Interest Rates in Trade Credit Markets

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Abstract

There is evidence that suppliers have private information about their customers’ credit risk. Yet, interest rates in trade credit markets are usually industry-not-firm specific. Why? If the demand for intermediate products is inelastic, suppliers should raise interest rates until they reach their customers’ outside option. By definition, this outside option cannot reflect information that is privy to suppliers. In contrast, a highly elastic demand may induce suppliers to extend credit to their customers at zero interest, making the suppliers’ private information once more irrelevant. By characterizing these two equilibria, we obtain implications on when trade credit rates shouldn’t vary with private information held by suppliers.

JEL: G30, G32

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1 Introduction

Trade credit is one of the most important sources of short-term external financing for firms in the G7 countries (Canada, France, Germany, Italy, Japan, U.K., and the U.S.).\footnote{See, for example, Rajan and Zingales (1995).} Smith (1987), Mian and Smith (1992) and Biais and Gollier (1997) argue that such prominence is due to an informational advantage: The sales effort of suppliers makes it easier for them to assess their customers’ credit risk. Accordingly, Petersen and Rajan (1997) show that, vis-à-vis banks, suppliers extend more credit to firms with current losses and positive growth of sales; a finding that they interpret as evidence that suppliers have comparative advantage in identifying firms with growth potential.

Yet, a supplier’s informational advantage is, at first glance, difficult to reconcile with a standard practice in the trade credit markets. Ng, Smith and Smith (1999) and Petersen and Rajan (1994) show that the terms of trade credit in the U.S. are industry-not-firm specific. Depending on the industry, suppliers usually waive interest in 30-day loans (net 30 loans) or charge an effective rate of 44 percent a year by combining a 30 day maturity with a two percent discount for early payment within 10 days of the invoice (2-10 net 30 loans). But if suppliers are informed lenders, why don’t they charge interest rates that reflect variations in the borrowers’ risk?

This paper explains when and why interest rates in the trade credit markets do not internalize private information held by suppliers. In a nutshell, suppliers should raise trade-credit rates until they reach their customers’ outside option, if the demand for the suppliers’ goods is sufficiently inelastic with respect to the financing costs. By definition, this outside option – e.g., the interest rate available in banking loans – cannot reflect information that is privy to suppliers. In contrast, a sufficiently elastic demand may induce suppliers to extend trade credit to their customers at zero interest, making the suppliers’ private information once more irrelevant to the equilibrium trade-credit rate. Trade credit rates do not vary with the suppliers’ private information, therefore, when the demand is either sufficiently inelastic or sufficiently elastic with respect to interest rates.

To understand the main ideas of the paper, consider an industry whose firms require financing to purchase inputs from a single supplier. In a fraction $f$ of these firms – the safe firms – the investment in the input will be paid back with probability one. In the remaining firms, fraction $1 - f$, the investment in the input may fail. We call these latter firms risky.
To finance the purchase of inputs, firms can borrow from banks or ask for trade credit. As such, we consider firms that, albeit possibly risky, are not credit constrained. In the model, banks act competitively (i.e., interest rates imply that the expected return on a loan equals the cost of funds), but they cannot distinguish between safe and risky firms. Hence, banks charge the same interest rate $r_B$ to all firms in the industry.

In contrast to the banks, the supplier knows whether a firm is safe or risky. Thanks to this informational advantage, the supplier may charge different interest rates to the two types of firms. Competition with banks constrains the supplier’s choice of interest rate, though. In particular, the supplier cannot extend trade credit at an interest rate higher than the customer’s outside option, which, in our model, is the banking rate $r_B$.

To be sure, competition with banks doesn’t prevent suppliers from extending credit at low interest rates. Is it in the supplier’s interest to undercut banks? This won’t be the case if the demand for the inputs is inelastic with respect to the financing cost. Intuitively, an inelastic demand induces the supplier to, regardless of the customer’s creditworthiness, raise interest to the banking rate, which is as high as a trade credit rate can be. An inelastic demand thus gives us a natural candidate for an equilibrium trade credit rate that does not vary with the supplier’s private information: the banking rate $r_B$. One problem remains for this candidate to be legitimate, though. Informed suppliers may be unwilling to lend to risky firms at an interest rate that is set by uninformed banks that fiercely compete with each other.

As it turns out, there is at least one good reason for suppliers to lend to risky firms at the banking rate. Frank and Maksimovic (1998) argue that suppliers are more efficient than banks in salvaging value from assets of financially distressed firms. If so, suppliers get a higher return than banks when a borrower becomes financially distressed; an advantage that may make it profitable for the supplier to extend credit to risky firms at the banking rate.

What happens if the demand for inputs is inelastic but suppliers are not more efficient than banks in lending to risky firms? The equilibrium at the banking rate breaks down. Petersen and Rajan (1997) show that suppliers extend less credit in industries that keep a high fraction of finished goods in inventory; a finding that they interpret as evidence that it is easier for suppliers to transform repossessed inputs (rather than finished goods) into liquid assets. Accordingly, our model predicts that suppliers are more likely to offer standardized rates that do not reflect private information in industries that keep a low fraction of finished goods in the inventory.

Our model builds on two recent papers: Biais and Gollier (1997) and Burkart and Ellingsen...
(2002). These articles explain why suppliers lend to firms that have exhausted their debt capacity with banks. In Biais and Gollier, suppliers can identify firms whose credit risks are overestimated by banks. Knowing that a firm’s credit line is unduly low, suppliers are willing to extend trade credit. In Burkart and Ellingsen, financially constrained firms have access to trade credit because it implies a lower risk of misuse of corporate funds than banking loans. In either Biais and Gollier’s or Burkart and Ellingsen’s models, the optimal trade-credit rate varies with the suppliers’ information.

Brennan, Maksimovic and Zechner (1988) is another related work. In this paper, a monopolist sells products to safe and risky customers, offering trade credit at a cost that is lower than the interest rate that banks charge to risky firms, but higher than the riskless rate. Hence, trade credit discriminates the demand by reducing the financial cost of the risky customers. As in Biais and Gollier (1997) and Burkart and Ellingsen (2002), the optimal trade-credit rate in Brennan, Maksimovic and Zechner would vary with the suppliers’ private information, had there been trade credit to customers in different classes of risk.

As in Brennan, Maksimovic and Zechner (1988), our supplier internalizes the impact of financial costs on the demand for inputs, possibly offering subsidized rates to their customers. In fact, we shall demonstrate that a sufficiently elastic demand makes low financing costs so important to sales that it is optimal for our supplier to waive interest in loans to both types of firms. A second equilibrium in which the suppliers’ private information is not relevant thus obtains: trade credit at zero interest when the demand for inputs is sufficiently elastic.

In our model, therefore, the interest-elasticity of demand plays an important role in determining interest rates in the trade credit markets. If the demand for inputs is sufficiently elastic, the equilibrium trade credit rate is at zero; if, instead, the demand for inputs is sufficiently inelastic, the equilibrium is at the customers’ outside option, that is, the interest rate that the less informed banks charge. In either case, the equilibrium interest rate does not internalize private information held by the suppliers.

These results yield the following interpretation of common trade-credit terms in the U.S. While the scale of production implies an optimal level of inventory that is unlikely to be too sensitive to interest rates, a temporary increase in the level of inventory probably is. Such temporary increases in inventory are good candidates to be financed by trade credit with very short maturity. As a result, our model predicts that, as often happens in the U.S., suppliers will waive interest in trade credit with very early repayments, e.g., 10 days.

Perhaps more interestingly, our model links the invariance of trade credit rates to whether
firms are credit constrained. Since waiving interest attracts all types of firms, there is no reason for the demand for trade credit at zero interest to consist mainly of credit constrained firms. It is easy to see, however, that, at positive trade-credit rates, the supplier’s private information matters when the customer is credit constrained. In this case, competition with banks does not constrain suppliers’ choices of trade credit rates, implying that the optimal terms of trade credit depend on any firm-specific information that suppliers may have: elasticity of demand, probability of default, etc. Another prediction of our model is thus that, in industries whose firms are not likely to be credit constrained, suppliers are more likely to offer standardized rates that do not reflect private information.

The remainder of the paper is organized as follows. After presenting the model in section 2, section 3 characterizes the equilibrium in which the trade credit rate does not vary with private information held by suppliers. In section 4, we discuss the empirical implications and exhibit sufficient conditions for uniqueness of equilibrium. Section 5 then concludes. Proofs of the propositions that are not in the text can be found in the appendix.

2 The Model

Consider an economy with two dates, $t = 0$ and $t = 1$, and an industry with three risk-neutral agents: firms, banks, and a supplier of the firms’ inputs. At $t = 0$, firms require financing to purchase inputs. Banks are always willing to finance the purchase of inputs at an interest rate that covers the cost of funds. Firms, however, may have a second source of financing: trade credit. With an exogenous probability $x$, the supplier has enough funds to finance its customers. Upon the purchase of inputs at $t = 0$, production takes place and firms sell the output at $t = 1$. At this time, firms repay the debt and distribute any remaining cash flow to shareholders. Below we describe the agents’ technologies and their information structures.

2.1 Firms

There are two types of firms, safe and risky, run by value-maximizing managers who know their firms’ types from the onset. The safe firms represent a fraction $f$ of the population and have a deterministic production function. With this safe technology, investing $I$ in inputs at $t = 0$ obtains $Q(I)$ at $t = 1$. We assume that $Q(I)$ is an increasing and concave function, with $Q(0) = 0$ and satisfying the following conditions: there exist investment levels $\underline{I}$ and $\overline{I}$ such that $Q'(\underline{I}) > (1 + r)/f$ and $Q'(\overline{I}) < 1$, where $r$ is the riskless interest rate. These
harmless technical assumptions on the marginal productivity of investment assure that firms buy a positive level of input. Without loss of generality, we assume that the input is the unit of account. Hence, \( Q(I) \) is the value of the output in units of inputs.

Risky firms are endowed with a stochastic production function. With this technology, purchasing \( I \) units of input at \( t = 0 \) yields \( \tilde{Q}(I) \) at \( t = 1 \), where:

\[
\tilde{Q}(I) = \begin{cases} 
Q(I), & \text{with probability } \pi \\
\delta I, & \text{with probability } 1 - \pi, \text{ and } \delta \in (0, 1)
\end{cases}
\]

Note that, with probability \( \pi \), the risky technology is as profitable as the safe technology. But, with probability \( 1 - \pi \), the risky technology gets into trouble; the fraction \( 1 - \delta \) of purchased inputs is lost and the only return on the investment is an amount \( \delta I \) of inputs that remained unused. We assume that both \( Q(I) \) and \( \tilde{Q}(I) \) are verifiable. As such, firms can write debt contracts that are contingent on the realization of outputs.

### 2.2 Banks

In the model, banks can neither distinguish between firms of different types nor observe terms of trade credit. Banks know only the proportion of safe and risky firms, and the amount of inputs \( I \) that firms purchase. Since banks operate in a competitive market, they will set an interest rate, \( r_B \), that yields their opportunity cost. Given risk neutrality, this opportunity cost is the riskless interest rate \( r \).

If a risky firm fails, the lender captures the firm’s output, \( \tilde{Q}(I) \), which is the fraction \( \delta \) of the input \( I \) originally purchased. It is unlikely, nonetheless, that banks can costlessly transform \( \delta I \) into liquid assets. In fact, one of the key assumptions of our paper is that banks are not as efficient as suppliers in transforming inputs into liquid assets. To emphasize this difference between banks and suppliers, and to facilitate the analysis, we assume that neither the banks nor the firms can rescue the unused inputs, \( \delta I \), if the technology fails.

### 2.3 The supplier

In our model, the supplier, who is a monopolist in the market for inputs, has a zero production cost.² Yet, as in Brennan, Maksimovic and Zechner (1988), we assume that the supplier cannot

²As we shall argue in section 4, assuming that the supplier is a monopolist with a zero production cost is not essential. The driving force of our results is the informational advantage of suppliers, which gives them
use the price of the input to discriminate the demand. We thus fix the input price input at one.\textsuperscript{3} In doing so, we can focus the analysis of the trade credit market on the interest rate.

A common view in the trade-credit literature is that suppliers have comparative advantage over banks in financing purchases of inputs. Biais and Gollier (1997) argue, for instance, that an ongoing sales effort makes it easier for suppliers to evaluate their customers’ credit risk; an argument that Petersen and Rajan (1997) find evidence for. Accordingly, we assume that, unlike the banks, the supplier knows whether a firm is risky or safe.

Ability to evaluate risk of credit is not the only reason for the existence of trade credit, though. Petersen and Rajan (1997) also find evidence that suppliers are more efficient than banks in transforming collateral into liquid assets. To model this advantage, we follow Frank and Maksimovic (1998) and assume that, unlike the banks, the supplier can costlessly resell inputs that they capture from bankrupted firms. Hence, when a risky investment of \( I \) units of input fails, the supplier captures the unused inputs, \( \delta I \), assuring some return on their trade credit. Since we have assumed that banks cannot rescue unused inputs, we can interpret \( \delta \) as a measure of suppliers’ comparative advantage over banks in financing purchases of inputs.

But, as Mian and Smith (1992) show, some suppliers do not have access to funds that can be used to provide trade credit. We model this potential constraint as follows. With a probability \( x \) in the interval \((0, 1)\), our supplier has access to funds at the same cost of banks, \( r \). In this event, the supplier can extend trade credit. With probability \( 1 - x \), however, the supplier has no access to funds, ruling out trade credit. Firms will then have to secure bank loans to purchase inputs. The supplier’s stochastic cost of funds assures an active role for banking credit, despite the supplier’s potential advantage as a lender.

\[ \text{2.4 The game in the extensive form} \]

Figure 1 describes the extensive form of the game. Nature acts first, determining the type of the firm (safe or risky) and whether the supplier can provide trade credit. If the supplier cannot extend trade credit, an event with probability \( 1 - x \), firms borrow from banks at a rate \( r_B \) before purchasing \( I_B \) from the supplier. If, instead, trade credit is available, firms choose the source of financing, that is, the supplier or the banks.

As the dotted lines in the tree show, banks do not know either the type of a firm that asks

\textsuperscript{3}As Petersen and Rajan (1997) point out, anti-trust laws can prevent the supplier from using prices to discriminate the demand.
a loan (dotted line in the lower part of the tree) or the availability of trade credit (dotted line in the upper part). In contrast, the supplier knows whether it can be a lender and the firm’s type. Accordingly, we let the supplier tailor the interest rate to the type of firm, charging $r^R_T$ to risky firms and $r^S_T$ to safe firms. These interest rates determine firms’ returns on the purchase of inputs, inducing risky firms to invest $I^R_T$ and safe firms to invest $I^S_T$.

The game has two types of equilibria. In the first one, the supplier lends to only one of the two types of firms, when trade credit is available. In the second type of equilibrium, suppliers lend to both types of firms, whenever possible. The first type of equilibrium is not interesting for the purposes of our work. If borrowers are all in the same class of risk, there is no scope for the supplier to vary the trade credit rate with its private information on the customers’ creditworthiness. Accordingly, our focus is on the equilibrium in which the supplier lends to both types, whenever trade credit is available. After characterizing this equilibrium in section 3, we exhibit conditions for it to be unique in section 4.

3 Equilibrium with Invariance of Interest Rates

3.1 Banking credit

In the equilibrium that we look for, the supplier extends trade credit to both types of firms, whenever possible. When trade credit is not available, an event with probability $1 - x$, firms finance purchases of inputs by borrowing from banks. Let us then start our analysis by deriving the demand for inputs of a safe firm that borrows from banks at an interest rate $r_B$.

By assumption, safe firms can always repay loans that are used to finance inputs.4 As a result, a safe firm’s optimal investment in inputs solves:

$$\max_I \frac{Q(I) - (1 + r_B)I}{(1 + r)}.$$  \hspace{1cm} (1)

Program (1) looks for the investment that maximizes the present value of a safe firm’s profit. By investing $I$ at $t = 0$, a safe firm obtains $Q(I)$ at $t = 1$ with probability one, from which the firm will use $(1 + r_B)I$ to pay principal plus interest to the bank (also at $t = 1$).

\hspace{1cm} 

4 For any finite interest rate, our assumptions on the marginal productivity of investment (see section 2.1) assure that a small purchase of inputs will more than offset the costs of servicing the debt, leaving a positive profit for the safe firm. Hence, an optimal choice of inputs must imply a positive profit as well. In the absence of uncertainty, a positive profit implies that any debt will be repaid with probability one.
Since the investment of a safe firm is riskless, we discount its payoff at the riskless interest rate \( r \). For an interest rate \( r_B \), the first order conditions, which are also sufficient, yield the demand for inputs of the safe firm, \( I^S_B(r_B) \), by setting the marginal product of input equal to the cost of financing:

\[
Q'(I^S_B) = (1 + r_B).
\]

(2)

Consider now a risky firm that borrows \( I \) to purchase inputs. With probability \( \pi \), the investment will yield the same return \( Q(I) \) of the safe firms. With probability \( 1 - \pi \), however, the investment will fail, leaving only \( \delta I \) units of inputs at \( t = 1 \). Regardless of the lender’s ability to transform the residual inputs into liquid assets, a failure of the risky technology implies that the firm loses all rights on the residual inputs. Given the assumption of risk neutrality, the demand for inputs of a risky firm, \( I^R_B(r_B) \), maximizes the present value of the expected payoffs, using the riskless interest rate as the discount rate, that is,

\[
\max_I \frac{\pi [Q(I) - (1 + r_B)I]}{1 + r}.
\]

(3)

Like the safe firms, a risky firm’s demand for inputs sets the marginal product of investment equal to the cost of financing, that is, \( Q'(I^R_B) = (1 + r_B) \). It then follows that the demand schedules of safe and risky firms are equal, that is, \( I^S_B = I^R_B = I_B \). Indeed, had the demand for loans varied across firms of different types, banks would have been able to infer the type of a firm that requests a loan. Banks and suppliers would then end up with the same information structure, and our model would not be fit to explain why interest rates in trade credit do not seem to reflect suppliers’ private information about their customers.

Of course, a request of a bank loan may convey information even if safe and risky firms have identical demands for inputs. For instance, in an equilibrium in which the supplier finances only safe firms, banks should expect that most of their loans go to risky firms. (Banks should not expect all firms to be risky because lack of trade credit might lead safe firms to look for banking credit.) In the equilibrium that we look for, though, the supplier finances both types of firms, whenever they can. Thus, banks know that lack of funds for trade credit is the only reason for firms asking for bank loans. Accordingly, requests of loans do not convey information, and banks do not update their priors about firms’ types.

Provided that requests for loans do not convey information, we can easily derive the equilibrium interest rate \( r_B \). Since the technologies of both types of firms are common knowledge,
the banks know that safe firms will pay principal plus interest with probability one, while risky firms will honor the debt contract with probability \( \pi \). (Risky firms do not pay anything with probability \( 1 - \pi \).) Therefore, the banks will collect principal plus interest at \( t = 1 \) if the firm is safe, probability \( f \), or if the firm is risky but the technology does not fail, probability \( \pi (1 - f) \). In other words, the probability that a borrower pays a bank at \( t = 1 \) is \( f + \pi (1 - f) \).

And the expected return of a bank that lends at a rate \( r_B \) is

\[
(1 + r_B) (f + \pi (1 - f)).
\]

Competition among banks drives the expected returns on banking loans to their opportunity cost, which, under our assumption of risk neutrality, is the riskless interest rate \( r \). As such, the interest rate that assures banks their opportunity cost is

\[
r_B = \frac{1 + r}{f + \pi (1 - f)} - 1. \tag{4}
\]

Having characterized the equilibrium banking rate and the demand for inputs of firms that borrow from banks, our next task is to introduce trade credit. Two questions then naturally arise. Is it optimal for the supplier to finance purchases of inputs? What is the optimal interest rate in the trade credit market? Answering these questions requires solving for the investment decision of a firm that has the option of using trade credit to finance purchases of inputs.

As it turns out, trade credit does not fundamentally change firms’ investment decisions. Whether a firm borrows from banks or from the supplier, all that matters is the cost of financing. It then follows that the investment decisions of safe and risk firms are still characterized by, respectively, programs (1) and (3), once we substitute the minimum cost of financing for the banking rate. We can, therefore, define an investment function that, for both types of firms, is implicitly defined by the equality of the marginal productivity of investment and the cost of financing. Formally,

\[
Q'(I(s)) = 1 + s, \tag{5}
\]

where \( s \) is the lowest between the banking rate and the trade credit rate. Under our assumptions, the investment function decreases with the cost of financing. Moreover, the investment function is concave on the interest rate if \( Q''''(I) < 0 \).

\footnote{To show that \( I(\cdot) \) is decreasing and concave (under \( Q''''(\cdot) < 0 \) on the cost of financing, apply the implicit function to \( Q'(I) = (1 + s) \) to obtain \( I' = -\frac{1}{Q'(I)} \), which is negative because, by assumption, \( Q''(I) < 0 \). Assuming \( Q''' < 0 \) and applying the implicit function theorem a second time yields \( I'' = -\frac{Q''''(I)}{Q'(I)^2} I' < 0. \)}
Equipped with the demand schedule, the next two sections characterize the supplier’s optimal strategies, starting with the optimal terms of trade credit of a supplier that faces a safe customer.

3.2 The supply of trade credit to safe firms

In our model, the supplier is not obliged to finance the purchase of inputs; it can let banks finance firms. Remember, however, that when banks cannot distinguish between safe and risky firms, the competitive rate \( r_B \) embeds a cross-subsidy between the two types of firms. Banks profit from safe firms to cover losses with risky firms. This means that, by financing the purchase of inputs of safe firms at a rate \( r_B \), the supplier fetches not only an operational profit (the sale of inputs), but also a financial profit. It then follows that, whenever possible, the supplier will undercut the banking rate (by a small amount \( \epsilon \) that we will ignore in the analysis) to finance a safe firm’s purchase of inputs.

The question then is: What terms of credit should the supplier offer to the safe firm? On the one hand, a low interest rate decreases the supplier’s financial profit. On the other hand, a low interest rate increases the demand for inputs, enhancing the operational profits. The optimal rate \( r^S_T \) weighs these two forces by solving the following program:

\[
\max_{r^S_T} \frac{(1+r^S_T)I(r^S_T)}{(1+r)} \quad \text{subject to} \quad 0 \leq r^S_T \leq r_B.
\]

The objective function in program (6) is the present value of the supplier’s profits. These profits are cashed at \( t = 1 \) and consist of two components: the operational revenue \( I(r^S_T) \) and the financial revenue \( r^S_T I(r^S_T) \). While the operational revenue is always associated to a profit (remember that, for simplicity, we assume that production cost is zero), the financial revenue will result in a financial loss if the interest rate \( r^S_T \) is lower than the discount rate \( r \) (the supplier’s cost of funds). The constraint of the program takes into account two restrictions. First, it rules out negative interest rates, which, being equivalent to a combination of zero interest and a discount in the price of input, amounts to assuring that the supplier cannot use prices to discriminate the demand. The second restriction takes into account that the bank
loans are outside options for the financing of the inputs. The interest rate in the trade credit market, therefore, cannot be larger than the banking rate.⁶

As discussed in the previous section, \(Q''(\cdot) < 0\) is a sufficient condition for the investment function to be concave. In this case, Program (6) is concave and its first order condition yields a very simple characterization of the optimal interest rate for loans to safe firms:

**Proposition 1** - Assume that the investment function is concave on the interest rate. Thus, the optimal interest rate for trade credit to safe firms, \(r_S^T\), is equal to the banking rate, \(r_B\), if the elasticity of demand is less than or equal to one at the interest rate \(r_B\). Otherwise, the optimal rate is less than the banking rate. In particular, \(r_S^T\) is equal to zero if and only if the elasticity is bigger than or equal to one at the rate zero.

The intuition for Proposition 1 is quite simple. While the demand for inputs is inelastic, monopoly power gives the supplier incentives to increase the interest rate. In particular, an inelastic demand at the banking rate \(r_B\) implies that a higher interest rate would have increased profits, had not it been for the firm’s outside option of borrowing from banks. Hence, an inelastic demand at the banking rate \(r_B\) implies that the supplier will increase the interest rate until it reaches the upper bound \(r_B\), beyond which firms prefer to borrow from banks.

What happens if the demand is elastic at \(r_B\)? The optimal trade rate is lower than \(r_B\), because the financial loss from a lower rate is more than offset by the operational gains from an increase in sales that a lower rate induces. In particular, the supplier will lower the interest while it remains in the elastic portion of the demand, reaching an optimal trade rate of zero when the demand is elastic at its lowest point of elasticity, that is, at the interest rate \(r_S^T = 0\).

From Proposition 1, we can derive the supplier’s profit function for loans to safe firms:

\[
\Pi_S^T(r_B) = \frac{(1 + r_S^T(r_B)) I (r_S^T(r_B))}{(1 + r)},
\]

where \(r_S^T(r_B)\) is the optimal interest rate of trade credit to safe firms. Defining the interest-elasticity of demand as \(\epsilon^D(r) = -\frac{(1+r)P'(r)}{I(r)}\), we can write the optimal trade credit rate as:

⁶The supplier can fetch an interest rate higher than \(r_B\) by denying inputs to firms that do not use trade credit. The analysis in the paper, therefore, ignores distortions in the trade credit markets that are driven by these types of bundling strategies.
Equation (8) gives us the optimal interest rate as a function of the elasticity of demand and the banking rate. In section 3.4, we will use this characterization to obtain conditions on the elasticity of demand that assure that the supplier will choose the same interest rate for safe and risky firms. But first we must derive the supply of trade credit to risky firms.

### 3.3 The supply of trade credit to risky firms

Let us now move to the risky firms. In general, riskier firms should pay higher interest rates. Yet, in our model, banks cannot distinguish between risky and safe firms. As it turns out, this friction has consequences to the trade credit market. Since risky firms can borrow from uninformed banks at the interest rate $r_B$, an informed supplier cannot charge a higher interest rate, even if informed banks would have required a higher interest rate from risky firms, had they known the firms’ types. It then follows that the interest rate of trade credit to risky firms, $r^R_T$, must satisfy:

$$r^R_T \leq r_B. \quad (9)$$

The inequality (9) may induce suppliers to forego loans to risky firms. Had banks known that a firm is risky, they wouldn’t lend at an interest rate $r_B$; at such rate, the expected return of lending to risky firms does not cover banks’ cost of funds. Banks are willing to lend at $r_B$ because they do not know the firm’s type and they count on profiting from loans to safe firms. But then, why should an informed supplier lend to risky firms at an interest rate that, due to competition with uninformed banks, cannot surpass $r_B$?

We have already discussed one reason for suppliers to lend to risky firms at the banking rate $r_B$. An elastic demand may induce the supplier to accept a loss in the financing of the input to increase sales. While this first reason is relevant for lending to safe firms as well, there is another reason that does not apply to safe customers. As you may recall, there is evidence that suppliers have a comparative advantage over banks in transforming collateral into liquid assets. If this advantage is sufficiently large, lending to risky firms at the interest rate $r_B$ may impose an expected loss to banks and yet assure an expected profit to the suppliers.
Of course, if the demand for inputs is too inelastic and the suppliers’ advantage in repossessing collateral is small, then it may be optimal for the supplier to forego loans to risky firms. As is often the case in game theoretical models, therefore, the equilibrium that we look for – one that the supplier lends to both types of firms at the same interest rate – obtains only in a certain range of parameters. To characterize this range, we will divide the analysis in two steps. In the first step, we obtain the interest rate that maximizes the supplier’s expected profits, under the assumption that it is optimal for the supplier to lend to risky firms. Then, in the second step, we exhibit conditions under which it is indeed optimal for the supplier to lend to both types of firms at the optimal interest rate.

3.3.1 Lending to risky firms

Assuming that it is optimal for the supplier to lend to risky firms, the optimal interest rate solves:

$$\max_{r_T^R} \pi \left[ \frac{(1+r_B^R)I(r_T^R)}{1+r} \right] + (1-\pi) \left[ \frac{\delta I(r_T^R)}{1+r} \right]$$  \hspace{1cm} (10)

subject to $0 \leq r_T^R \leq r_B$.

In program (10), the objective function is the present value of expected profits, which, from the risky-neutrality assumption, is discounted at the riskless interest rate. With probability $\pi$, the risky firm succeeds, allowing the repayment of the debt at $t = 1$, as in loans to safe firms. In this event, the risky firm pays the principal $I(r_T^R)$, which corresponds to the demand for inputs at the interest rate $r_T^R$, and interest, $r_T^RI(r_T^R)$. But, with probability $1 - \pi$, the risky firm fails and the supplier rescues the unused inputs, $\delta I(r_T^R)$. In maximizing expected profits, the supplier faces two constraints. The interest rate cannot be larger than the banking rate (or else the firm borrows from a bank) or negative.

As in program (6), $Q''(.) < 0$ is a sufficient condition for both the investment function and program (10) to be concave, in which case the first order condition yields a very simple characterization of the optimal interest rate for loans to risky firms.

**Proposition 2** - Assume that the investment function is concave on the interest rate. Thus, the optimal interest rate for trade credit to risky firms, $r_T^R$, is equal to the banking rate, $r_B$, if the elasticity of demand is less than or equal to $\frac{\pi}{\pi+(1-\pi)\delta}$ at $r_B$. Otherwise, the optimal rate is less than the banking rate. In particular, $r_T^R$ is equal to zero if and only if the elasticity is bigger than or equal to $\frac{\pi}{\pi+(1-\pi)\delta}$ at the rate zero.
In loans to safe firms (see equation (8)), an inelastic demand induces the supplier to increase the interest rate as much as possible. In contrast, the supplier will extend credit to risky firms at a lower interest rate if, at the banking rate \( r_B \), the elasticity of demand is lower than one (inelastic demand) but higher than \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} < 1 \). Likewise, while the optimal interest rate to safe firms reaches zero if the demand is elastic at zero, the optimal rate to risky firms reaches zero if the elasticity of demand at the rate zero is larger than \( \frac{\pi}{\pi+(1-\pi)\delta} < 1 \).

The incentives to offer lower interest rates to risky firms follow from the assumption of zero cost to produce inputs. When the risky firm fails, the contracted interest is irrelevant to the supplier. Still, the rescuing of the unused inputs, \( \delta I \), in default leaves a positive profit, as assumed, the cost of producing inputs is zero. Hence, the supplier has incentives to offer lower interest rates to risky firms because the risk of default increases the relative importance of the operational profits vis-à-vis interest payments.

These incentives break down if default imposes an operational loss on the supplier. In this case, default reduces the supplier’s expected benefits from an increase in sales, raising the ex-ante importance of the financial earnings. In particular, if the costs of production are sufficiently large, an elastic demand for inputs at \( r_B \) may not induce the supplier to undercut the banking rate. It can be shown, however, that, with operational losses in default, the optimal interest rate is the banking rate \( r_B \) if the demand is sufficiently inelastic, and zero if the demand is sufficiently elastic. This is all we need for the trade-credit rate not to vary with private information held by the supplier.

From Proposition 2, the supplier’s profit function for loans to risky firms is

\[
\Pi^R_T (r_B) = \pi \left[ \frac{(1 + r_B^R (r_B))}{1 + r} \frac{I (r_B^R (r_B))}{I(r)} \right] + (1 - \pi) \left[ \frac{\delta I (r_B^R (r_B))}{1 + r} \right],
\]

where \( r_B^R (r_B) \) is the optimal interest of trade credit to risky firms. Using the definition of the interest-elasticity of demand, \( \epsilon^D (r) = \frac{(1+r)I'(r)}{r I(r)} \), we can write the optimal trade credit rate as follows:

\[
r_T^R (r_B) = \begin{cases} r_B, & \text{if } \epsilon^D (r_B) \leq \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}; \\ r_T^R \in (0, r_B), & \text{if } \epsilon^D (0) \leq \frac{\pi}{\pi+(1-\pi)\delta} \text{ and } \epsilon^D (r_B) > \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}; \\ 0, & \text{if } \epsilon^D (0) \geq \frac{\pi}{\pi+(1-\pi)\delta}. \end{cases}
\]
Under the assumption that it is optimal for the supplier to lend to risky firms, equation (12) gives us the optimal interest rate as a function of the elasticity of demand and the banking rate. Next, we obtain a necessary and sufficient condition for lending to risky firms to be optimal.

3.3.2 Should suppliers lend to risky firms?

It may be optimal for the supplier to finance risky firms, for two reasons. First, the banking rate may imply a positive expected profit for a supplier that has a sufficiently large comparative advantage in rescuing unused inputs, even if this interest rate imposes an expected loss on banks. Second, as Schwartz and Whitcomb (1997) and Brennan, Maksimovic and Zechnier (1998) point out, financing risky firms at a low interest rate may increase sales to a point that the increase in operational profits more than offsets an expected loss in the loan contract. These two effects are summarized in condition (13),

\[
\frac{\pi (1 + r^R_T(r_B)) I(r^R_T(r_B)) + (1 - \pi) \delta I(r^R_T(r_B))}{(1 + r)} \geq I(r_B). 
\]

Condition (13) is intuitive. It simply requires that extending trade credit to the risky firm increases the supplier’s expected profit. More precisely, the left-hand-side of the condition (13) is the present value of the expected profits when the supplier finances the risky firm, under the optimal interest rate \( r^R_T(r_B) \) (see equation (12)). Lending to risky firms is optimal if and only if this present value is larger than \( I(r_B) \), which is the supplier’s profit when the risky firm borrows from banks at the rate \( r_B \) to pay for the inputs at \( t = 0 \).

In the next section, we combine Propositions 1 and 2 with condition (13) to characterize an equilibrium in which the supplier extends trade credit to both types of firms at the same interest rate.

3.4 Characterizing the equilibrium

A relevant equilibrium in which the trade credit rate does not vary with private information held by the supplier requires two main ingredients. First, the supplier must have incentives to finance the two types of firms, or else we rule out variations of interest rates across firms from the onset. The condition (13) summarizes this first ingredient. Second, the invariant
interest rate must satisfy the necessary and sufficient conditions for loans to safe and risky firms, as summarized in equations (8) and (12).

Assume for a while that it is optimal for the supplier to finance both types of firms (i.e., condition (13) is satisfied). A quick inspection of the optimal interest rates (see equations (8) and (12)) shows two natural candidates for an invariant interest rate: the banking rate \( r_B \) and a zero interest rate.

Consider first the banking rate \( r_B \). From equation (8), the optimal interest rate of loans to safe firms is the banking rate if, at \( r_B \), the demand for inputs is inelastic. In turn, equation (12) shows that the banking rate is optimal for trade credit to risky firms if, at \( r_B \), the elasticity of demand is less than or equal to \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} < 1 \). It then follows from the necessary and sufficient conditions of the supplier’s problem that the banking rate is the only candidate for an invariant interest rate, if the elasticity of demand at \( r_B \) is less than or equal to \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} \).

Is it in the interest of the supplier to finance risky firms at the banking rate when the elasticity of demand at \( r_B \) is less than or equal to \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} \)? Substituting \( r_B \) for \( r_T \) (\( r_B \)) in the left-hand side of condition (13) and using \( \pi \frac{1+r_B}{f+(1-f)\pi} - 1 = r_B \) yields

\[
\pi (1 + r_B) + (1 - \pi) \delta \geq (1 + r) \Rightarrow \delta \geq \frac{(1 + r)f}{f + (1 - f)\pi}. \tag{14}
\]

It then follows that it is optimal for the supplier to finance risky firms at the banking rate if the fraction of the purchased inputs that the supplier rescues in default, \( \delta \), is bigger than or equal to \( \frac{(1+r)f}{f + (1 - f)\pi} \). Since \( \delta \) is a fraction in the interval \((0, 1)\), this condition can be satisfied only if \( \frac{f}{f+(1-f)\pi} \leq \frac{2}{\pi} \). In other words, the probability that a risky firm repays the debt, \( \pi \), must be high relative to the fraction of safe firms in the industry, \( f \). If this necessary condition is satisfied and the elasticity of demand at \( r_B \) is less than or equal to \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} \), then there is a cut-off for \( \delta \) such that, for values larger than this cut-off, there is an equilibrium in which the trade credit rate is the banking rate. Moreover, since the first order conditions of the supplier’s problem are also sufficient (under a technical assumption that assures that the investment function is concave), the equilibrium at the banking rate is unique in the class of equilibria in which the supplier finances both types of firms.\footnote{The role of the banking rate as an outside option thus determines the equilibrium level of interest rates in the trade credit markets, even for suppliers that provide all of their customers’ financing requirements. Felli and Harris (1996) explore this role of outside options in a model of investment decisions in human capital.}
Proposition 3 below summarizes the conditions that assure the invariance of the trade-credit rate at the banking rate.

**Proposition 3** - Suppose that (i) the investment function is concave on the interest rate, \( f \leq \frac{\pi}{1-f} \), and (ii) the fraction of unused inputs that the supplier rescues in case a risk firm fails satisfies \( \delta \geq \frac{(1+r)f}{f+(1-f)\pi} \). Then there is an equilibrium in which the supplier finances both types of firms at an interest rate that is equal to the banking rate \( r_B \) if and only if the interest-elasticity of the demand for inputs is smaller than or equal to \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} \) at the rate \( r_B = \frac{1+r}{f+\pi(1-f)} - 1 \).

What happens if the elasticity of demand is larger than \( \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} \) at the banking rate? From Proposition 3, the invariance at the banking rate breaks down; the interest-elasticity of demand makes it optimal for the supplier to lower the interest rate to increase operational profits. As we have already shown, nonetheless, a sufficiently elastic demand may induce the supplier to extend trade credit to both types of firms at zero interest. Accordingly, we now exhibit conditions for existence of an equilibrium at this second candidate for invariant trade-credit rates.

From equations (8) and (12), a zero interest rate is optimal for loans to safe firms if the demand is elastic at the rate zero and, for loans to risky firms, if the elasticity at zero is bigger than or equal to \( \frac{\pi}{\pi+(1-\pi)\delta} < 1 \). Of course, this latter condition is automatically satisfied by an elastic demand at zero. Hence, a necessary condition for an equilibrium with invariant rates at zero is an elastic demand at a zero interest rate. This condition is also sufficient if, rather than letting risky firms borrow from banks to purchase inputs, the supplier is better off extending credit to risky firms at a zero interest rate. This happens if condition (13) is satisfied at \( r_B^{T} = 0 \), that is, if

\[
\pi I (0) + (1 - \pi) \delta I (0) \geq (1 + r) I (r_B) .
\]  

Proposition 4 exhibits conditions that assure that condition (15) is satisfied. In particular, lending to risky firms is optimal for the supplier only if its advantage (relative to banks) in lending to risky firms, \( \delta \), is large enough to allow for positive expected profits, in spite of the fact that \( r_B \) imposes a loss on the uninformed banks. The proposition also shows that there is

They show that an employee’s productivity in a rival firm matters, even when an investment in firm-specific human capital reduces the chances that the employee changes jobs.
an equilibrium in which the supplier extends trade credit to both types of firm at no interest if and only if the demand for inputs is elastic at the zero interest rate.

**Proposition 4** - Suppose that the investment function is concave on the interest rate and that the fraction of unused inputs that the supplier rescues when a customer becomes financially distressed, $\delta$, is bigger than or equal to $\frac{(1+r)I(0)}{(1-\pi)(1+(1-f)(1-\pi))}$ - $\frac{\pi}{(1-\pi)}$. Then there is an equilibrium in which the supplier extends trade credit to both types of firms at a zero interest rate if and only if the demand is elastic at the zero interest rate.

Proposition 4 takes into account that higher operational profits from increased sales may not be enough to induce the supplier to waive interest. If so, the supplier’s advantage in lending to risky firms, $\delta$, must be larger than or equal to $\frac{(1+r)I(0)}{(1-\pi)(1+(1-f)(1-\pi))}$ - $\frac{\pi}{(1-\pi)}$. Economic intuition suggests, nonetheless, that a large $\delta$ might not be necessary if the demand is sufficiently elastic. After all, in this case, a reduction in the cost of financing to zero should induce an increase in operational profits that offsets the loss of interest. Proposition 5 formalizes this intuition.

**Proposition 5** - Suppose that the investment function is concave on the interest rate and that the interest-elasticity of demand for inputs is greater than $\left(1 - \frac{\pi}{1+r}\right) \frac{f+(1-f)\pi}{1+r-f(1-f)\pi}$ at a zero interest rate. Then there is an equilibrium in which the supplier extends trade credit to both types of firms at a zero interest rate, even if the supplier cannot rescue any input when a customer becomes financially distressed (i.e., $\delta = 0$).

Elliehausen and Wolken (1993), Petersen and Rajan (1994) and Ng, Smith and Smith (1999) all report that interest rates in trade credit markets are often standardized. And that suppliers waive interest whenever their customers repay the loans within 10 days. Waiving interest rates is consistent with Propositions 4 and 5 if the demand for inputs is highly elastic with respect to interest rates of loans that must be repaid in a very short term.

Note, however, that while Propositions 4 and 5 impose lower bounds on the elasticity of demand for the invariance at zero exist, Proposition 3 imposes an upper bound on the elasticity to obtain an equilibrium at the banking rate. It then follows that, for a given set of parameters of the economy, the invariant equilibria at the banking rate and at the zero rate cannot simultaneously exist. To see why, recall from Proposition 4 that the equilibrium with no interest requires an elastic demand at zero. Since an increase in the interest rate raises
the interest-elasticity of the demand for inputs, an elastic demand at zero implies an elastic demand at \( r_B > 0 \), which, from Proposition 3, breaks down the invariant equilibrium at the banking rate. Hence, the interest-elasticity of demand plays an important role in determining the equilibrium set of interest rates in the trade-credit markets.

Still, one might wonder whether there might be alternative equilibria in which the supplier extends trade credit at an invariant rate that is neither zero nor the banking rate. Proposition 6 shows that these other equilibria do not exist.

**Proposition 6** - *If we restrict attention to equilibria in which the supplier extends trade credit to both types of firms at the same interest rate, then there are only two possible levels for the trade-credit rate: zero and the banking rate.*

The intuition for Proposition 6 is straightforward. When the banking rate is the optimal trade-credit rate, the supplier would actually like to raise interest further, tailoring the optimal interest rate to firm-specific characteristics. The supplier stops at the banking rate only because banks provide an outside option for firms. Yet, such constraint does not exist if the supplier starts at an interest rate in the open interval \((0, r_B)\). Here, the supplier can marginally change the interest rate to tailor the trade credit rate to, for example, the riskiness of each firm’s technology.

4 Empirical Implications and Discussion

4.1 Do suppliers have incentives to release information truthfully?

Consider an equilibrium with invariant interest rates and suppose that banks request information on the credit standing of a supplier’s customer. Is it in the supplier’s interest to release the information truthfully? As it turns out, announcing that the customer is a risky firm is a weakly dominating strategy for a supplier that can extend trade credit.

To see why, consider first the equilibrium in which the interest rate in the trade credit market is equal to the banking rate. In this equilibrium, the banking rate is an outside option for the firms that prevents the supplier from further increasing the interest rate. If the supplier can convincingly announce that the customer is a risky firm, banks will increase the interest rate accordingly, letting the supplier increase the interest rate as well. The higher interest

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8 This is true under the assumption that the investment in inputs is concave on the interest rate.
rate moves the supplier closer to its unconstrained optimal.\textsuperscript{9} In turn, the banking rate is not relevant to the supplier in the equilibrium in which the supplier waives interests. Hence, a supplier that can extend trade credit is either strictly or weakly better off announcing that a safe customer is risky.

Consider now a supplier that cannot extend trade credit. Here, the incentives to release information are reversed. If the supplier can convince the banks that its customers are safe, the banking rate will decrease accordingly, and the customer will demand more inputs. It is then a dominating strategy for the supplier to announce that their risky customers are safe.

To be sure, banks can offer some revelation mechanism to suppliers. For instance, profit-sharing mechanisms between a bank and a supplier should provide incentives for the supplier to credibly reveal private information. Still, we are not aware of any study that documents revelation mechanisms between banks and suppliers in standard trade-credit transactions. It is conceivable, though, that some sort of revelation mechanism is in place in project loans that are typically structured around very complex contracts. In these types of transactions, we do not expect an equilibrium with invariant interest rates.

### 4.2 Monopoly power and informational advantage

So far, we have assumed that the supplier enjoys monopoly power in the market for inputs. Are our results robust to competition among suppliers? To answer this question, we assume in this section that suppliers have access to a technology with constant returns to scale and a unit cost \( c \) of producing inputs.

Competition in the market for inputs drives prices to the marginal cost, that is, \( p = c \). In the absence of operational profits, there is no reason for a supplier to finance firms at a subsidized interest rate. The question then is whether a competitive market for inputs drives financial profits to zero as well.

Suppose first that the suppliers are all equally informed. In this case, there is no scope for

\[ \text{More formally, let } r_B^R \text{ be the interest rate that banks would have offered to a known risky firm. Competition among banks implies that the expected return of the loan at the interest rate } r_B^R \text{ equals the cost of funds } r. \text{ Hence, } \pi(1 + r_B^R) = (1 + r) \text{ which implies that } r_B^R = \frac{1 + r}{r} - 1 > r_B. \text{ This means that the interest rate in banking loans increases if a supplier convinces the banks that the customer is a risky firm. As a result, the constraint in the supplier’s program changes from } 0 \leq r_T^R \leq r_B \text{ to } 0 \leq r_T^R \leq r_B^R. \text{ Since } r_B^R > r_B, \text{ the constraint is relaxed, implying an increase in expected profits because a concave investment function implies that the supplier’s profit function is concave.} \]
a supplier to profit by lending to a safe firm. Competition will drive down the interest rate of loans to safe firms to the riskless rate $r$. Note, though, that the equilibrium interest rate in loans to risky firms will not be equal to the riskless rate. In these loans, the supplier takes into account the probability $1 - \pi$ that the debt contract will not be honored and that, in default, only $\delta I < I$ will be collected. As such, the interest rate $r^{R_T}$ that equals the expected return on the loan to the riskless rate is larger than the riskless rate $r$. And we conclude that competition among equally informed suppliers breaks down the equilibrium with invariance of interest rates in trade credit markets.

It is unlikely, nonetheless, that all suppliers of inputs are equally informed about their customers. It should be easier to learn private information about your best customers. Hence, although a threat to buy inputs from an alternative supplier may force a competitive price for the inputs, it may not break down the current supplier’s informational advantage. It then follows that the analysis of the pure monopoly case still applies. In particular, the informed supplier has the option of reducing the interest rate of loans to safe firms, but it cannot increase the interest rate beyond the firm’s outside option, which may be the interest rate that a rival supplier offers. As before, a sufficiently inelastic demand makes it optimal for the current supplier to offer financing to both types of firms at their outside option, while an elastic demand may induce the supplier to waive interest.

4.3 Invariance of interest rates and efficiency in default

As Proposition 3 demonstrates, an equilibrium with invariance at the banking rate requires that suppliers be more efficient than banks in lending to risky firms. In this paper, we model this relative efficiency as a greater ability to salvage assets of financially distressed firms. This is not the only reason for suppliers to be more efficient lenders to risky firms. Cuñat (2002) points out that the threat of stopping the supply of vital intermediate goods may induce firms to prioritize payments to suppliers. Still, a low comparative advantage in salvaging assets of financially distressed firms makes it more difficult for the threat of terminating the supply of inputs to be strong enough to assure that trade credit to risky firms is profitable. As we argue below, a testable implication of our model then follows.

Petersen and Rajan (1997) show that suppliers offer larger lines of credit to firms with a low fraction of their inventory in finished goods. Petersen and Rajan interpret their finding as evidence that suppliers’ advantage is salvaging assets is stronger in firms that hold a low fraction of their inventory in finished goods. Intuitively, once firms transform intermediate
goods into finished goods, suppliers can no longer use their regular sales force to sell the firms’ inventory. In this spirit, our model predicts that suppliers are more likely to offer standardized interest rates in industries with a low fraction of finished goods in inventory.

4.4 Credit constraint and insurance against liquidity shocks

In a sample of small firms in the National Survey of Small Business Finances (NSSBF), Petersen and Rajan (1997) find evidence that the demand for trade credit is inelastic. Given the high interest rate implicit in the standard discounts for early payments (e.g., 44 percent a year for the 2-10 net 30 loans), Petersen and Rajan argue that credit constrained firms are more likely to use trade credit. Yet, credit constraint cannot account for the invariance of the terms of credit at positive rates. In the absence of alternative sources of financing, suppliers are free to charge the interest rate that maximizes expected profits, which should vary with the borrowers’ credit standing. The model thus predicts that, although suppliers commonly publicize a single trade credit policy for all firms, they charge varying interest rates from the credit constrained customers. Standard terms of trade credit should be available mostly for customers that are not credit constrained.

Likewise, the evidence that the terms of trade credit are industry-not-firm specific is at odds with arguments that relate high discounts in early payments (i.e., a high interest rate) with an insurance against liquidity shocks. For instance, Wilner (2000) points out that, to preserve long-term business relationships, suppliers have incentives to bail-out financially distressed customers. According to this argument, suppliers provide insurance against liquidity shocks that may lead their customers into financial distress. Anticipating these incentives, suppliers should embed the cost of the insurance in the trade-credit rate. Note, however, that the expected cost of this insurance premium should vary with the customer’s risk, implying that, contrary to the existing evidence, the optimum trade-credit rate is firm specific.

4.5 Uniqueness of equilibrium with invariant rates

So far, we have restricted attention to equilibria in which, whenever possible, suppliers offer trade credit to both types of firms. In this class, Proposition 6 shows that the equilibrium with invariant rates is either at the banking rate (if the demand for inputs is sufficiently inelastic) or at a zero interest rate (if the demand is sufficiently elastic). If the conditions on
the elasticity of demand are not satisfied, it can be shown that there may be an equilibrium with varying interest rates.

The reason for focusing on equilibria in which the supplier extends trade credit to both types of firms is quite clear. There cannot be variation of interest rates if the supplier offers trade credit to customers that are all in the same class of risk. Yet, as often happens in game theoretical models, there exists an equilibrium in which the supplier extends trade credit to safe firms only.\footnote{There is no equilibrium in which the supplier finances risky firms only. To see why, suppose by contradiction that such equilibrium exists. Still, a risky firm would borrow from banks when trade credit is not available. Hence, the banking rate must be higher than the riskless rate to assure banks their cost of funds. However, this very same banking rate, which is the safe firm’s outside option, assures expected profits to trade credit to safe firms, breaking down the candidate for equilibrium. A similar argument breaks down equilibria in mixed strategies.} In this section, we exhibit conditions under which this alternative equilibrium breaks down while preserving the equilibrium with invariance of interest rates.

To break down equilibria in which the supplier extends trade credit to safe firms only, it suffices to make it optimal for the supplier to offer trade credit to risky firms. As before, the net benefits of lending to risky firms depends on the banking rate. Our first task, therefore, is to characterize the banking rate in the alternative equilibrium.

Assume then that the supplier denies trade credit to risky firms. If so, risky firms borrow from banks to purchase inputs. Understanding the equilibrium strategies, banks update their beliefs on a firm that requests a banking loan as follows. If the supplier can offer trade credit (an event with probability \( x \)) the firm that asks a banking loan is certain to be risky, and the loan will be repaid with probability \( \pi \). If, however, the supplier cannot offer trade credit (an event with probability \( 1 - x \)), a firm that requests a banking loan is safe with probability \( f \) – and the loan will be repaid with probability one – while it will be risky with probability \( 1 - f \), in which case the loan will be repaid with probability \( \pi \). The expected return on a banking loan at an interest rate \( \tilde{r}_B \) is thus \((1 + \tilde{r}_B)[(f + (1 - f)\pi)(1 - x) + \pi x]\). And the interest rate that makes the loan’s expected return equal to the cost of funds \( r \) solves:

\[
(1 + \tilde{r}_B)[(f + (1 - f)\pi)(1 - x) + \pi x] = (1 + r). \tag{16}
\]

Solving for the banking rate in equation (16) yields
\[
\tilde{r}_B = \frac{1 + r}{(f + (1 - f) \pi)(1 - x) + \pi x} - 1. \quad (17)
\]

One can easily verify that the interest rate \(\tilde{r}_B\) is bigger than the banking rate \(r_B\) of the equilibrium in which the supplier finances both types of firms (see equation (4)). Intuitively, the banking rate \(\tilde{r}_B\) of the alternative equilibrium must take into account that a larger number of risk firms borrow from banks because they do not have access to trade credit.

We now have all the necessary ingredients to break down the equilibrium in which the supplier denies trade credit to risky firms. To do this, we make it optimal for the supplier to finance risky firms. This condition is satisfied if

\[
\frac{\pi (1 + r^R_T(\tilde{r}_B)) I (r^R_T(\tilde{r}_B)) + (1 - \pi) \delta I (r^R_T(\tilde{r}_B))}{(1 + r)} \geq I^R_B(\tilde{r}_B). \quad (18)
\]

The left-hand-side of condition (18) is the present value of the expected profit of a supplier that finances the risky firm. This expected profit depends on the probability \(\pi\) that the risky firm repays the debt, the supplier’s ability to rescue inputs of financially distressed customers, \(\delta\), and the interest rate \(r^R_T(\tilde{r}_B)\) that, conditioned on lending to a risky firm that can borrow from banks at an interest rate \(\tilde{r}_B\), maximizes expected profits. As it turns out, the maximization program that yields \(r^R_T(\tilde{r}_B)\) is identical to the program (10) that solves for the optimal rate to risky firms in the equilibrium that the supplier finances both types of firms. Hence, we can characterize \(r^R_T(\tilde{r}_B)\) by equation (12), once we substitute \(\tilde{r}_B\) for \(r_B\).

Consider now the right-hand-side of condition (18). \(I(\tilde{r}_B)\) is the expected profit at \(t = 0\) of a supplier that does not offer trade credit to the risky firm. This expected profit consists of the sale of inputs (recall that the cost of producing the input is zero), which is the demand for inputs of the risky firm at the banking rate \(\tilde{r}_B\). Since, in this case, the supplier is paid at \(t = 0\), we do not need to discount the profit \(I(\tilde{r}_B)\). It then follows that condition (18) assures that financing the risky firm adds value to the supplier. If this condition is satisfied, the equilibrium breaks down because the supplier would have incentives to finance risky firms.

Condition (18) is basically identical to condition (13), which makes it optimal for the supplier to lend to risky firms in the equilibrium that both types of firms have access to trade credit. The sole difference in the two conditions is the banking rate. Since the necessary and sufficient conditions for the equilibrium with invariance assure that condition (13) holds, it
follows from Proposition 4 that condition (18) holds when \( \tilde{r}_B \) is equal to \( r_B \). In other words, \( \tilde{r}_B \) equal to \( r_B \) implies that the conditions that assure existence for the equilibrium with invariance also rules out the equilibrium in which only safe firms have access to trade credit.

When is \( \tilde{r}_B \) equal to \( r_B \)? A sufficient condition is that a supplier’s decision to deny trade credit does not change the equilibrium banking rate. This additional condition will be satisfied, for example, if we assume that the firm’s investment consists of purchases of several inputs from different monopolists, and that at least one of these suppliers is certain to be credit constrained. In this case, asking for banking loans to finance purchase of inputs does not convey information on the firm’s type. More formally, the probability \( x \) that trade credit satisfies all financing needs becomes arbitrarily close to zero. And, as one can easily check in equation (17), \( \tilde{r}_B \) converges to \( \frac{1+r}{f+\pi(1-f)} - 1 = r_B \).

5 Conclusions

As several studies have recently documented (Elliehausen and Wolken (1993), Petersen and Rajan (1994) and Ng, Smith and Smith (1999)), the terms of trade credit in the U.S. are industry-not-firm specific. Since there is evidence that, vis-à-vis banks, suppliers are better informed about the economic health of their customers, a lack of firm-specific variation in the terms of trade credit is surprising.

This paper provides a reason for why interest rates in trade credit markets do not reflect suppliers’ private information about their customers’ credit risk. We argue that if the demand for intermediate goods is sufficiently elastic with respect to interest rates, waiving interest may lead to an increase in the operational profit that more than offsets financial losses in trade credit transactions. If so, it is optimal for suppliers to waive interest when they extend trade credit. In the other extreme, an inelastic demand gives suppliers no incentives to undercut banks, regardless of the firm’s type. Thus, under a sufficiently inelastic demand, the trade-credit rate converges to the cost of the customers’ outside option (a banking loan), which, by its very nature, cannot reflect private information held by suppliers. It then follows that private information held by suppliers is not conveyed to the trade credit rates, whenever the demand is highly elastic or highly inelastic.

Our model does not account for all the reported rigidity of interest rates in trade credit markets, though. In particular, under an inelastic demand, our model predicts different terms of trade credit to firms that, from the banks’ perspective, are in different classes of risk. One
possible reason for the seemingly excessive variation of trade credit rates in our model is that we have ignored alternative mechanisms to vary the cost of trade credit. For instance, as Petersen and Rajan (1994) point out, suppliers allow for some variation in the actual trade credit rate by selectively granting discounts for payments after the due date. Yet, rather than counting on costly ex-post renegotiations to change contracted interest rates, one would expect that trade credit contracts embeds interest rates that vary with the borrower’s credit standing, as normally happens in banking loans (see Petersen and Rajan (1995)).

We find it likely, however, that some suppliers use the price of their products – rather than interest rates – to discriminate their customers. In this case, the invariance of the trade credit rate may simply reflect its redundancy: the freedom to selectively set prices may let the supplier extract all of the consumer surplus. To be sure, it is hard to imagine that a supplier can enjoy such an extensive degree of monopoly power for a long time. Still, some suppliers may have leeway to increase prices when, for example, the standard 44 percent rate is too low for a customers’ risk. An interest topic for future research, therefore, is to understand when suppliers should change prices, trade-credit terms, or both, in response to shocks in their cost of funds, demand for products, or the financial health of their customers.
Appendix

Proof of Proposition 1: Differentiating the supplier’s profit, $\Phi_T^S (r_T^S) = (1 + r_T^S)I (r_T^S)$, with respect to $r_T^S$ yields

$$\frac{\partial}{\partial r_T} \Phi_T^S (r_T^S) = (1 + r_T^S)I' (r_T^S) + I (r_T^S),$$

$$\frac{\partial^2}{\partial r_T^2} \Phi_T^S (r_T^S) = (1 + r_T^S)I'' (r_T^S) + 2I' (r_T^S).$$

From equation (20), the profit function $\Phi_T^S$ is concave on $r_T^S$ because the investment function is concave and decreasing on $r_T^S$. Concavity of $\Phi_T^S$ implies that the first order condition of the supplier’s problem is also sufficient for optimality. We analyze three cases:

Case 1: Suppose first that $\frac{\partial}{\partial r_T} \Phi_T^S (r_B) \geq 0$. This first case happens when $(1 + r_B)I' (r_B) + I (r_B) \geq 0$, which is equivalent to $\frac{1 + r_B}{I (r_B)}I' (r_B) \geq -1$. Noting that the absolute value of the left-hand side of this latter inequality is our definition of the interest-elasticity of the demand for inputs at $r_B$, $\epsilon^D (r_B)$, it follows that $\frac{\partial}{\partial r_T} \Phi_T^S (r_B) \geq 0$ if and only if $\epsilon^D (r_B) \leq 1$. By concavity of the profit function, the marginal profits decrease with the interest rate. Hence, $\frac{\partial}{\partial r_T} \Phi_T^S (r_B) \geq 0$ implies that profits increase with interest in the interval $0 \leq r_T^S \leq r_B$. We thus conclude that $\epsilon^D (r_B) \leq 1$ if and only if it is optimal for the supplier to set $r_T^S = r_B$.

Case 2: Suppose now that $\frac{\partial}{\partial r_T} \Phi_T^S (0) \leq 0$, which, from the arguments of the previous case, is equivalent to $\epsilon^D (0) \geq 1$. Since marginal profits decrease with the interest rate, $\frac{\partial}{\partial r_T} \Phi_T^S (0) \leq 0$ implies that the supplier’s profit decreases with interest in the interval $0 \leq r_T^S \leq r_B$. Thus, it is optimal for the supplier to set $r_T^S = 0$. And we conclude that $\epsilon^D (0) \geq 1$ if and only if it is optimal for the supplier to set $r_T^S = 0$.

Case 3: Suppose that $\frac{\partial}{\partial r_T} \Phi_T^S (0) > 0$ and $\frac{\partial}{\partial r_T} \Phi_T^S (r_B) < 0$. This case happens when $I'(0) + I (0) > 0$ and $(1 + r_B)I' (r_B) + I (r_B) < 0$, or equivalently, $\epsilon^D (0) < 1$ and $\epsilon^D (r_B) > 1$. Here, the supplier’s profit reaches its maximum in the interval $(0, r_B)$. Thus, $\epsilon^D (0) < 1$ and $\epsilon^D (r_B) > 1$ are necessary and sufficient conditions for $r_T^S \in (0, r_B)$, as we wanted to prove.

Proof of Proposition 2: We can write the expected profit of the supplier as $\Phi_T^R (r_T^R) = \pi [(1 + r_T^R)I (r_T^R)] + (1 - \pi) [\delta I (r_T^R)]$. The first and second derivatives of the expected profit with respect to the interest rate $r_T^R$ are:

$$\frac{\partial}{\partial r_T} \Phi_T^R (r_T^R) = \pi [(1 + r_T^R)I' (r_T^R) + I (r_T^R)] + (1 - \pi) \delta I' (r_T^R)$$

$$\frac{\partial^2}{\partial r_T^2} \Phi_T^R (r_T^R) = \pi [(1 + r_T^R)I'' (r_T^R) + 2I' (r_T^R)] + (1 - \pi) \delta I'' (r_T^R)$$
The expected profit $\Phi^R_T$ is concave on $r_T^S$, because the investment function is concave and decreasing on $r_T^S$. Concavity of $\Phi^R_T$ then implies that the first order condition of the supplier’s problem is also sufficient for optimality. Rearranging terms in $\frac{\partial}{\partial r} \Phi^R_T (r_T^R)$ as in the proof of Proposition 1 yields:

**Case 1:** $\epsilon^D (r_B) \leq \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$. The expected profit increases with interest rates $(\frac{\partial}{\partial r} \Phi^R_T \geq 0)$ in the interval $[0, r_B]$. Hence, $r_B$ is the optimal interest rate.

**Case 2:** $\epsilon^D (r_B) > \frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$ and $\epsilon^D (0) < \frac{\pi}{\pi+(1-\pi)\delta}$. From the first inequality, the expected profit decreases with the interest rate at $r_B$. From the second inequality, the expected profit increases with the interest rate at the rate zero, reaching its maximum in the open interval $(0, r_B)$.

**Case 3:** $\epsilon^D (0) \geq \frac{\pi}{\pi+(1-\pi)\delta}$. The expected profit decreases with the interest rate $(\frac{\partial}{\partial r} \Phi^R_T \leq 0)$ in $[0, r_B]$, reaching its maximum at $r_T^R = 0$. ■

**Proof of Proposition 4:** We showed in section 3.4 that suppliers are willing to finance risky firms at a zero interest rate if $\pi I (0) + (1-\pi)\delta I (0) \geq (1+r) I (r_B)$. This condition can be rewritten as $\delta \geq \frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$. Since the demand for investment decreases with the interest rate, $I' (\cdot) < 0$, $r_B > 0$ implies that $I (r_B) < I (0)$. Define $\frac{I(r_B)}{I(0)} = \beta(r_B) \in (0, 1)$. Since $r_B$ is a function of $\pi$ and $r$, so is $\beta(r_B)$ and we can write $\beta(\pi, r)$. Hence, we can once more rewrite the condition on $\delta$ as $\delta \geq \beta(\pi, r) \frac{(1+r)}{(1-\pi)} - \frac{\pi}{(1-\pi)}$.

Call $\psi (\pi) = \beta(\pi, r) \frac{(1+r)}{(1-\pi)} - \frac{\pi}{(1-\pi)}$. Since $\psi(0) = \beta(0, r)(1+r)$, for values of $\pi$ sufficiently close to zero, there is an equilibrium with an invariant rate at zero if the elasticity of demand at zero satisfies the condition in the statement of the proposition and if $\beta(0, r)(1+r) < 1$, which assures that there is $\delta \in (0, 1)$ satisfying $\delta \geq \beta(\pi, r) \frac{(1+r)}{(1-\pi)} - \frac{\pi}{(1-\pi)}$. We claim that the set of parameters that satisfy $\beta(0, r)(1+r) < 1$ is not empty. Plugging $r_B = \frac{1+\pi}{1+\pi(1-f)} - 1$ into the definition of $\beta(\pi, r)$ and making $\pi = 0$ in $\beta(0, r)(1+r) < 1$ yields $I \left( \frac{1+r}{1+r} - 1 \right) (1+r) < I (0)$. For $r = 0$, this inequality holds because $I(\cdot)$ decreases with the interest rate and $f < 1$. To prove that the inequality holds for any $r \geq 0$, define the left-hand side of the inequality as a function of $r$: $\varphi (r) = I \left( \frac{1+r}{1+r} - 1 \right) (1+r)$. Thus, it suffices to show that $\varphi$ decreases with $r$. Differentiating $\varphi$ obtains $\varphi' (r) = I(r_B) + I'(r_B) \frac{1+r}{1+r}$, after taking into account that $\frac{1+r}{1+r} - 1$ is $r_B$ evaluated at $\pi = 0$. Hence, after some algebra, $\varphi' (r) \leq 0$ if and only if $\epsilon^D (r_B) \geq 1$; an inequality that holds trivially because, by assumption of the proposition, the demand is elastic at zero, which implies an elastic demand at $r_B$ from concavity of the investment function. An equilibrium thus exists at zero interest.

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Finally, if $e^D(0) \geq 1$, equations (8) and (12) imply that it is strictly optimal for the supplier to waive interest, regardless of the firm’s type. As such, the equilibrium at a zero interest is unique in the class of equilibria in which the supplier extends trade credit to both types of firms. 

**Proof of Proposition 5:** In Proposition 4, the equilibrium with invariance at a zero interest rate obtains if the supplier’s advantage in default satisfies $\delta \geq \frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$. This restriction is irrelevant if $\frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)} \leq 0$, or equivalently,

\[(1+r)\frac{I((1+r)(f + (1+f)\pi)^{-1})}{I(0)} \leq \pi. \tag{23}\]

By the concavity of $I(.)$, any $x \geq 0$ implies that $I(x) \leq I(0) + I'(0)x$, which, at $x = (1+r)(f + (1+f)\pi)^{-1}$ implies equation (23) if $\frac{1+r}{\pi} \leq \frac{1}{1-e^{\delta}(0)(1+r)(f+(1+f)\pi)^{-1}}$. This latter inequality is equivalent to $e^D(0) \geq (1 - \frac{\pi}{1+r}) \frac{f+(1+f)\pi}{1+r(f+(1+f)\pi)}$, as we wanted to prove. 

**Proof of Proposition 6:** From Proposition 3, the banking rate $r_B$ is the only candidate for an invariant equilibrium if, at this rate, the elasticity of demand is smaller than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$. Consider first that, at $r_B$, the elasticity of the demand for inputs lies in the open interval $\left(\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}, 1\right)$. In this interval, the demand is inelastic, implying, from equation (8), that the supplier should extend trade credit to safe firms at the banking rate $r_B$. From equation (12), nonetheless, an elasticity of demand bigger than $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$ implies that the optimal trade-credit rate is lower than the banking rate $r_B$. Hence, there cannot be an equilibrium with invariant interest rates if the elasticity at $r_B$ belongs to the interval $\left(\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}, 1\right)$.

Suppose now that the demand is elastic at the banking rate $r_B$. From equations (8) and (12), the optimal interest rate is smaller than the banking rate $r_B$, regardless of the firm’s type. One can easily check from the first order conditions of programs (6) and (10) that the optimal interest rates of the two types of firms cannot be equal if they lie in the open interval $(0, r_B)$. Hence, zero is the only candidate for an interest rate that does not vary with the firm’s type, if the elasticity of the demand for inputs at $r_B$ is larger than $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$. 


REFERENCES


Figure 1: Game in the extensive form