $\sum_{i,j\ (i\neq j)} \omega_i\omega_j = \sum_{i,j} \omega_i\omega_j - \sum_i {\omega_i}^2$. Since $\sum_{i,j} \omega_i\omega_j = \sum_i \omega_i \sum_j \omega_j$ and $\sum_i \omega_i = 1$, then $\sum_{i,j\ (i\neq j)} \omega_i\omega_j = 1 - \sum_i {\omega_i}^2 = \lambda_D$. Therefore, $0 \leq \Delta \leq \lambda_D < 1$.

[73] Equations (3.5) and (3.6) yields, respectively: $\lambda = 2\gamma(1-\gamma)$ and $\Delta = d_{12}2\gamma(1-\gamma) = d_{12}\lambda$.

3.5.1 Correlation coefficient

The correlation coefficient is equal to: $\rho^{S}_{(1-\sigma)} = \frac{\sum_{i=1}^{m}\left(\mathrm{E}^{S}_{(1-\sigma)}(\gamma_i)-\overline{\mathrm{E}^{S}_{(1-\sigma)}}\right)(\lambda(\gamma_i)-\bar{\lambda})}{\sqrt{\sum_{i=1}^{m}\left(\mathrm{E}^{S}_{(1-\sigma)}(\gamma_i)-\overline{\mathrm{E}^{S}_{(1-\sigma)}}\right)^2 \sum_{i=1}^{n}(\lambda(\gamma_i)-\bar{\lambda})^2}}$, where m is the number of observations of γ, that is, $m = 2001$, $\overline{\mathrm{E}^{S}_{(1-\sigma)}} = \frac{\sum_{i=1}^{m}\mathrm{E}^{S}_{(1-\sigma)}(\gamma_i)}{m}$ is the mean of the degree of energy security and $\bar{\lambda} = \frac{\sum_{i=1}^{m}\lambda(\gamma_i)}{m}$ is the mean of the diversity index.

Figure 3.1 shows the values obtained for the correlation coefficient for each scenario and dependence ratio. We can see that, when the dependence ratio is small (i.e., $0.05 < (1-\sigma) < 0.197$)[74], there is a positive correlation between energy security and the supplier diversity of energy imports. That is, in this case, a greater degree of greater energy security is associated with greater supplier diversity. On the other hand, when the dependence ratio is large (i.e., $0.197 < (1-\sigma) \leq 1$), the correlation is negative. In other words, in this case, a higher degree of energy security is associated with a lower supplier diversity, which means that it is associated with a greater concentration of suppliers. Moreover, when the dependence ratio is close to 0.197, the correlation is insignificant. This means that, in this case, there is no clear relationship between supplier diversity of energy imports and energy security.

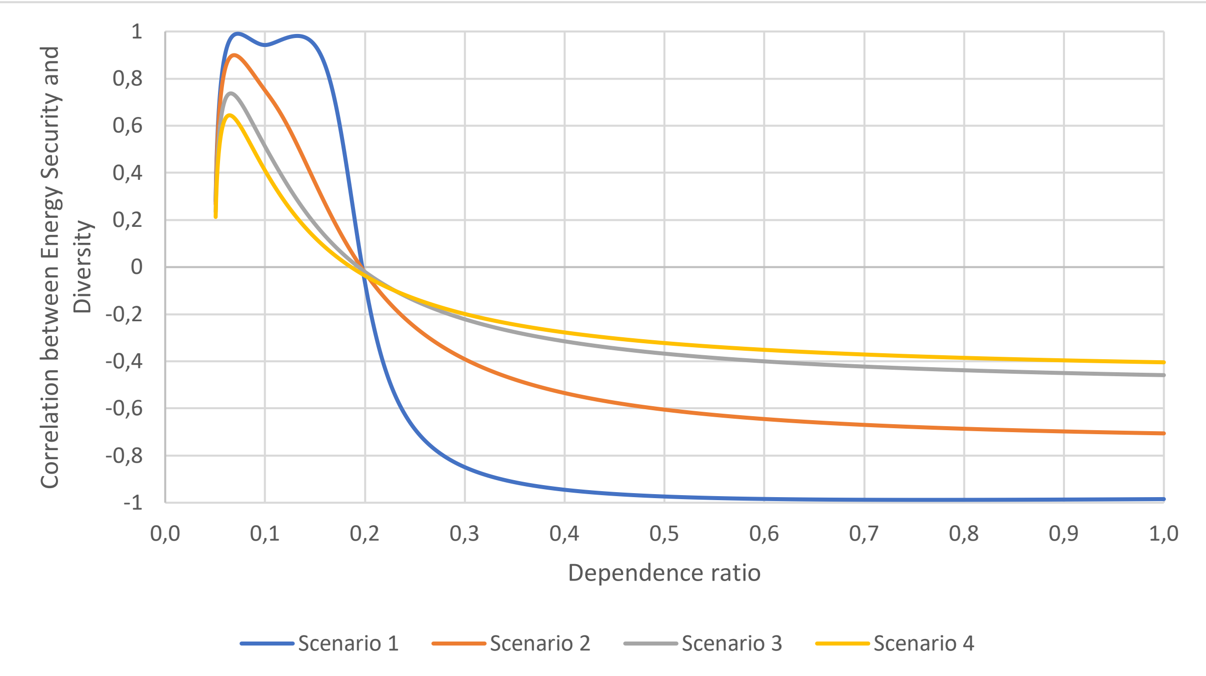

Figure 3.1 – Correlation coefficient between supplier diversity of energy imports and degree of energy security for each scenario and dependence ratio

[74] The value 0.1970 refers to scenario 1. The respective values for scenarios 2, 3, and 4 are 0.196, 0.1935, and 0.186. This means that the range in which there is a positive correlation between the supplier diversity of energy imports and energy security does not change significantly between the scenarios.

Source: Author's own elaboration.

Figure 3.1 also shows that, for small and large dependence ratio values, scenario 1 presents a strong correlation, reaching, in some cases, absolute values very close to 1. However, the lower the relative security of supplier 2 to supplier 1, the weaker the correlation is. That is, although in scenario 1 the correlation coefficient, in absolute values, reaches values close to 1, its value decreases progressively in scenarios 2, 3, and 4.

Therefore, the assumption, widely used in the literature, of a positive relationship between energy security and supplier diversity of energy imports holds only when energy import dependency is small and the level of threats is similar between energy suppliers. Furthermore, supplier diversity does not become more relevant to energy security if the import dependency is high. On the contrary, its relevance decreases, since, in this case, a greater degree of energy security is associated with a greater concentration (i.e., less diversity) of suppliers.

3.5.2 Changes in γ

Although greater energy security is associated with greater diversity in situations of low energy import dependency, this does not mean that diversifying energy imports necessarily improves the degree of energy security in this case. Figure 3.2 shows the diversity index ($\lambda(\gamma)$) and the degree of energy security ($\mathrm{E}^{S}_{(1-\sigma)}(\gamma)$) for selected values of the dependence ratio ($(1-\sigma) = (0.12, 0.2, 0.9)$ and for each scenario. We can see that, in scenario 1 even when the dependence ratio is equal to 0.12 and consequently $\rho^{1}_{0.12} = 0.9706$ (see Figure 3.1), there are some situations where diversifying imports worsen the degree of energy security. For example, if $\gamma = 0.417$, then $\mathrm{E}^{1}_{0.12}(0.417) = 0.9964$ and $\lambda(0.417) = 0.4862$. If we diversify imports by changing the value of γ to 0.5, although the diversity index reaches its maximum (i.e., $\lambda(0.5) = 0.5$), the degree of energy security is reduced, that is, $\mathrm{E}^{1}_{0.12}(0.5) = 0.9962$.

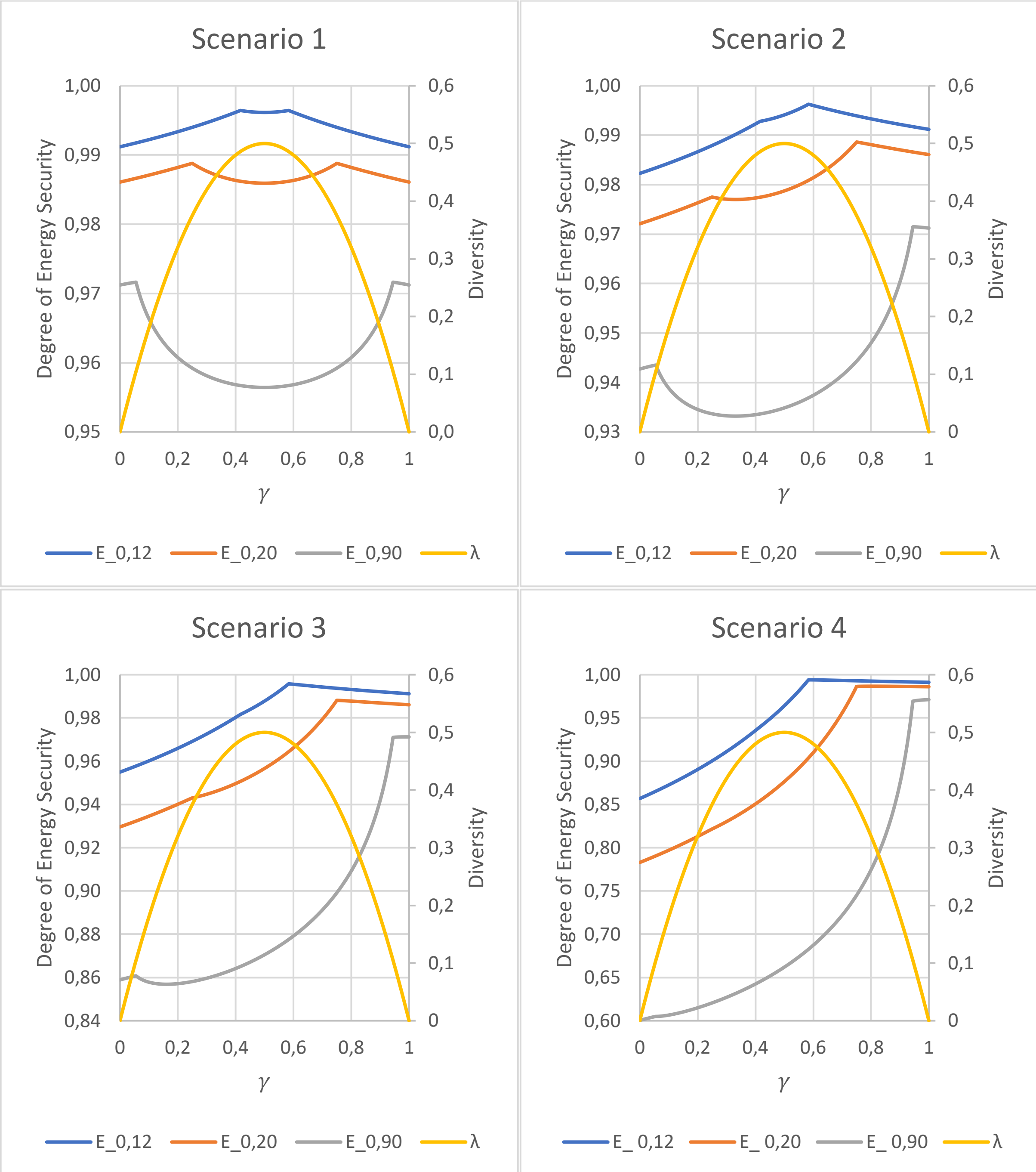

Figure 3.2 – Diversity index and degree of energy security for each scenario and selected values of the dependence ratio
Source: Author's own elaboration.

The analog is also true for cases where the import dependency is high. In this case, greater energy security is associated with greater concentration, however, this does not mean that concentrating energy imports necessarily improves the degree of energy security. Figure 3.2 shows in scenario 1 that, even when the dependence ratio is equal to 0.9 and consequently $\rho^1_{0.9} = -0.9866$ (see Figure 3.1), there are some cases where concentrating imports decreases the degree of energy security. For example, if $\gamma = 0.9445$, then $\lambda(0.9445) = 0.1048$ and

$\mathrm{E}_{0.9}^{1}(0.9445) = 0.9716$. Nevertheless, if we concentrate imports on supplier 1, that is, $\gamma = 1$, albeit the diversity index reaches its minimum (i.e., $\lambda(1) = 0$)[75], the degree of energy security is reduced, that is, $\mathrm{E}_{0.9}^{1}(1) = 0.9712$.

Nevertheless, in both cases, the situations in which the diversification (or concentration) of energy imports worsens the degree of energy security are scarce and with very limited effect. Therefore, when the level of threats of each energy supplier is similar, the diversity index can be used as a *proxy* for the ordinal measure of energy security (or at least part of it). It should be noted that, when the energy import dependency is small, the diversity index (λ) must be used, on the other hand, when the energy import dependency is high, the concentration index (HHI) must be used. However, even in these situations of strong correlation, the diversity index or the concentration index remains an inaccurate guide to energy security.

Figure 3.2 also shows that, as the relative security of supplier 2 in relation to supplier 1 decreases (i.e., scenarios 2, 3, and 4), the number of cases in which the diversification (or concentration) of energy imports worsens the degree of energy security increases and its effects become more relevant. Thus, when the level of threats between the energy suppliers is different (i.e., the correlation is weak), the diversity index (or the concentration index) is not a good measure for the ordinal meaning of energy security. Moreover, Figure 3.2 demonstrates that, in all cases, the diversity index does not represent a cardinal measure of energy security. That is, although changes in γ may result in variations with the same sign (i.e., increase or decrease) in the degree of energy security and diversity index, the magnitudes of these variations are, in general, different.

3.5.3 Optimal value

If the diversification of energy imports is not a guarantee of energy security, then what is the best strategy for energy security? To answer this question, we will obtain the optimal value, γ^*, that solves: $\max_{\gamma} \mathrm{E}_{(1-\sigma)}^{1}(\gamma)$.

Figure 3.3 shows the optimal values of the share of imports from suppliers 1 (γ^*) for the degree of energy security, by scenario and dependence ratio. It is possible to note that, when the dependence ratio is small (i.e., $0.05 < (1-\sigma) \leq 0.10$), then γ^* is very close to 0.5.[76] This value is also the optimal value of the diversity index (i.e., $\gamma^* = 0.5$ is the optimal value that

[75] This means that the HHI concentration index reaches its maximum, that is, $HHI(1) = 1$.
[76] It is noteworthy that for scenario 1, when $0.05 < (1-\sigma) \leq 0.10$, then $\gamma^* = 0.5$. However, the others scenarios may have optimal values slightly greater than 0.5. That is, when $0.05 < (1-\sigma) \leq 0.10$, scenarios 2, 3, and 4 presents, respectively, $\gamma^* \in [0.5, 0.5005]$, $\gamma^* \in [0.5, 0.503]$, and $\gamma^* \in [0.5, 0.5115]$.

solves $\max_{\gamma} \lambda(\gamma)$). In other words, in this case, γ^* maximizes both the degree of energy security and the diversity index. This means that, when energy import dependency is low, the best strategy for energy security is the total diversity of energy imports.

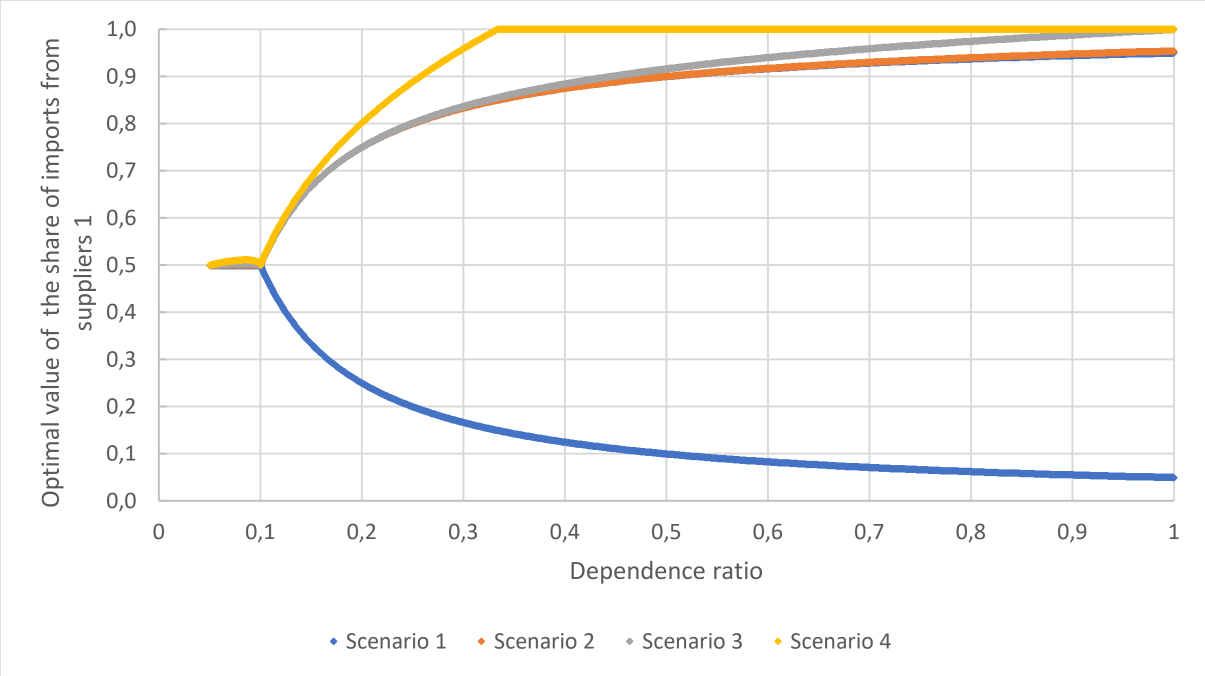

Figure 3.3 – Optimal values of the share of imports from suppliers 1 for the degree of energy security, by scenario and dependence ratio
Source: Author's own elaboration.

On the other hand, when the dependence ratio is high, the optimal value γ^* is different from 0.5. That is, in this case, γ^* is associated with a certain degree of concentration, in such a way that this value is different from that which maximizes the diversity index. In addition, the comparison of the scenarios shows that the optimum value, γ^*, is associated with the concentration of energy imports in the most secure supplier (i.e., in supplier 1).[77] Furthermore, as the dependence ratio increases, the optimal value γ^* increases and the speed of this increase is more pronounced in scenarios where the relative security of supplier 1 in relation to supplier 2 is greater (i.e., in scenarios 2, 3, and 4). This means that, when the energy import dependency is high, the best strategy for energy security is not the total diversity of energy imports. On the contrary, in this case, the best strategy for energy security is to concentrate energy imports, to a certain degree, on the most secure supplier (i.e., on the supplier that has the lowest level of

[77] It should be noted that scenario 1 is a peculiar situation because $t_{M_2} = t_{M_1} = 0.01$. When $t_{M_2} = t_{M_1}$, then $\mathrm{E}^S_{(1-\sigma)}(\gamma) = \mathrm{E}^S_{(1-\sigma)}(1-\gamma)$. Thus, in scenario 1, if γ^* is an optimal value of $\max_{\gamma} \mathrm{E}^1_{(1-\sigma)}(\gamma)$, hence $1-\gamma^*$ is also an optimal value. Therefore, in scenario 1, it does not matter whether the concentration occurs on supplier 1 or supplier 2, since both are equally secure, that is, both present the same level of threats.

threats) and that concentration must be adjusted whenever energy import dependency increases, that is, less secure suppliers should be replaced by more secure ones.

3.6 Conclusion

This article used the definition of energy security developed in the second essay of this Thesis and the definition of diversity proposed by Stirling (2007, 2010a) to analyze the relationship between energy security and supplier diversity of energy imports. The article found that this relationship is not straightforward; it depends on the level of threats (i.e., relative security) between suppliers and the level of energy import dependency.

The article argued that, in the literature, the defense of the supplier diversity of energy imports in favor of energy security is based on an assumption, rather than a result, or conclusion, of the methodological frameworks for energy security. We showed that this assumption of a positive relationship between energy security and supplier diversity of energy imports holds only when energy import dependency is small and when the level of threats of each energy supplier is similar. In addition, supplier diversity does not become more relevant to energy security if the energy import dependency is high. On the contrary, its relevance decreases, since the correlation is negative in this case. That is, when the energy import dependency is high, a greater degree of energy security is associated with a greater concentration of suppliers.

Also, the article showed that, although the diversity index, or concentration index, is an inaccurate guide, it can be used as a *proxy* for the ordinal measure of energy security (or at least part of it), when the level of threats between suppliers is similar. We emphasize that, when the energy import dependency is small, the diversity index must be used, on the other hand, when the energy import dependency is high, the concentration index must be used. However, when the levels of threats are different, the diversity index and the concentration index are not a good measure for the ordinal meaning of energy security. Moreover, in all cases, these indices do not represent a cardinal measure of energy security.

Since diversification is not a guarantee of energy security, our results also provided policy recommendations of the utmost importance. In general, when the energy import dependency is low, suppliers should be diversified in energy import. That is, in this case, the best strategy for energy security is total diversity. On the other hand, when the energy import dependency is high, energy imports should be concentrated, to a certain degree, in the most secure supplier. In addition, as the energy import dependency increases, less secure suppliers should be replaced by more secure ones.

It is noteworthy that the analysis carried out to verify the relationship between supplier diversity of energy imports and energy security has limitations. In addition to the limitations of the energy security model exposed in the second essay of this Thesis, some caveats are still needed. First, we assume that the random variable that represents energy supply disruptions in the energy supply chain are independent, however, they may not be. For example, as highlighted by Huntington (2011), the revenue received by oil-exporting countries may finance terrorism or belligerent dictators controlling oil resources. Thus, a certain degree of concentration of energy imports in oil-exporting countries with such characteristics could destabilize oil-exporting regions or other exporting countries that are located nearby. Therefore, the joint probability of energy supply disruptions would become greater. Second, we assume that the levels of threats are exogenous. Nevertheless, the concentration, to some extent, of energy imports in a particular supplier could increase their bargaining power, in such a way that this energy supplier could use energy as a political weapon more effectively. Therefore, increasing the level of threats from this supplier and so making the energy supply chain less secure. In this case, the level of threats would be endogenous to the model.

Appendix 3.A Assumptions on energy security

In this appendix, we will derive the degree of energy security expressed in equation (3.2). It is noteworthy that D_{M_1} and D_{M_2} are independents, $0 \leq \sigma, \gamma, D_{M_1}, D_{M_2} \leq 1$, and $e < 1$. As $P\left(D_{E_s} = 0\right) = 1$, equation (3.2) can be rewritten as:

$$P\left(D_{M_1}\gamma(1-\sigma) + D_{M_2}(1-\gamma)(1-\sigma) \leq e\right) \tag{3.8}$$

Note that $0 \leq D_{M_1}\gamma(1-\sigma) + D_{M_2}\gamma(1-\sigma) \leq 1$, since $0 \leq \sigma, \gamma, D_{M_1}, D_{M_2} \leq 1$. Thus, when $e < 0$, equation (3.8) yields:

$$P\left(D_{M_1}\gamma(1-\sigma) + D_{M_2}(1-\gamma)(1-\sigma) \leq e\right) = 0 \tag{3.9}$$

Now, assume that $0 \leq e < 1$. When $\sigma = 1$, it is easy to see that equation (3.8) is equal to:

$$P(0 \leq e) = 1 \tag{3.10}$$

On the other hand, when $0 \leq \sigma < 1$, equation (3.8) yields:

$$P\left(D_{M_1}\gamma + D_{M_2}(1-\gamma) \le e_\sigma\right) \tag{3.11}$$

where $e_\sigma = \frac{e}{(1-\sigma)}$.

Since $0 \le D_{M_1}\gamma + D_{M_2}(1-\gamma) \le 1$, thus, when $e_\sigma \ge 1$, equation (3.11) is equal to:

$$P\left(D_{M_1}\gamma + D_{M_2}(1-\gamma) \le e_\sigma\right) = 1 \tag{3.12}$$

Note that $e_\sigma \ge 1$, if and only if $e \ge (1-\sigma)$. Therefore, using equations (3.10) and (3.12), it is possible to see that equation (3.2) is equal to 1 when $1 > e \ge (1-\sigma) \ge 0$. Thereby, we will derive equation (3.11) for $0 \le e < (1-\sigma) \le 1$ (i.e., $0 \le e_\sigma < 1$).

Let $\Gamma(\alpha)$ the gamma function, $B(x;\alpha,\beta)$ the incomplete beta function, $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ the beta function and $I_x(\alpha,\beta) = \frac{B(x;\alpha,\beta)}{B(\alpha,\beta)}$ the cumulative distribution function of the beta distribution. We are assuming that D_{M_1} and D_{M_2} have beta distribution, that is, $D_{M_i} \sim Beta(\alpha_i, 1)$, where $\alpha_i = \frac{t_i}{1-t_i}$. This means that the probability density functions are equal to $f_{D_{M_i}}(x|t_i) = \begin{cases} \alpha_i x^{\alpha_i - 1} & x \in [0,1] \\ 0 & x \notin [0,1] \end{cases}$ and that cumulative distribution function is equal to $F_{D_{M_i}}(x|t_i) = I_x(\alpha_i, 1)$. Note that $\alpha_i > 0$, since $t_i \in (0,1)$. It is easy to see that when $\gamma = 1$ and $\gamma = 0$, equation (3.11) yields respectively:

$$P\left(D_{M_1} \le e_\sigma\right) = I_{e_\sigma}(\alpha_1, 1) \tag{3.13}$$

$$P\left(D_{M_2} \le e_\sigma\right) = I_{e_\sigma}(\alpha_2, 1) \tag{3.14}$$

In this way, we must find the value of equation (3.11) when $0 < \gamma < 1$. Let $A = \left\{\left(x_{M_1}, x_{M_2}\right) \middle| x_{M_1}\gamma + x_{M_2}(1-\gamma) \le e_\sigma, 0 \le x_{M_1} \le 1, 0 \le x_{M_2} \le 1\right\}$ the integration area of equation (3.11). Figure 3.4 shows that $A = A_1 \cup A_2$ and $A_1 \cap A_2 = \emptyset$, where $A_1 = \left\{\left(x_{M_1}, x_{M_2}\right) \middle| x_{M_1}\gamma + (1-\gamma) \le e_\sigma, 0 \le x_{M_1} \le 1, 0 \le x_{M_2} \le 1\right\}$ and $A_2 = \left\{\left(x_{M_1}, x_{M_2}\right) \middle| e_\sigma - (1-\gamma) < x_{M_1}\gamma \le e_\sigma - x_{M_2}(1-\gamma), 0 \le x_{M_1} \le 1, 0 \le x_{M_2} \le 1\right\}$. In this way, equation (3.11) can be rewritten as: $P\left(D_{M_1}\gamma + D_{M_2}(1-\gamma) \le e_\sigma\right) = P(A_1) + P(A_2)$. When $0 \le e < (1-\sigma) \le 1$ (i.e., $0 \le e_\sigma < 1$) and $0 < \gamma < 1$, it is possible to see through Figure 3.4 that:

$$P(A_1) = \int_0^{\hat{\varphi}} \alpha_1 \left(x_{M_1}\right)^{\alpha_1 - 1} \left[\int_0^1 \alpha_2 \left(x_{M_2}\right)^{\alpha_2 - 1} dx_{M_2} \right] dx_{M_2} \tag{3.15}$$

$$P(A_2) = \int_{\varphi\left(\frac{e_\sigma}{\gamma}\right)}^{\psi\left(\frac{e_\sigma}{\gamma}\right)} \alpha_1 \left(x_{M_1}\right)^{\alpha_1 - 1} \left[\int_0^{\frac{e_\sigma - \gamma x_{M_1}}{(1-\gamma)}} \alpha_2 \left(x_{M_2}\right)^{\alpha_2 - 1} dx_{M_2} \right] dx_{M_1} \tag{3.16}$$

where $\varphi = \max\left(0, \frac{e_\sigma - (1-\gamma)}{e_\sigma}\right) = \frac{\gamma}{e_\sigma} \max\left(0, \frac{e_\sigma - (1-\gamma)}{\gamma}\right)$, $\hat{\varphi} = \varphi\left(\frac{e_\sigma}{\gamma}\right) = \max\left(0, \frac{e_\sigma - (1-\gamma)}{\gamma}\right)$ and $\psi = \min\left(\frac{\gamma}{e_\sigma}, 1\right) = \frac{\gamma}{e_\sigma} \min\left(1, \frac{e_\sigma}{\gamma}\right)$.

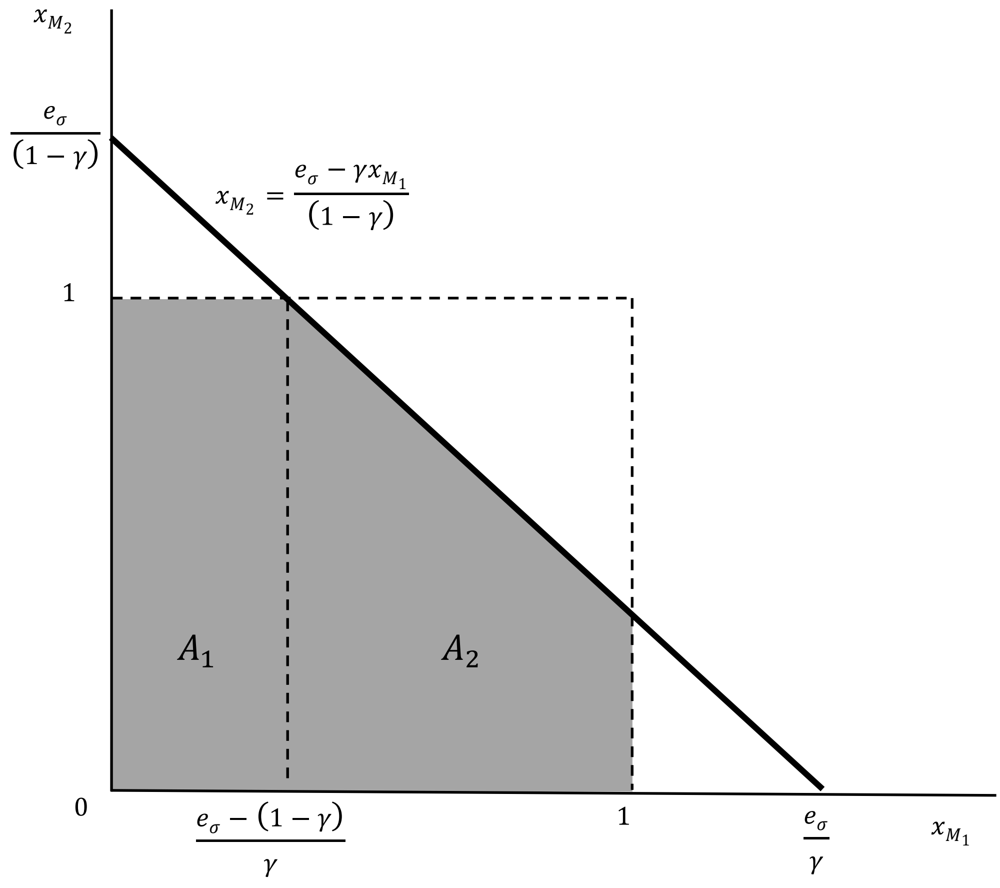

Figure 3.4 – Integration area for the probability of the degree of energy security
Source: Author's own elaboration.

Since $\int_0^1 \alpha_2 \left(x_{M_2}\right)^{\alpha_2 - 1} dx_{M_2} = 1$, then equation (3.15) is equal to:

$$P(A_1) = F_{D_{M_1}}(\hat{\varphi} | t_1) = I_{\hat{\varphi}}(\alpha_1, 1) \tag{3.17}$$

Since $\int_0^{\frac{e_\sigma-\gamma x_{M_1}}{(1-\gamma)}} \alpha_2\left(x_{M_2}\right)^{\alpha_2-1} dx_{M_2} = \left(\frac{e_\sigma-\gamma x_{M_1}}{(1-\gamma)}\right)^{\alpha_2}$, then it is easy to see that equation (3.16) yields:

$$P(A_2) = \alpha_1\left(\frac{e_\sigma}{(1-\gamma)}\right)^{\alpha_2} \int_{\varphi\left(\frac{e_\sigma}{\gamma}\right)}^{\psi\left(\frac{e_\sigma}{\gamma}\right)} \left(x_{M_1}\right)^{\alpha_1-1}\left(1-\frac{\gamma}{e_\sigma}x_{M_1}\right)^{\alpha_2} dx_{M_1} \quad \textbf{(3.18)}$$

Using the method of integration by substitution, we have: $u = \frac{\gamma}{e_\sigma}x_{M_1}$ and $du = \frac{\gamma}{e_\sigma}dx_{M_1}$. Note that $\psi\left(\frac{e_\sigma}{\gamma}\right)\frac{\gamma}{e_\sigma} = \psi$ and $\varphi\left(\frac{e_\sigma}{\gamma}\right)\frac{\gamma}{e_\sigma} = \varphi$. Thus, equation (3.18) is equal to:

$$P(A_2) = \alpha_1 \mathrm{Z} \int_{\varphi}^{\psi} (u)^{\alpha_1-1}(1-u)^{\alpha_2} du \quad \textbf{(3.19)}$$

where $\mathrm{Z} = \left[\frac{(e_\sigma)^{\alpha_1+\alpha_2}}{(\gamma)^{\alpha_1}(1-\gamma)^{\alpha_2}}\right]$.

It is easy to see that $\int_\varphi^\psi (u)^{\alpha_1-1}(1-u)^{\alpha_2}du = B(\psi;\alpha_1,\alpha_2+1) - B(\varphi;\alpha_1,\alpha_2+1)$. In addition, note that $\alpha_1 B(\alpha_1,\alpha_2+1) = \frac{\Gamma(\alpha_1+1)\Gamma(\alpha_2+1)}{\Gamma(\alpha_1+\alpha_2+1)}$. Thus, multiplying and dividing equation (3.19) by $B(\alpha_1,\alpha_2+1)$ yields:

$$P(A_2) = \mathrm{Z}\hat{\Gamma}\left[I_\psi(\alpha_1,\alpha_2+1) - I_\varphi(\alpha_1,\alpha_2+1)\right] \quad \textbf{(3.20)}$$

where $\hat{\Gamma} = \left[\frac{\Gamma(\alpha_1+1)\Gamma(\alpha_2+1)}{\Gamma(\alpha_1+\alpha_2+1)}\right]$.

Therefore, using equations (3.17) and (3.20) (i.e., $P(A_1) + P(A_2)$), we have that, when $0 \le e < (1-\sigma) \le 1$ and $0 < \gamma < 1$, equation (3.11) yields:[78]

$$I_{\hat{\varphi}}(\alpha_1,1) + \mathrm{Z}\hat{\Gamma}\left[I_\psi(\alpha_1,\alpha_2+1) - I_\varphi(\alpha_1,\alpha_2+1)\right] \quad \textbf{(3.21)}$$

[78] Note that when $e = 0, \mathrm{Z} = 0, \psi = 1, \varphi = 0$, and $\hat{\varphi} = 0$. This implies that $I_\psi(\alpha_1,\alpha_2+1) = 1, I_\varphi(\alpha_1,\alpha_2+1) = 0$, and $I_{\hat{\varphi}}(\alpha_1,1) = 0$. Therefore, when $e = 0$ and $0 < (1-\sigma) \le 1$, hence $P(A_1) = 0$ and $P(A_2) = 0$.

Thus, using equations (3.9), (3.10), (3.12), (3.13), (3.14), and (3.21), it is possible to see that equation (3.2) is equal to:

$$\begin{cases} 0 & e < 0 \\ 1 & 1 \geq e \geq (1-\sigma) \geq 0 \\ I_{e_\sigma}(\alpha_1, 1) & 0 \leq e < (1-\sigma) \leq 1; \gamma = 1 \\ I_{e_\sigma}(\alpha_2, 1) & 0 \leq e < (1-\sigma) \leq 1; \gamma = 0 \\ I_{\widehat{\varphi}}(\alpha_1, 1) + \mathrm{Z}\hat{\Gamma}\left[I_\psi(\alpha_1, \alpha_2 + 1) - I_\varphi(\alpha_1, \alpha_2 + 1)\right] & 0 \leq e < (1-\sigma) \leq 1; \gamma \in (0,1) \end{cases}$$

CONCLUSION

Through three essays, this Thesis sought to broaden the theoretical foundations of the rebound effect and energy security. The first essay, in expanding Wei's (2010) general equilibrium model, has attempted to contribute to broadening the theoretical foundation of macroeconomic rebound effects. It demonstrated several theoretical mechanisms that govern the rebound effect (i.e., direct effect, cross-price effect, input price effect, output price effect, and energy price effect), highlighting when they can amplify or mitigate the rebound size. Regarding the reallocation effect, we point out the importance of the direct effect. That is, the greater the price elasticity of demand for energy service i (direct effect), the greater the chances of backfire. Nevertheless, due to the indirect effects, however great this elasticity is, the rebound effect can still fit into any of the five rebound conditions (backfire, full rebound, partial rebound, zero rebound, or super-conservation). We also show the importance of the energy supply for the rebound magnitude. That is, the more price-inelastic is the energy supply, the rebound effect will be closer to the full rebound condition and, in the extreme case when the energy supply is fixed, this condition will be checked. Furthermore, we find that the number of energy services is relevant to the rebound size. This is because, when the model includes only a single energy service, super-conservation is not allowed.

The first essay also showed under what circumstances the long-run effect is greater or less than the short-run effect. Regarding the reallocation effect, we show that, in the simplest model, the long-run effect will always be greater than or equal to the short-run effect. This is explained by the Le Chatelier principle. When the model includes more than one energy service and/or endogenizes the output price or energy price, the long-run effect may be greater or less than the short-run effect. Furthermore, the first essay showed that, whenever the production function is homogeneous, this effect will generate backfire. In addition, we found that when the production function is homogeneous and the non-energy inputs are gross substitutes for the energy service, the short-run innovation effect will generate a more powerful backfire than the long-run effect. On the other hand, if the non-energy inputs are gross complements for the energy service, the short-run effect will be less than the long-run effect and may not generate backfire. Finally, we point out that backfire is definitely a problem in terms of welfare in situations where energy consumption is based on highly polluting energies (e.g., fossil fuels) and where output is highly energy-intensive.

The second essay seeks to contribute to the broadening of the theoretical foundation of energy security, by clarifying some points of the two main gaps in the literature, namely: the lack of consensus in its definition and the lack of a rigorous methodological framework. It developed a simplified energy security model that combines economic theory and the concept of security in a probabilistic framework. This model shows that energy security is a universal concept, but it has several meanings. Therefore, energy security can be defined in several ways. The agents' choices about the minimum desired level of utility and the maximum values for insecurity probability, as well as the forms of aggregation of utilities and probabilities, make energy security a highly subjective concept. For example, depending on the social welfare function, the concept of energy security may include the environmental and energy poverty dimensions in some cases, while these dimensions may be omitted in others. This means that personal judgments are an integral part of the energy security definition.

However, energy security is not just a matter of opinion; there is consistency in its reasoning, ranging from premises to conclusions and then to prescriptions. In this way, the model developed in the second essay incorporates the multidimensionality of energy security in a rigorous methodological framework, in such a way that it allows an integration of different dimensions of energy security. Although the model does not include all dimensions, as they are numerous, it presents a logical mechanism that determines how the different dimensions interact with each other and consequently how they affect energy security. This is illustrated by the relationship between energy price and energy security. Despite this relationship cannot be determined in general, it is possible to determine, via the concept of preferable energy security strategy, in which situations energy price increases improve or worsen the degree of energy security. Therefore, the operationalization of the model can guide energy policies to improve energy security.

The third essay has attempted to analyze the relationship between energy security and supplier diversity of energy imports. It used the definition of diversity proposed by Stirling (2007, 2010a) and the definition of energy security proposed in the second essay. The third essay found that the relationship between energy security and supplier diversity of energy imports is not straightforward; it depends on the level of threats (i.e., relative security) between suppliers and the level of energy import dependency. This relationship is positive only when energy import dependency is small and when the level of threats of each energy supplier is similar. In addition, supplier diversity does not become more relevant to energy security if the energy import dependency is high. On the contrary, its relevance decreases, since, in this case, the correlation is negative. That is, when the energy import dependency is high, this relationship is negative.

Also, the third essay showed that, although the diversity index, or concentration index, is an inaccurate guide, it can be used as a *proxy* for the ordinal measure of energy security (or at least part of it), when the level of threats between supplier is similar. We emphasize that, when the energy import dependency is small, the diversity index must be used, on the other hand, when the energy import dependency is high, the concentration index must be used. However, when the level of threats is different, the diversity index and the concentration index are not a good measure for the ordinal meaning of energy security. Moreover, since diversification is not a guarantee of energy security, our results also provided policy recommendations of the utmost importance. In general, when the energy import dependency is low, suppliers should be diversified in energy import. That is, in this case, the best strategy for energy security is total diversity. On the other hand, when the energy import dependency is high, energy imports should be concentrated, to a certain degree, in the most secure supplier. In addition, as the energy import dependency increases, less secure suppliers should be replaced by more secure ones.

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