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## **Abstract**

The objective of the present article is to build a *Stock-and-Flow Consistent* version of the IS-LM model for a closed economy with endogenous money supply in order to analyze the dynamic properties of the model. We show that once the capacity effect of investment is taken into account, the equilibrium of the model is no longer characterized by a stationary level of real output; but by *a constant growth rate of real output*. The level of activity is represented in the model by the variable capacity utilization, which is constant and lower than one over the equilibrium path. This means that the SFC version of the IS-LM model reproduces the traditional Keynesian result of *underemployment equilibrium*. Moreover, in steady-state disposable income, capital stock and private wealth all grow at a same constant rate. Regarding the comparative dynamics of the model, we had performed some numerical simulations about the dynamic effects of shocks over the time path of endogenous variables. We showed that some traditional results of Keynesian Theory as the “paradox of thrift” and the demand-led nature of economic growth still hold in a SFC framework.

**Keywords:** Post-Keynesian Growth, Stock-Flow Consistency, Simulation Models

**JEL Classification** – E12, E37, P10

# 1 Introduction

Since the seminal paper “Mr. Keynes and the Classics: a suggested interpretation” written by John Hicks in 1937, IS-LM model has become the main theoretical framework for the exposition and spreading of Keynesian ideas all over the world. Despite the strong criticism of some Post-Keynesian economists as Luigi Pasinetti, who claimed that IS-LM model deformed the central message of Keynes’s General Theory, neutralizing its revolutionary spirit (Pasinetti and Mariutti, 2008, p.59); the IS-LM model has been perceived ever since as a true piece of Keynesian economics.

One of the major weaknesses of the IS-LM apparatus is the clear inconsistency between flows and stocks. Indeed, not only the capacity effect of investment is not taken into account, but also the effect of private and government savings over the stock of public bonds is completely neglected. In words of Godley and Lavoie (2007) the IS-LM model has a huge “black hole” inside it, since nothing is said about what families do with their savings or how government finance the excess of public expenditures over taxes. Some important flows of income (as savings) or expenditure (as investment) have no consequence over the magnitude of the existing stocks. They simply disappear from the model, as if they had fallen in a “black hole”.

The theoretical problem posed by this inconsistency is that the results of comparative statics of the IS-LM model – for instance, the partial derivatives of equilibrium level of output and interest rates relative to changes in government expenditures and money supply – could be completely different if the relation between flows and stocks are taken into account. This is a very weak basis for Keynesian ideas.

The objective of this article is precisely to correct this deficiency of the IS-LM apparatus. We will build a *Stock-and-Flow Consistent* version of the IS-LM model for a closed economy with endogenous money supply in order to analyze the dynamic properties of the model. Although the traditional specification of IS-LM model considers a constant stock of money supply, recent developments in both theory and practice of monetary policy emphasizes the role of short-term nominal interest rate as the basic policy instrument at the hands of monetary authority. This means that nominal interest rate, instead of money supply, must be taken as a policy parameter in the structure of the IS-LM model, as it is done in Romer (2000).

Once the capacity effect of investment is taken into account, the equilibrium of the model is no longer characterized by a stationary level of real output; but by *a constant growth rate of real output*. The level of activity is represented in the model by the variable capacity utilization, which is constant and lower than one over the equilibrium path. This means that the SFC version of the IS-LM model reproduces the traditional Keynesian result of *underemployment equilibrium*. Moreover, in steady-state disposable income, capital stock and private wealth all grow at a same constant rate. Finally, the distribution of wealth between money and bonds is also constant in steady-state growth.

Regarding the comparative dynamics of the model, we had performed some numerical simulations about the dynamic effects of shocks over the time path of endogenous variables. We have tested changes in the autonomous rate of capital accumulation, the propensity to consume out of disposable income, the coefficient of profit retention, the tax rate, the nominal interest rate and the wage share. One important and interesting result is that the qualitative effects of changes in the parameters of investment and consumption functions are completely different. Although an increase in the propensity to consume out of disposable income generated an increase in the level of capacity utilization, an increase in the growth rate of capital stock and private wealth and a decrease in the public debt/GDP ratio; an increase in the autonomous rate of capital accumulation generated just the opposite effects. This means that in the SFC version of IS-LM model the level and growth rate of economic activity can be stimulated only by expenditures that do not create productive capacity.

Another interesting result is about the old “paradox of thrift”. An increase in the coefficient of profit retention – which is equivalent to an increase in the propensity to save out of profits – resulted in a reduction in the level of capacity utilization, a reduction in the growth rate of capital stock and private wealth and an increase in the ratio of public debt to GDP<sup>1</sup>. This means that in our version of IS-LM model any attempt of firms to increase its rate of expansion by means of increasing retained profits is self-defeating,

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<sup>1</sup> This result replicates the “paradox of thrift” in the growth and distribution model of Joan Robinson. See Harcourt (2006, p. 29).

since it will result in a decrease in the rate of capital expansion for the economy as a whole.

Regarding the effects of changes in fiscal policy over the dynamic behavior of the economy, the model showed that an increase in the tax rate – in a possible attempt of the government to reduce fiscal deficit and the ratio of public debt to GDP - is followed first by an increase in the public debt to GDP ratio, which is reversed after some periods. This strange behavior of the ratio of public debt to GDP resulted from the fact the immediate impact of the increase in taxes is to reduce the level of capacity utilization and the growth rate of capital stock due to its negative impact over disposable income and, therefore, over consumption and aggregate demand. Over time, however, as output growth stabilize at a lower level, the effect of a higher tax rate over government receipts tends to become dominant, producing a reduction in fiscal deficit and hence in the public debt to GDP ratio.

Finally, the model showed a clear *wage-led* regime of accumulation<sup>2</sup>, since an increase in the wage share resulted in an increase in the growth rate of both capital stock and private wealth and also an increase in the level of capacity utilization.

The article is organized in six sections, including the present introduction. In section two we presented the accounting structure and the theoretical assumptions of the SFC version of IS-LM model. In section three we will present the behavior equations of the model, that is, its formal theoretical structure. Section four is dedicated to the calibration of the model and the performing of the basic numerical simulation. In section five we perform the comparative dynamic exercises, evaluating the effects of exogenous shocks in some behavior and policy parameters over the dynamic path of the endogenous variables. In section six, we do some final remarks.

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<sup>2</sup> This is not a surprising result since we are supposing the existence of a strong accelerator effect in the investment function. If investment spending was sensible to changes in the profit share, as in Bhaduri and Marglin (1990), then this result could be reversed to a *profit-led* regime.

## 2 Accounting Structure and Theoretical Assumptions

In our model, the economy is closed (there is no import, export or capital flows) and consist of Households, Firms, Government, Central Bank. The balance sheet of these sectors is summarized in table 1 below.

**Table 1: Balance Sheet of Model IS-LM**

|                     | Households | Firms  | Government | Central Bank | $\Sigma$ |
|---------------------|------------|--------|------------|--------------|----------|
| Fixed Capital       |            | $+K_f$ |            |              | $+K_f$   |
| Money               | $+H_h$     |        |            | $-H$         |          |
| Bills               | $+B_h$     |        | $-B$       | $+B_{cb}$    |          |
| Balance (net worth) | $-V_h$     | $-V_f$ | $+V_g$     |              | $-K_f$   |
| $\Sigma$            | 0          | 0      | 0          | 0            | 0        |

Note: Positive variables are assets, while negative ones are liabilities.

Looking at balance sheet, you should notice that the only asset own's by Firms is the fixed capital (tangible goods). Thus all their funds are used to finance the purchase of new fixed capital equipment. We don't consider commercial banks in the composition of monetary system nor the issuance of corporate bonds. All funds used to finance firms comes from the portion of retained profits. Households accumulate financial wealth, which can be allocated in the form of money or buying bills issued by the government. The Central Bank is considered as an institution in its own right. The central bank purchases bills from government, thereby adding to its stock of assets. On its liability side, the central bank provides money to households. This money can take the form of either cash or deposits at the central bank. In either case, it would be high powered money. It is assumed that have zero net worth. The value of bills insured by government is the public debt. As usual all rows and columns must sum zero. The exception is the fixed capital row.

Table 2 shows the transactions-flow matrix of our model. Once again, all columns and all rows must sum zero to ensure that all transactions are taken into account. Thus we avoid black holes in the system. The government pays interest arising from government debt both to households and central bank. Interest payments each period are generated by stocks of assets in existence at the end of previous period. Because of this time lag, the

rate of interest on bills relevant in period  $t$  is the rate of interest that was set at the end of previous period, at time  $t_{-1}$ .

**Table 2: Transactions-flow matrix of Model IS-LM**

|                         | Households            | Firms   |         | Government              | Central Bank            |                  | $\Sigma$ |
|-------------------------|-----------------------|---------|---------|-------------------------|-------------------------|------------------|----------|
|                         |                       | Current | Capital |                         | Current                 | Capital          |          |
| Consumption             | $-C$                  | $+C$    |         |                         |                         |                  | 0        |
| Government Expenditures |                       | $+G$    |         | $-G$                    |                         |                  | 0        |
| Investment              |                       | $+I$    | $-I$    |                         |                         |                  | 0        |
| Wages                   | $+W$                  | $-W$    |         |                         |                         |                  | 0        |
| Taxes                   | $-T$                  |         |         | $+T$                    |                         |                  | 0        |
| Interest Payments       | $+r_{t-1} * B_{ht-1}$ |         |         | $-r_{t-1} * B_{t-1}$    | $+r_{t-1} * B_{bc t-1}$ |                  | 0        |
| Central Bank Profits    |                       |         |         | $+r_{t-1} * B_{bc t-1}$ | $-r_{t-1} * B_{bc t-1}$ |                  | 0        |
| Firms Profits           | $+P_h$                | $-P$    | $+P_f$  |                         |                         |                  | 0        |
| Change in Money         | $-\Delta H$           |         |         |                         |                         | $+\Delta H$      | 0        |
| Change in Bills         | $-\Delta B_h$         |         |         | $+\Delta B$             |                         | $-\Delta B_{bc}$ | 0        |
| $\Sigma$                | 0                     | 0       | 0       | 0                       | 0                       | 0                | 0        |

Note: Positive figures denote sources of funds, while negative ones denote uses of fund.

The government savings is the difference between government revenues and expenditures. The increasing of public debt is financed by insurance of new bills. In the opposite way, the fiscal surplus should be used to decrease public debt. By the way, the column sum of government must be zero.

We have taken the central bank's net worth to be zero, which implies that any profit it makes is always distributed to government. Here, the central bank certainly does make profits since it owns bills which yield interest payments, whereas its liabilities (money) pay no interest.

The households revenues are: wages from firms, interest payments from bills and profits distributed from firms. With all these revenues, households pay taxes, purchases goods and services from firms.

Goods sales are the only source of revenues to firms. Households buys part of goods  $C$  and government buy another part  $G$ . The current account represents the income flows within the sector, while the capital account represents sources to finance firms. For the model be consistent, the column sum must be zero. The entire resource flows entering should be spent. The firms spend their resources paying wages to household. The



difference between all flows constitutes the profit. It is assumed that part of the profits is retained to finance new investments, while the other part is distributed to the households.

### 3 Model Behavioral Structure

Social account is no able for forecast, by itself, the path taken by the economy. For this purpose, this section will present the behavior equations that explain decision making by economic agents. The behavior of firms, households, government and central bank will be displayed. Decision making is thus represented by aggregate behavior equations (like the consumption and investment function) instead of *Euller* equations coming from some problem of utility maximization. This means that rationality in the model to be presented rationality is better represented by the concept of procedural rationality in the sense of Simon (1982)<sup>3</sup>. It is also shown the calculation of short-term output and portfolio decisions. Following the logic of a CFS model, the model follows the proposal not present black holes. Everything that comes from a place is going somewhere else.

#### 3.1 Firms

To begin, equation (1) defines the level of investment in the current period. The firms must choose the investment level, defining how many capital goods must be added to the actual capital stock. The capital accumulation rate is given by (2), being  $\gamma_0$  the parameter that represents the animal spirit of entrepreneurs,  $\gamma_1$  the sensibility for level of capacity utilization and the last one  $\gamma_2$  the sensibility for interest rate. The current level of capacity utilization is given by equation (3), and full capacity output level is given by equation (4). The retained profit to finance new investments is given by (5).

$$I_d = \Delta K = GR_k * K_{-1} \quad (1)$$

$$GR_k = \gamma_0 + \gamma_1 * u_{-1} - \gamma_2 r \quad (2)$$

$$u = \frac{Y_s}{Y_{fc}} \quad (3)$$

$$Y_{fc} = \frac{K}{\sigma} \quad (4)$$

$$P_f = d * P \quad (5)$$

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<sup>3</sup> For a discussion of alternative concepts of rationality see Silvia Possas (1995).

Where:  $I_d$  is desired investment,  $\Delta K$  is the change in capital stock,  $GR_k$  is the desired growth of capital stock,  $u$  is the rate of capacity utilization,  $Y_{fc}$  is the full-capacity level of output,  $\sigma$  is the capital-output ratio,  $Y_s$  is the actual level of output,  $P_f$  is the amount of retained profits,  $d$  is the profit retention coefficient,  $P$  is the amount of aggregate profits,  $r$  is the nominal interest rate,  $K$  is the level of capital stock.

### 3.2 Households

The households receive three different types of revenues. First, as declared on equation (6), all residual profits (difference between retained profit and full profit) is distributed to households. Second, as a labor force payment, they receive wages, as can be noticed in equation (9), (9a) and (10). The last one is the interest received for holding bills, as a portfolio decision. This still could be noticed looking at equation (9) and (9a). The equation (8) and (12) declares, by the side of expenditures, the households pay taxes and consume. The households wealth's is defined in period  $t$  by the stock accumulated in  $t - 1$  plus the savings (difference between disposable income and consumption) in  $t$ .

$$P_h = P - P_f \quad (6)$$

$$V = V_{-1} + (YD - C) \quad (7)$$

$$C_d = \alpha_1 * YD + \alpha_2 * V_{-1}, \quad 0 < \alpha_2 < \alpha_1 < 1 \quad (8)$$

$$YD = W + r_{-1} * B_{h-1} + P_h - T_s \quad (9)$$

$$YD = (1 - \theta) * (W + r_{-1} * B_{h-1} + P_h) \quad (9a)$$

$$W = w * Y \quad (10)$$

Where:  $P_h$  is the amount of profits distributed to households,  $V$  is the stock of private wealth,  $YD$  is the level of disposable income,  $C_d$  is the desired consumption by households,  $W$  is the wage bill,  $w$  is the wage share,  $\theta$  is the income tax rate,  $T_s$  is the amount of taxes collected from households,  $B_{h-1}$  is the value of government bonds held by households in the last period,  $\alpha_1$  is the propensity to consume out of disposable income,  $\alpha_2$  is the propensity to consume out of total wealth.

### 3.3 Government and Central Bank

Here we show the government and central bank equations. First, in equation (11), we define how the government expenditures increase over the time. We have some positive and small parameter that allows the increasing of expenditures along the time follow the increasing of productive capital. The equation (12) shows that all taxes revenues come from income taxes out of disposable income from households. The implicit idea is that only households are taxed, in other words, profits that remain on the firm to finance news machines aren't taxed. As show in equation (13), if the public budget is on deficit, the government needs to finance through the issuance of new bills. They offer this news bills to the market. The households will demand some part of those bills. The difference between the supply and demand will be bought by central bank. The basic idea is that central bank is a residual buyer as show equation (14) and (15), since it operates monetary policy by means of fixing nominal interest rate in the money market, as it is show in equation (16). This means that money supply is fully endogenous.

$$G = \gamma * K_{-1} \quad (11)$$

$$T_d = \theta * (W + r_{-1} * B_{h-1} + P_h), \quad \theta < 1 \quad (12)$$

$$\Delta B_s = B_s - B_{s-1} = (G + r_{-1} * B_{s-1}) - (T + r_{-1} * B_{bc-1}) \quad (13)$$

$$\Delta H_s = H_s - H_{s-1} = \Delta B_{bc} \quad (14)$$

$$B_{bc} = B_s - B_h \quad (15)$$

$$r = \bar{r} \quad (16)$$

Where:  $G$  is the real flow of government expenditures,  $\gamma$  is the desired ratio between government expenditures and the stock of capital,  $\Delta B_s$  is the change in the stock of public bonds,  $\Delta H_s$  is the change in the stock of high powered money,  $B_{bc}$  is the stock of public bonds in Central Bank's portfolio,  $B_s$  is the total stock of public bonds,  $B_h$  is the stock of public bonds in the households' portfolio,  $\bar{r}$  is the nominal interest rate set by Central Bank,  $\Delta B_{bc}$  is the change in the stock of public bonds in Central Bank's portfolio.

### 3.4 Output determination

A simple way to solve the model is established by the calculation of short-term GDP. We can see in equation (17) short-term output is determined by the demand side. The superior bound for the short-term GDP is given by the production own limit set in equation (4). If the demand is greater than the product of full capacity, firms provide only the full capacity of product. Once you have short-term GDP, we can calculate all other model variables and have the desired trajectory of all variables. The next equations (18), (19) and (20) shows, by demand side, how we get the equation (20).

$$Y_{sr} = \begin{cases} C + I + G & \text{if } C + I + G < Y_{fc} \\ Y_{fc} & \text{if } C + I + G \geq Y_{fc} \end{cases} \quad (17)$$

Where we have:

$$C_t = \alpha_1 * (1 - \theta) * [w + (1 - d) * (1 - w)] * Y_t + \alpha_1 * (1 - \theta) * r * B_{h_{t-1}} + \alpha_2 * V_{t-1} \quad (18)$$

$$I_t = (\gamma_0 + \gamma_1 * u_{t-1} - \gamma_2 * r) * K_{t-1} \quad (19)$$

$$G_t = \gamma * K_{t-1} \quad (20)$$

After some algebraic manipulation, we found:

$$Y_{sr} = \frac{\alpha_1 * (1 - \theta) * r * B_{h_{t-1}} + \alpha_2 * V_{t-1} + (\gamma_0 + \gamma_1 * u_{t-1} - \gamma_2 * r) * K_{t-1} + \gamma * K_{t-1}}{1 - \alpha_1 * (1 - \theta) * [w + (1 - d) * (1 - w)]} \quad (21)$$

Equation (21) shows the determination of the short-run equilibrium value of real output as a function of the exogenous and pre-determined variables of the model.

### 3.5 Portfolio Decisions

The next equations (ADUP 1,2,3 and 4) defines portfolio restrictions. The ADUP.1 simply says that the total of the shares of each asset must sum to unity, whatever the actual values taken by the rates of return and disposable income. The next two ADUP equations imply that the vertical sum of the coefficients in the rates of return matrix must be zero. The last ADUP says that an increase or a decrease in disposable income must result in a symmetrical change in the assets. The equations of the portfolio decisions are based on

the originally proposed approach Tobin (1969). Thus, we present the equation (22), in matrix form, that households keeps part of their expected wealth in the form of asset  $i$ , and these proportions varies by variation on interest rates and the level of disposable income. In this economy, expectations are adaptive as could be seen in equations (23) and (24).

$$\lambda_{10} + \lambda_{20} = 1 \quad (\text{ADUP.1})$$

$$\lambda_{11} + \lambda_{21} = 0 \quad (\text{ADUP.2})$$

$$\lambda_{12} + \lambda_{22} = 0 \quad (\text{ADUP.3})$$

$$\lambda_{14} + \lambda_{24} = 0 \quad (\text{ADUP.4})$$

$$\begin{bmatrix} H_d \\ B_d \end{bmatrix} = \begin{bmatrix} \lambda_{10} \\ \lambda_{20} \end{bmatrix} * V^e + \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} * \begin{bmatrix} 0 \\ r_b \end{bmatrix} * V^e + \begin{bmatrix} \lambda_{14} \\ \lambda_{24} \end{bmatrix} * YD_r^e \quad (22)$$

$$V^e = V_{-1} \quad (23)$$

$$YD^e = YD_{-1} \quad (24)$$

## 4 Calibration and Basic Simulation

The model was simulated in MATLAB 2013 software environment. We calibrate our model in order to make it as close as possible to what we find in the literature<sup>4</sup>. We have on the table 3 below the values used and the parameter description.

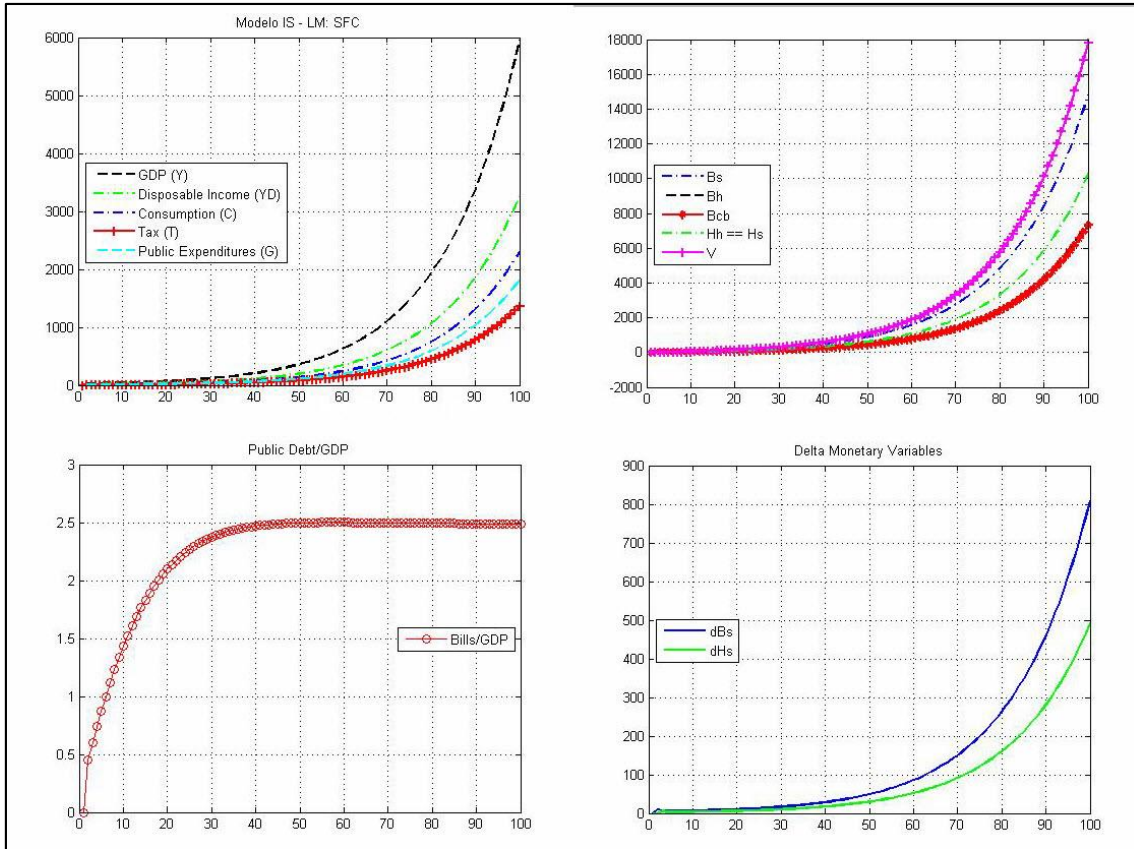
**Table 3: Description of Variables and Their Initial Calibration**

| Variable       | Description  | Numerical Value |
|----------------|--|-----------------|
| $\alpha_1$     | Propensity to consume out of disposable income               | 0.6             |
| $\alpha_2$     | Propensity to consume out of current wealth                  | 0.02            |
| $\bar{d}$      | Profit retention coefficient                                 | 0.75            |
| $\gamma_0$     | Rate of autonomous growth of capital stock                   | 0.02            |
| $\gamma_1$     | sensitivity of investment to changes in capacity utilization | 0.2             |
| $\gamma_2$     | sensitivity of investment to changes in interest rate        | 0.2             |
| $\sigma$       | Capital-output ratio   | 1.5             |
| $\theta$       | Income tax rate  | 0.3             |
| $r$            | Nominal interest rate  | 0.03            |
| $\varphi$      | Capital Stock Depreciation Rate                              | 0.1             |
| $\gamma$       | Desired ratio of government expenditures to capital stock    | 0.15            |
| $w$            | wage-share   | 0.6             |
| $\lambda_{10}$ | sensitivity of money demand to expected wealth               | 0.5             |
| $\lambda_{20}$ | sensitivity of bonds demand to expected wealth               | 0.5             |
| $\lambda_{11}$ | sensitivity of money demand to expected wealth               | -0.4            |
| $\lambda_{12}$ | sensitivity of money demand to nominal interest rate         | 0.3             |
| $\lambda_{21}$ | sensitivity of bonds demand to expected wealth               | 0.4             |
| $\lambda_{22}$ | sensitivity of bonds demand to nominal interest rate         | -0.4            |
| $K_{initial}$  | Initial level of capital stock                               | 100             |

In the figure 1, we have four quadrants. The first quadrant (top/west) shows the trajectory of the main aggregates of the real economy. In the second quadrant (top/east) we have the behavior of the monetary variables stocks. The third quadrant (bottom/west) shows the trajectory of the debt/GDP. The fourth quadrant (bottom/east) shows the variations in monetary variables. The first chart shows that we have a growth model. An important point is that the public debt / GDP converges to a positive constant rather than keep growing indefinitely.

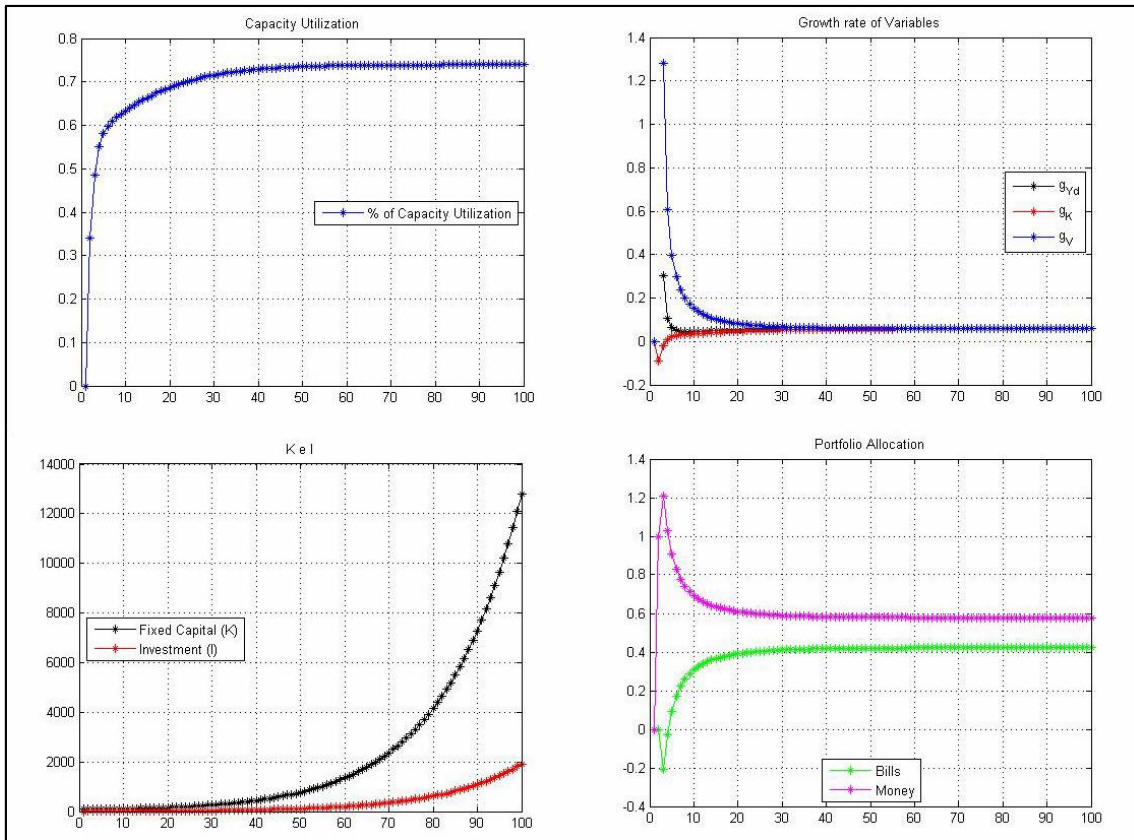
<sup>4</sup> For a detailed description of calibration methodology see Oreiro and Ono (2007);.

**Figure 1: Standard Simulation of IS-LM SFC Model**



In the figure 2, we have four quadrants. The first quadrant (top/west) shows the trajectory of the production capacity utilization. As can be seen, capacity utilization converges to a lower than one value and that means the model reproduces the traditional Keynesian result of *underemployment equilibrium*. In the second quadrant (top/east) we have the trajectory of growth rate variables. As can be seen, all rates converge to the same positive constant. This characterizes this model as a growth model. The third quadrant (bottom/west) shows the trajectory of the capital and investment levels. The fourth quadrant (bottom/east) shows the behavior of the agents in the asset portfolio composition. As can be seen, the agents after a certain time keep its assets composition constant in their portfolio.

**Figure 2: More results of IS-LM simulated model.**



As we can see in figure 2, along the steady-state growth path, disposable income, capital stock and private wealth all grow at a same constant rate. We also see that distribution of wealth between money and bonds is also constant in steady-state growth.



## 5 Comparative Dynamics

Here, we present the main effects in macroeconomic variables after some shocks. In table 4, we present these results. The shocks were given as follows: it was chosen deliver a shock at the time the model had reached its steady state, in other words, the time which the rates have converged to grow at the same value. Thus, we wait until the period 70 to add a shock. Thus, Table 3 shows the observed values of the key variables in the period 100.

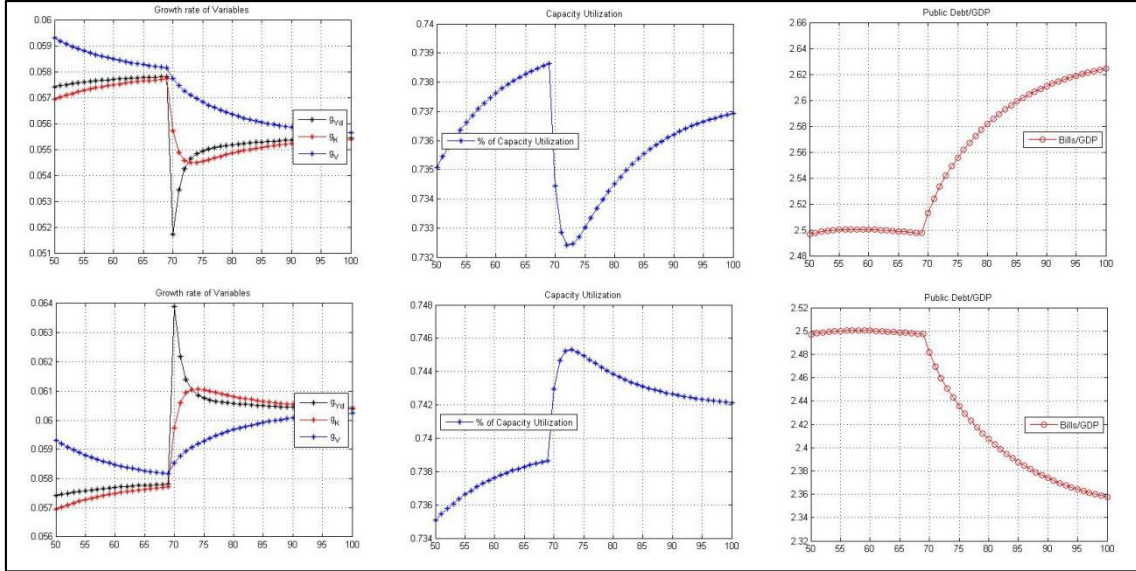
**Table 4: Results of the main variables after shocks.**

| Shock       | $g_{Y_d}$ | $g_V$ | $g_K$ | Debt/GDP | Capacity Utilization |
|-------------|-----------|-------|-------|----------|----------------------|
| Alpha1 +10% | 6.37%     | 6.25% | 6.39% | 1.94     | 76.95%               |
| Alpha1 -10% | 5.30%     | 5.42% | 5.27% | 3.10     | 76.50%               |
| d +10%      | 5.34%     | 5.34% | 5.34% | 2.97     | 71.69%               |
| d -10%      | 6.27%     | 6.27% | 6.27% | 2.04     | 76.33%               |
| Gama_0 +10% | 6.04%     | 6.02% | 6.04% | 2.36     | 74.21%               |
| Gama_0 -10% | 5.54%     | 5.56% | 5.54% | 2.62     | 73.69%               |
| r +10%      | 5.77%     | 5.78% | 5.77% | 2.55     | 74.35%               |
| r -10%      | 5.81%     | 5.80% | 5.81% | 2.43     | 73.56%               |
| Theta +10%  | 5.31%     | 5.31% | 5.31% | 2.50     | 71.53%               |
| Theta -10%  | 6.30%     | 6.30% | 6.30% | 2.47     | 76.50%               |
| w +10%      | 6.52%     | 6.52% | 6.52% | 1.83     | 77.59%               |
| w -10%      | 5.12%     | 5.13% | 5.12% | 3.23     | 70.61%               |

In Figure 3 we present the results of shocks in  $\gamma_0$ . In the upper quadrant we have the result of a 10% increase in the parameter that presents the animal spirits of entrepreneurs. As discussed earlier, the shock was given in the period 70. We had a fall in the growth rate of all the variables in its steady state. We had a drop in capacity utilization and an increase in the ratio public debt / GDP. This means that an increase in the rate of autonomous growth of capital stock is followed by a decrease in the growth rates of capital and private wealth, a decrease in the level of capacity utilization and an increase in the ratio of public debt to GDP. That is output growth is negatively related with the growth of expenditures that create productive capacity. This surprising result is in line with some previous work on demand led growth models as the one of Serrano (1995). Already in the lower quadrant, we have the result of a decrease of 10% over the same parameter. We can see

that there is an increase in the growth rate of all the variables in its steady state. We had an increase in capacity utilization and a fall in the ratio public debt / GDP.

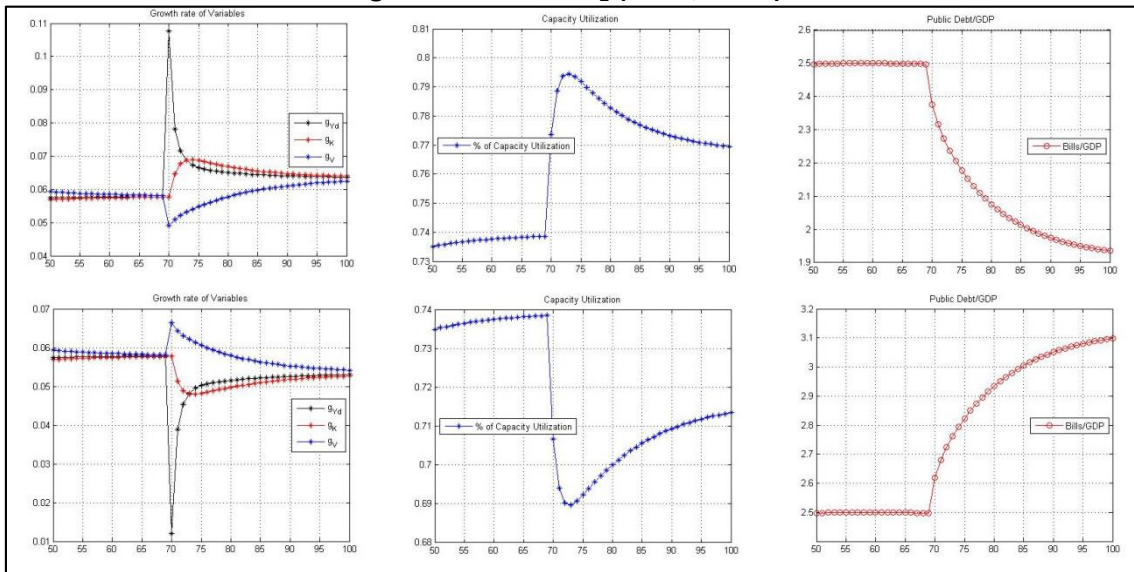
**Figure 3: Shocks in  $\gamma_0$  (+10%; - 10%).**



Note: Positive shocks on top line and negative ones in bottom line.

In figure 4 we show the results of a positive and negative shocks in  $\alpha_1$ . Upper quadrant there was a +10% shock in the parameter and the lower quadrant -10%. We can see that the positive shock in the propensity to consume out of disposable income led to an increase in growth rates of the variables, an increase in the capacity utilization and a drop in the public debt / GDP. The negative shock, led to a fall in growth rates, a drop in capacity utilization and an increase in the public debt / GDP.

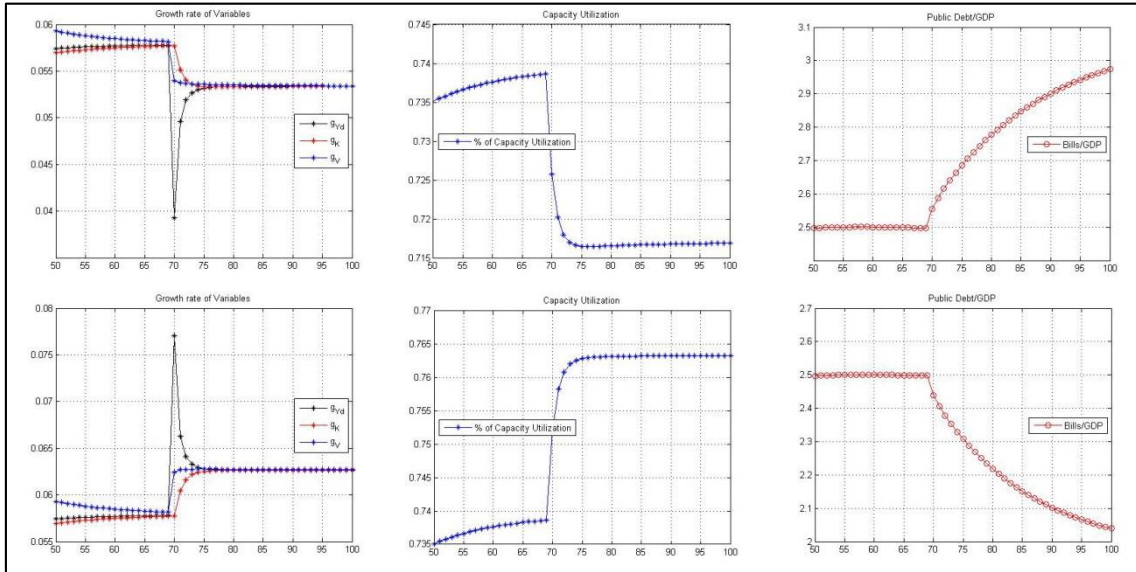
**Figure 4: Shocks in  $\alpha_1$  (+10%; - 10%).**



Note: Positive shocks on top line and negative ones in bottom line.

In Figure 5 we show the effects of a shock in  $d$ , the profit retention coefficient. The upper quadrant shows the results of a shock of +10% in  $d$  and in the lower quadrant a shock of -10% in  $d$ . The positive shock led to a fall in growth rates, a drop in capacity utilization and a rise in the public debt / GDP. The negative shock resulted in a rise in growth rates, an increase in capacity utilization and a fall in the ratio public debt / GDP. This result clearly shows that an increase in propensity to save out of profits – represented by an increase in the profit retention coefficient - is associated with a decrease in growth rate of capital stock and private wealth, a decrease in the level of capacity utilization and an increase in the ratio of public debt to GDP. This resembles the traditional Keynesian result of the “paradox or thrift”.

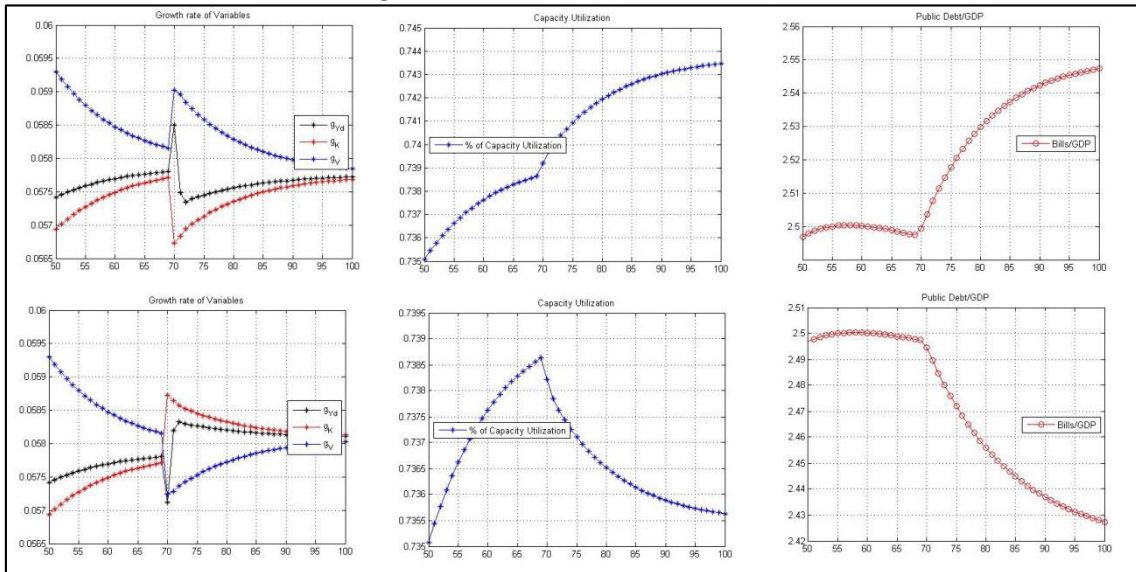
**Figure 5: Shocks in  $d$  (+10%; - 10%).**



Note: Positive shocks on top line and negative ones in bottom line.

In Figure 6 we show the effects of a shock in  $r$ , the nominal interest rate. The upper quadrant shows the results of a shock of + 10% in  $r$  and in the lower quadrant a shock of -10% in  $r$ . The positive shock led to a fall in growth rates (to a value that is almost the same before the shock), a rise in capacity utilization and a rise in the ratio public debt / GDP. The negative shock resulted in an increase in growth rates (to a value that is almost the same before the shock), an increase in capacity utilization and a fall in the ratio public debt / GDP.

**Figure 6: Shocks in  $r$  (+10%; - 10%).**

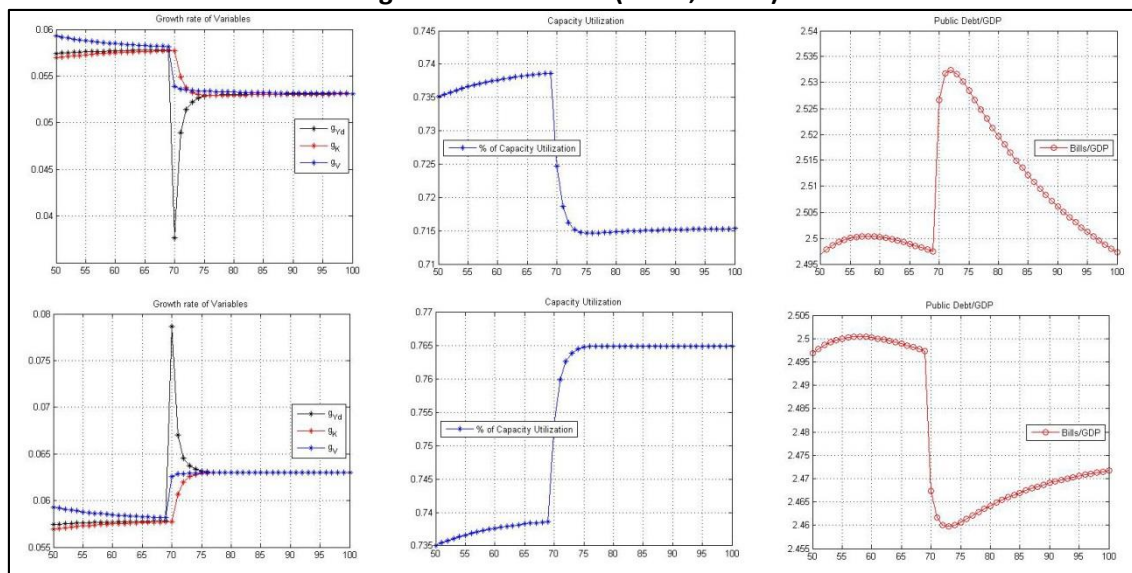


Note: Positive shocks on top line and negative ones in bottom line.

In Figure 7 we show the effects of a shock in  $\theta$ , the income tax rate. The upper quadrant shows the results of a shock of + 10% in  $\theta$  and in the lower quadrant a shock of -10% in  $\theta$ . The positive shock led to a fall in growth rates (to a value that is almost the same before the shock), a rise in capacity utilization and an initial rise in the ratio public debt / GDP followed by a fall, reproducing a clear hump shaped pattern. The negative shock resulted in an increase in growth rates (to a value that is almost the same before the shock), an increase in capacity utilization and a fall in the ratio public debt / GDP.

This strange behavior of the ratio of public debt to GDP resulted from the fact the immediate impact of the increase in taxes is to reduce the level of capacity utilization and the growth rate of capital stock due to its negative impact over disposable income and, therefore, over consumption and aggregate demand. Over time, however, as output growth stabilize at a lower level, the effect of a higher tax rate over government receipts tends to become dominant, producing a reduction in fiscal deficit and hence in the public debt to GDP ratio.

**Figure 7: Shocks in  $\theta$  (+10%; - 10%).**



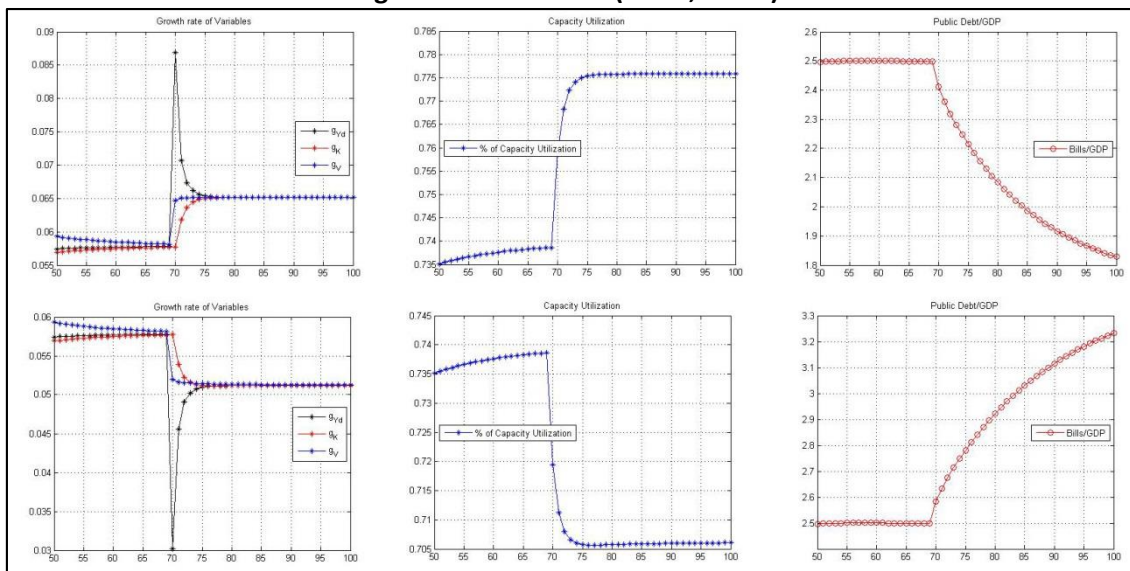
Note: Positive shocks on top line and negative ones in bottom line.

In Figure 8 we show the effects of a shock in  $w$ , the wage share. The upper quadrant shows the results of a shock of +10% in  $w$  and in the lower quadrant a shock of -10% in  $w$ . The positive shock led to a rise in growth rates (this was the shock that led to higher growth rates), a rise in capacity utilization and a fall in the public debt / GDP. The



negative shock resulted in a fall in growth rates (this was the shock that led to lower growth rates), a rise in capacity utilization and a rise in the ratio public debt / GDP.

**Figure 8: Shocks in  $w$  (+10%; - 10%).**



Note: Positive shocks on top line and negative ones in bottom line.

From the above results we can see that the model showed a clear *wage-led* regime of accumulation, since an increase in the wage share resulted in an increase in the growth rate of both capital stock and private wealth and also an increase in the level of capacity utilization.

## 6 Final Remarks

The idea of this paper is to build a Stock-and-Flow Consistent version of the IS-LM model for a closed economy with endogenous money supply, in order to analyze the dynamic properties of the model. We show that once the capacity effect of investment is taken into account, and once the investment rate exceeds depreciation rate, the equilibrium of the model is no longer characterized by a stationary level of real output, but by a constant growth rate of real output.

The level of activity is represented in the model by the variable capacity utilization, which is constant and lower than one over the equilibrium path. Furthermore, any shock generated was unable to put the economy at full capacity utilization. This means that the SFC version of the IS-LM model reproduces the traditional Keynesian result of underemployment equilibrium. Moreover, in steady-state disposable income, capital stock and private wealth all grow at a same constant rate.

Regarding the comparative dynamics of the model, we had performed some numerical simulations about the dynamic effects of shocks over the time path of endogenous variables. The most sensible parameter was the wage share. The shocks in the wage share generated the highest and lowest values in growth rates, capacity utilization and public debt/GDP ratio. Moreover, the model shows that some traditional results of Keynesian Theory as the “paradox of thrift” and the demand-led nature of economic growth still hold in a SFC framework.

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