Monetary Policy and Exchange Rate Shocks in Brazil: Sign Restrictions versus A New Hybrid Identification Approach

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Abstract

This paper analyzes the impacts of several (monetary policy, exchange rate, demand, and supply) exogenous disturbances on the Brazilian economy using a structural vector autoregression (SVAR) model identified by two alternative methodologies. The first uses sign restrictions on impulse responses based on an open-economy macroeconomic model. The second (hybrid) is a new methodology, which we develop, that combines the first with restrictions on the contemporaneous causal interrelationships among variables, derived by DAGs (Directed Acyclic Graphs).

The algorithms we employ to identify the exogenous shocks are based on the algorithm developed by Rubio-Ramírez, Waggoner and Zha (2007), which is currently the most efficient algorithm when several independent exogenous shocks are identified by sign restrictions on impulse responses. We estimate our SVARs using the Bayesian method proposed by Sims and Zha (1998) and Waggoner and Zha (2003), which allows for consistent estimation of overidentified models, present in the hybrid methodology.

The results of the hybrid identification show a delayed response of the price level to monetary policy shocks, consistent with the presence of some price rigidity. A comparison of the results shows that while the effects of exchange rate shocks are nearly the same, the effects of monetary policy shocks depend on the methodology adopted. We find a higher contribution of monetary policy shocks to output, prices, and exchange rate fluctuations when using sign restrictions only. There is a strong response of the exchange rate to demand shocks and to shocks originating in the foreign exchange market. Exchange rate shocks have an important role in explaining short-run fluctuations of prices and output. We conclude that the exchange rate is an independent source of shocks and a shock absorber.

Keywords: Structural VAR, Hybrid Identification, Directed Acyclic Graphs, Sign Restrictions.


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1. Introduction

Uncovering some stylized facts about short run fluctuations of the Brazilian economy after the Real Plan, Céspedes, Lima, and Maka (2008) [CLM] find that unanticipated movements of the exchange rate not only have large inflationary effect, but also large real effects. This suggests that the exchange rate is an important component in any model aimed toward explaining the behavior of the Brazilian economy after the Real Plan. CLM analysis is concerned mainly with the identification of the effects of monetary policy shocks, so not much attention was given to the identification of the exchange rate shocks or any other shock.

In this article we extend CLM analysis by identifying several (monetary policy, exchange rate, demand, and supply) exogenous disturbances on the Brazilian economy using a structural vector autoregression (SVAR) model identified by two alternative methodologies. The first methodology uses sign restrictions on impulse responses of the shocks based on short-run dynamics of a stochastic open-economy macroeconomic model. The second methodology (hybrid) is a new methodology, which we develop, that combines sign restrictions with restrictions on the contemporaneous causal interrelationships among variables, derived by DAGs (Directed Acyclic Graphs). The hybrid identification strategy pursued in this article consists of two steps\(^1\). In the first step, we use DAGs to select over-identifying restrictions on the contemporaneous coefficients based on the conditional independence relations between the variables. These over-identifying restrictions allow us to identify a monetary policy shock and to restrict the covariance matrix of the reduced-form residuals. In the second step, maintaining restricted the covariance matrix of reduced-form residuals, we keep the identified monetary policy shock and impose sign restrictions on the impulse response functions of the other three shocks to identify the demand, supply, and exchange rate shocks.

The algorithms we employ to identify the exogenous shocks are based on the algorithm developed by Rubio-Ramírez, Waggoner and Zha (2007), which is currently the most efficient algorithm when several independent exogenous shocks are identified by sign restrictions on impulse responses. We estimate our SVARs using the Bayesian method proposed by Sims and Zha (1998) and Waggoner and Zha (2003), which allows for consistent estimation of overidentified models, present in the hybrid methodology.

A comparison of the results of the two identification approaches shows that while the effects of exchange rate shocks are nearly the same, the effects of monetary policy shocks depend on the methodology adopted. We find a higher contribution of monetary policy shocks to output, prices, and exchange rate fluctuations when using sign restrictions only. There is a strong response of the exchange rate to demand shocks and to shocks originating in the foreign exchange market. Exchange rate shocks have an important role in explaining short-run fluctuations of prices and output. We conclude that the exchange rate is an independent source of shocks and a shock absorber.

The article is organized as follows. Section 2 presents a brief discussion of the identification methodologies that we combine in our hybrid approach. Section 3

\(^1\) Dungey and Fry (2008) propose a different hybrid identification approach that combines sign restrictions, cointegration and traditional exclusion restrictions within a system which explicitly models stationary and non-stationary variables and accounts for both permanent and temporary shocks.
explains the methodology adopted for identifying and estimating the VARs. Section 4 describes the empirical model. Section 5 presents the hybrid identification procedure that combines short-run restrictions on the contemporaneous coefficients with sign restrictions on the impulse response functions. Section 6 shows an alternative identification procedure based on sign restrictions only. Finally, section 7 offers some concluding remarks.

2. Identification of SVARs

Identifying restrictions are necessary in order to give a meaningful interpretation to residuals in vector autoregressions (VARs) models. Without such restrictions impulse response functions typically do not trace out the effects of exogenous structural disturbances such as monetary policy or exchange rate shocks. Instead, they typically pick up the effects of a linear combination of these structural shocks. Typical restrictions employed in the literature include constraints on the short run or long run impact of certain shocks on variables or informational delays (e.g. inflation is not contemporaneously observed by Central Banks when deciding interest rates).

The identification of structural shocks is, in general, a highly controversial enterprise because researchers imposing different identifying assumptions may reach different conclusions about interesting economic questions (e.g. the sources of business cycle fluctuations). Criticisms to the nature of identification process have repeatedly appeared in the literature. For example, Cooley and LeRoy (1985) criticize Choleski decompositions because contemporaneous recursive structures are hard to obtain in general equilibrium models. Faust and Leeper (1997) argue that long run restrictions are unsatisfactory as they may exclude structures that generate perfectly reasonable short run dynamics but fail to satisfy long run constraints by infinitesimal amounts. Cooley and Dwyer (1998) indicate that long run restrictions may also incompletely disentangle permanent and transitory disturbances. Canova and Pina (2004) show that standard dynamic stochastic general equilibrium (DSGE) models almost never provide the zero restrictions employed to identify monetary disturbances in structural VAR systems and that misspecification of the features of the underlying economy can be substantial.

Recently, a new identification approach emerged based on the procedures of Faust (1998), Canova and De Nicoló (2002), and Uhlig (2005), where identification is achieved by restricting the sign (and/or shape) of structural responses. Such sign restrictions are attractive for several reasons. First, while (log)-linearized versions of DSGE models rarely deliver the $m(m-1)/2$ set of zero restrictions needed to recover $m$ structural shocks, they contain a large number of sign restrictions usable for identification purposes. Second, sign restrictions make explicit restrictions that are often used implicitly by researchers when identifying VARs. Third, they can be robust in the sense that they hold across several structural models or parameterizations of the same model. However, these advantages come at a cost: identification of the structural shocks is not exact. There are multiple matrices defining the linear mapping from orthogonal structural shocks to VAR residuals. All of these matrices satisfy the sign restrictions and imply the same reduced form covariance matrix of VAR residuals. In other words, they are observationally equivalent and equally consistent with the economic theory imposed with sign restrictions. Paustian (2007) evaluates the sign restrictions method based on
two DSGE models and concludes that sign restrictions can be a useful tool to recover structural shocks from VAR residuals. However, two conditions must be met for the method to unambiguously deliver the correct sign of unconstrained impulse responses. First, a sufficiently large number of restrictions must be imposed; more than what is typically employed in applied work. Second, the variance of the shock under study must be sufficiently large.

Following the growing interest on graphical models and in particular on those based on directed acyclic graphs (DAGs) as a general framework to describe and infer causal relations, several authors have applied this methodology to identify SVARs. Swanson and Granger (1997) were the first to apply graphical models to identify contemporaneous causal order of a SVAR, followed by Bessler and Lee (2002), Demiralp and Hoover (2003) and Céspedes, Lima, and Maka (2008) [CLM]. The last three articles adopt a procedure that uses statistical properties of the sample – more specifically, conditional independence relations between the variables to select over-identifying restrictions to estimate structural VARs. These restrictions follow from DAGs estimated by the TETRAD software developed by Spirtes, Glymour, and Scheines (2000) using as input the covariance of reduced form VAR disturbances. However, the use of DAGs for making causal inferences is subject to an important caveat: as Robins et al. (2003) have shown, causal procedures based on associations of non-experimental data under weak conditions, are not uniformly consistent. That means that for any finite sample, there are no guarantees that the results of the causality tests will converge to the asymptotic (correct) results. Zhang (2002), Zhang and Spirtes (2003) showed that, under the hypothesis that small partial correlations among variables indicate small direct causal effects, it is possible to guarantee convergence to the asymptotic correct results. Throughout this article we assume that small partial correlations indicate small direct causal effects.

In this article we develop a new methodology that combines sign restrictions with DAGs. The next section describes this new methodology.

3. Methodology

Let $y_t$ be the data vector – there are 6 variables in the model, therefore $y_t$ has dimension $n \times 1$ ($n=6$) for each period $t$:

$$y_t = \begin{bmatrix} y_{t1} & y_{t2} & \cdots & y_{tn} \end{bmatrix},$$

where:

$y_{t1} = \log(\text{Gross annualized Selic interest rate}),$  
$y_{t2} = \log(\text{Nominal exchange rate(R$/US$))},$  
$y_{t3} = \log(\text{IPCA index}),$  
$y_{t4} = \log(\text{180 days Swap rate (PRE x CDI - annualized considering 252 working days))},$  
$y_{t5} = \log(\text{Industrial Production Index}),$  
and  
$y_{t6} = \log(\text{M1}).$

---

2 The methodology developed here builds on Ramírez, Waggoner and Zha (2007), Sims and Zha (1998), and Waggoner and Zha (2003).
The structural VAR model has the general form:

\[ y_t' A' = \sum_{r=1}^{p} y_{t-r}' A' + z_t' D' + \varepsilon_t', \quad \text{for} \quad t = 1, \ldots, T, \tag{1} \]

where

- \( y_t \) is an \( n \times 1 \) column vector of endogenous variables at time \( t \),
- \( A \) and \( A_t \) are \( n \times n \) parameter matrices;
- \( D \) is an \( n \times h \) parameter matrix,
- \( z_t \) is an \( h \times 1 \) column vector of seasonal dummies and constant term at time \( t \),
- \( \varepsilon_t \) is an \( n \times 1 \) column vector of structural disturbances at time \( t \);
- \( p \) is the lag length, and \( T \) is the sample size (\( p=6 \) and \( T=113 \)).

The parameters of individual equations in (1) correspond to the columns of \( A', A_t' \) and \( D' \).

The structural disturbances have a Gaussian distribution with

\[ \begin{align*}
E(\varepsilon_t | y_1, \ldots, y_{t-1}, z_t, \ldots, z_T) &= 0 \\
E(\varepsilon_t \varepsilon_{t}^\prime | y_1, \ldots, y_{t-1}, z_t, \ldots, z_T) &= I
\end{align*} \]

and are normalized to have an identity covariance matrix. Right multiplying the structural form (1) by \( (A')^{-1} \), we will obtain the usual representation of a reduced-form VAR with the reduced-form variance matrix being \( \Omega = (A'A)^{-1} \).

Unlike typical unrestricted VAR models, \( \Omega \) will be restricted when the contemporaneous parameter matrix \( A \) is overidentified.

The structural VAR models (1) can be rewritten in the compact form:

\[ y_t' A' = x_t' F' + \varepsilon_t' \]

where

\[ x_t'[y_{t-1}' \ldots y_{t-p}' \ z_t], \quad F'=[A_1 \ldots A_p \ D] \]

and \( k = np + h \). We will refer to \( F' \) as lagged parameters even though \( F' \) may also contain exogenous parameters.

For \( 1 \leq i \leq n \), let \( a_i \) be the \( i'th \) column of \( A' \), let \( f_i \) be the \( i'th \) column of \( F' \) and let \( T_i \) be an \( n \times n \) matrix of rank \( q_i \). The linear restrictions of interest can be summarized as follows:

\[ T_i a_i = 0, \quad i = 1, \ldots, n, \tag{2} \]
The restrictions given by (2) are said to be non-degenerate if there exists at least one non-singular matrix $A'$ satisfying them. In this paper, all restrictions are assumed to be non-degenerate.

When VAR models are large and degrees of freedom are low, the likelihood function itself can be ill behaved and there is the well-known tendency of estimates to become unreliable. To deal with these problems, Litterman (1986) introduces a widely used Bayesian prior distribution for reduced-form models to down-weight models with large coefficients on distant lags and explosive dynamics. Sims and Zha (1998) incorporate Litterman’s idea in the structural framework by specifying the prior distribution of $\alpha_i$ and $\phi_i$ as

$$a_i \sim N(0, S_i) \quad \text{and} \quad f_i|a_i \sim N(\bar{P} a_i, \bar{H}_i), \quad (3)$$

Where $\bar{H}_i$ is a $k \times k$ diagonal and positive definite (SPD) matrix:

$$\bar{H}_i = \begin{bmatrix}
\lambda_0 \lambda_1 \\
\sigma_i & 0 & \ldots & 0 \\
0 & \lambda_2 \lambda_1 \\
\sigma_i & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \frac{\lambda_n \lambda_1}{\sigma_i} \\
\end{bmatrix}_{36x36}$$

The standard deviation of the conditional prior of $f_i$ (subset of parameters of equation $i$) for the coefficient on lag $l$ of the variable $j$, is given by

$$\frac{\lambda_0 \lambda_1}{\sigma_l \lambda_l}$$

Where: the hyperparameter $\lambda_0$ controls the tightness of beliefs on $A'$; $\lambda_1$ controls what Litterman called overall tightness of beliefs around the random walk prior; $\lambda_2$ controls the rate at which prior variance shrinks for increasing lag length; $\lambda_n$ is the tightness for the constant term and seasonal dummies, i.e., for the last 12 rows of each column of $F'$. We give it a conditional prior mean of zero and a standard deviation controlled by $\lambda_0 \lambda_4$.

The vector of parameters $\sigma_1, \ldots, \sigma_n$ (one for each equation) are scale factors, allowing for the fact that the units of measurement or scale of variation may not be uniform across variables. The scale factors are taken as the sample standard deviations.
of residuals from univariate autoregressive models, with lag length \( p \), fit to the individual series in the sample.

The diagonal matrix \( S_i \) is a \( n \times n \) positive definite matrix, the individual elements in the \( i^{th} \) column of \( A' \) are assumed independent, with prior standard deviations set to \( \lambda_i / \hat{\sigma}_i \) (parameters defined above):

\[
\begin{pmatrix}
\lambda_i / \hat{\sigma}_i & 0 & 0 & \cdots & 0 \\
0 & \lambda_i / \hat{\sigma}_2 & 0 & \cdots & \vdots \\
\vdots & 0 & \lambda_i / \hat{\sigma}_3 & \ddots & 0 \\
0 & 0 & \cdots & 0 & \lambda_i / \hat{\sigma}_n
\end{pmatrix}
\]

We use the following values for the hyperparameters:

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_0 )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>1</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\( \bar{P}_i \) is a \( k \times n \) matrix defined as:

\[
\bar{P}_i = \begin{bmatrix}
1_{6 \times 6} \\
0_{42 \times 6}
\end{bmatrix}
\]

The prior form summarized above represents a class of existing Bayesian priors that have been widely used for structural VAR models. Combining the prior form (3) with the restriction (2), we wish to obtain the functional form of the conditional prior distribution:

\[
q(a_i, f_i | T a_i = 0).
\]  

(4)

In our case, the following matrices are the restricted \( A \) and \( A' \) matrices obtained by the application of the TETRAD software:
Then, we can obtain the \( T_i \) matrices which satisfy the constraints for each column \( i \) of \( A' \):

\[
T_i a_i = 0
\]

Each matrix \( T_i \) reproduces the restrictions present in column \( i \) of \( A' \), given by TETRAD. All element of \( T_i \) off the diagonal are zero. At the diagonal, there are zeros in the position of free parameters and ones in the position of parameters restricted to be equal zero. Therefore, for example

\[
T_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\quad \text{and} \quad
T_6 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Let \( U_i \) be an \( n \times q_i \) matrix whose columns form an orthonormal basis for the null space of \( T_i \). The column \( a_i \) will satisfy the restriction (2) if and only if there exist a \( q_i \times 1 \) vector \( b_i \) ( \( q_i = \text{number of free parameter at column } i \) of matrix \( A' \)) such that

\[
a_i = U_i b_i,
\]

The column vector \( b_i \) contains the free parameters of column \( i \) of matrix \( A' \) given by TETRAD. For this matrix \( A' \) the \( U_i \)'s are given by,
For example,

\[
\begin{bmatrix}
U_1' \\
U_2' \\
U_3' \\
U_4' \\
U_5' \\
U_6'
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

The distributions of \( b_i \) and \( f_i \) are given by

\[
b_i \sim N(0, \tilde{S}_i) \quad \text{and} \quad f_i | b_i \sim N(\tilde{P}_i b_i, \tilde{H}_i),
\]

(6)

where,

\[
\tilde{H}_i = \Pi_i, \quad \tilde{P}_i = \Pi_i U_i, \quad \text{and} \quad \tilde{S}_i = (U_i \tilde{S}_i^{-1} U_i)^{-1}.
\]

Note that \( \tilde{S}_i \) is a \( q_i \times q_i \) SPD matrix, \( \tilde{H}_i \) is an \( r_i \times r_i \) SPD matrix, and \( \tilde{P}_i \) is an \( r_i \times q_i \) matrix. It can be verified that the prior distribution (6) for \( b_i \) is equivalent to the prior distribution (4) for \( a_i \). For the most part of this paper, we work directly with \( b_i \) with the understanding that the original parameters \( a_i \) can be easily recovered via the linear transformations \( U_i \).

Let \( b = [b_1 \cdots b_n] \), \( f = [f_1' \cdots f_n'] \), \( X = [x_1 \cdots x_T] \), and \( Y = [y_1 \cdots y_T] \).

The likelihood function for \( b \) and \( f \) \( L((b, f) | X, Y) \) is proportional to
\[
|\text{det}[U_i b_i | \cdots | U_n b_n]|^r \exp \left( -\frac{1}{2} \sum_{i=1}^{n} b_i Y Y U_i b_i - 2 f_i Y Y U_i b_i + f_i X X f_i \right).
\]

Combining the priors on \(b\) and \(f\) given by (6) with the likelihood function given by (7) leads to the following joint posterior p.d.f for \(b\) and \(f\):

\[
p(b_1, \ldots, b_n | X, Y) \prod_{i=1}^{n} p(f_i | b_i, X, Y),
\]

where

\[
p(b_1, \ldots, b_n | X, Y) \propto |\text{det}[U_i b_i | \cdots | U_n b_n]|^r \exp \left( -\frac{T}{2} \sum_{i=1}^{n} b_i S_i^{-1} b_i \right),
\]

\[
p(f_i | b_i, X, Y) = \phi(P_b b_i, H_i),
\]

with

\[
H_i = (X X + \tilde{H}_i^{-1})^{-1},
\]

\[
S_i = \left( \frac{1}{T} (U_i Y Y U_i + \tilde{S}_i^{-1} + \tilde{P}_i \tilde{H}_i^{-1} \tilde{P}_i - \tilde{P}_i H_i^{-1} \tilde{P}_i) \right)^{-1}.
\]

Since (8) has an unknown distribution, we must take draws from the posterior distribution of \(b\) by Gibbs Sampling and, given each draw of \(b\), take draws of \(f\) from the Gaussian conditional distribution (9). The notation \(\phi(P_b b_i, H_i)\) in (9) denotes the Gaussian density with mean \(P_b b_i\) and covariance matrix \(H_i\).

In many works with VARs, only the likelihood function (i.e., proportional to the posterior density under a flat prior for \(b\) and \(f\)) is considered. Because (7) is the same as (8) and (9) when the prior variances (diagonal elements in \(\tilde{S}_i\) and \(\tilde{H}_i\)) approach infinity, the posterior density specified in (8) and (9) includes the likelihood as a special case.

To obtain small-sample inferences of \(b\) and \(f\) or for functions of them (e.g., impulse responses), it is necessary to simulate the joint posterior distribution of \(b\) and \(f\). This simulation involves two consecutive steps. First, simulate draws of \(b\) from the marginal posterior distribution (8). Second, given each draw of \(b\), simulate draws of \(f\) from the conditional posterior distribution (9). The second step is straightforward because it requires draws only from a multivariate normal distribution. The first step, as mentioned earlier, can be challenging when linear restrictions on \(A\) imply a restricted reduced-form covariance matrix.

The following algorithm was designed to obtain a sample of the impulse response functions, which satisfy the sign restrictions.
**Algorithm.** The following steps compose the algorithm for simulating draws from the
posterior distribution of $b, f$ and, given these draws, draws of the impulse responses that
satisfy the sign restrictions.

1. Get the values at the peak of the posterior density function.
2. For $s = 1, \ldots, N_1$ and given $b_1^{(s-1)}, \ldots, b_n^{(s-1)}$, obtain $b_1^{(s)}, \ldots, b_n^{(s)}$ by
   a. simulating $b_1^{(s)}$ from the distribution $b_1 | b_2^{(s-1)}, \ldots, b_n^{(s-1)}$,
   b. simulating $b_2^{(s)}$ from $b_2 | b_1^{(s)}, b_3^{(s-1)}, \ldots, b_n^{(s-1)}$,
   $\vdots$
   c. simulating $b_n^{(s)}$ from $b_n | b_1^{(s)}, \ldots, b_{n-1}^{(s)}$.
3. Keep $b_1^{(N_1)}, \ldots, b_n^{(N_1)}$.
4. For $s = N_1 + 1, N_2$ and given $b_1^{(s-1)}, \ldots, b_n^{(s-1)}$, obtain $b_1^{(s)}, \ldots, b_n^{(s)}$ by
   d. simulating $b_1^{(s)}$ from the distribution $b_1 | b_2^{(s-1)}, \ldots, b_n^{(s-1)}$,
   e. simulating $b_2^{(s)}$ from $b_2 | b_1^{(s)}, b_3^{(s-1)}, \ldots, b_n^{(s-1)}$,
   $\vdots$
   f. simulating $b_n^{(s)}$ from $b_n | b_1^{(s)}, \ldots, b_{n-1}^{(s)}$.
   g. Given $b_1^{(s)}, \ldots, b_n^{(s)}$ simulate $f_1^{(s)}, \ldots, f_n^{(s)}$ from the conditional normal
distribution described in equation (9).
   h. Given $b_1^{(s)}, \ldots, b_n^{(s)}$ and $f_1^{(s)}, \ldots, f_n^{(s)}$ obtain $A^{(s)}$ and $B^{(s)} = F^{(s)} A^{(s)-1}$ ($A$
   and $F$ were described previously — $B$ contains the reduced form
parameters).
   i. Draw an independent standard normal $n \times n$ matrix $\tilde{X}$ and let
   $\tilde{X} = \tilde{Q} \tilde{R}$ be the $QR$ decomposition of $\tilde{X}$ with the diagonal $\tilde{R}$ normalized
to be positive.
   j. Let $P = \tilde{Q}$ and generate the impulse responses $IRF^{(s)}$ from
   $A^{(s)} P$ and $B^{(s)} = F^{(s)} A^{(s)-1} P$.
   k. If $IRF^{(s)}$ satisfies the sign restrictions keep it, otherwise discard it.
   l. If the number of accepted IRF is equal to 1000 stop.
5. Collect all the IRF that were not discarded in step 4

In step 2 and 4 of the Algorithm, all simulations are carried out according
Theorem 2 of Waggoner and Zha (2002). The central result of Theorem 2 states that
drawing from the distribution of $b_i$ conditional on $b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n$ is equivalent to
drawing from a multivariate Gaussian distribution and a special univariate distribution.

For a fixed $i_*$, where $1 \leq i_* \leq n$, let $w$ be an non-zero $n \times 1$ vector perpendicular
to each vector in $\{U_i b_i | i \neq i_*\}$. Since the restrictions are assumed to be non-degenerate,
the n-1 vectors $U_i b_i$ for $i \neq i_*$ will almost surely be linearly independent and $U_{i_*} w$ will
be non-zero. Define $w = T_n' U_{i_*} w / \|T_n' U_{i_*} w\|$ where $T_n$ is a $q_h \times q_h$ matrix such that
$T_i T_i' = S_i$, and choose $w_2, \ldots, w_{q_i}$ so that $w_1, w_2, \ldots, w_{q_i}$ form an orthonormal basis for $R^{q_i}$. Then the random vector $b_i$ conditional on $b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n$ can be represented as

$$b_i = \beta_i U_i T_i^{-1} w_i + \sum_{j=2}^{q_i} \beta_j U_j T_j^{-1} w_j. $$

The random variable $\beta_j$, for $2 \leq j \leq q_i$, is normally distributed with mean zero and variance $1/T$ and is straightforward to simulate. The density function for $\beta_i$, the special univariate distribution, is proportional to $|\beta_i|^2 \exp(-T \beta_i^2 / 2)$. Waggoner and Zha (2002) show how to simulate from this latter distribution.

i. **Hybrid Identification**

Suppose we want to keep the identification of the first shock obtained by TETRAD (the monetary policy shock). Then we have to modify matrix $P$ employed in step 4-j of the previous algorithm. It will take the hybrid form:

$$P = \tilde{Q} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \tilde{Q}_{22} & \tilde{Q}_{32} & \cdots & \tilde{Q}_{62} \\ 0 & \tilde{Q}_{23} & \tilde{Q}_{33} & \cdots & \tilde{Q}_{63} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \tilde{Q}_{26} & \tilde{Q}_{36} & \cdots & \tilde{Q}_{66} \end{bmatrix}$$

Where the submatrix $Q_s$ is obtained by a draw of an independent standard normal $(n-1) \times (n-1)$ matrix $\tilde{X}$, and $Q_s$ is obtained by the QR decomposition of $\tilde{X}$ ($\tilde{X} = Q_s \tilde{R}$, with the diagonal $\tilde{R}$ normalized to be positive).

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3 The hybrid methodology adopted here has some similarities with the one used by Dungey and Fry (2008). However, they use “Givens rotation” and we use the QR decomposition. Furthermore, different from them, we use DAGs to impose restrictions on the contemporaneous causal interrelationships among variables.
4. Empirical Model

The model is estimated using monthly data and it is composed of the following variables: the short-term interest rate (SELIC), the nominal exchange rate (e), the price index (IPCA), the medium-term interest rate (SWAP), output (y), and a monetary aggregate (M1), a constant, and seasonal dummies. Based on the exchange rate regime and monetary policy operational procedures we decided to start our sample period on 1999:03, going until 2008:07. This is a period characterized by a free-floating exchange rate and an explicit SELIC targeting. The lag length chosen is six months. The model identifies four independent sources of exogenous disturbances: monetary policy, demand, supply, and exchange rate shocks.

5. Model Identification: A Hybrid Approach

The hybrid identification strategy pursued in this article consisted of two steps. In the first step, we use directed acyclic graphs (DAGs) to select over-identifying restrictions on the contemporaneous coefficients based on the conditional independence relations between the variables. These over-identifying restrictions allow us to identify a monetary policy shock and to restrict the covariance matrix of the reduced-form residuals. In the second step, maintaining restricted the covariance matrix of reduced-form residuals, we keep the identified monetary policy shock and impose sign restrictions on the impulse response functions of other three shocks to identify the demand, supply, and exchange rate shocks.

Step 1: Selection of the Over-Identifying Restrictions to Identify Monetary Policy Shocks

Applying the software TETRAD at the 20% significance level we obtain a graphical representation of the DAG containing the contemporaneous causal ordering of the variables, displayed on figure 1. According to figure 1, the SELIC and the exchange rate affect contemporaneously the SWAP rate, while the price index responds to output shocks within the current period. None of the variables affect contemporaneously the SELIC rate, even the price level and the output. The fact that output and prices have no contemporaneous effect over the SELIC rate may be associated with the difficulty of obtaining information on the current level of output and price level by the time policy makers have to make their decisions.

4 A detailed description of the data and its sources can be found in Appendix I.
5 For an introduction on how to use DAGs to identify VARs, see CLM.
6 TETRAD suggests that the contemporaneous causality can go either from IPCA to output or from output to IPCA. In what follows we chose the causality going from output to IPCA but the adoption of the alternative direction doesn’t change the results.
7 This is an identification assumption made, for example, by Sims and Zha (2006).
It is interesting to compare figure 1 with the patterns obtained by CLM in their alternative model that includes money (see figures 20-22 of their article). The differences between our results and CLM’s can be explained by three factors. First of all, our sample period is longer than CLM (CLM ends in 2004:12, while ours goes until 2008:07). Second, the lag lengths are different (CLM analyses 1-3 lags, while we use six lags). Third, CLM employs a Classical estimation procedure while ours is Bayesian.

The causal ordering between the variables of the VAR can be represented by matrix $A$ that establishes a relationship between reduced form and structural form residuals. The DAG pictured on figure 1 can be represented by the following matrix:

$$
A = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & A_{35} & 0 \\
A_{41} & A_{42} & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
$$

where $A_{ij}$ are parameters to be estimated and the vector of endogenous variables that multiplies $A$ is given by $[\text{SELIC, exchange rate, IPCA, SWAP, output, M1}]$.

The contemporaneous causal ordering resulting from the application of DAGs implies restrictions on the covariance matrix of the reduced-form residuals, meaning that we now have an overidentified model. Structural VAR models that are overidentified can be consistently estimated only by Bayesian estimation methods that introduce these restrictions on the covariance matrix of reduced form residuals. These restrictions are considered when Bayesian estimation methods are applied to the parameters of a structural VAR (and not to the parameters of a reduced form VAR). The method developed by Sims and Zha (1998), and adopted in this article, is one of these methods.

Using the contemporaneous causal ordering of figure 1 to identify the SVAR, we obtained the impulse response functions of economic variables to exogenous and independent shocks, displayed on figure 2. We identify SELIC shocks as monetary
policy shocks, leaving the exchange rate shocks to be identified by sign restrictions in the next step, when we identify also demand and supply shocks in order to better identify exchange rate disturbances.

According to figure 2, after a positive SELIC innovation the stock of M1 falls and output decreases temporarily, taking near 15 months to recover. The direction of the exchange rate response is not clear, but it is more likely that it will appreciate. The price level goes down, but it takes near six months until the price level starts to fall despite the contraction of economic activity. The price level temporarily increases in response to a positive SELIC shock, a result known in the literature as the “price puzzle”, since it is at odds with most theoretical models’ prediction that restrictive monetary policy should reduce the price level\(^8\).\(^9\).

---

\(^8\) Barth and Ramey (2001) argue that a temporary price puzzle may not be a puzzle at all once one takes into account the possibility that the monetary transmission mechanism itself has cost effects. Prices should go up in the short-run following an unanticipated monetary contraction if the cost effects of the monetary transmission mechanism dominate the demand effects.

\(^9\) The temporary price puzzle appears as a common feature in several alternative identification schemes tested. With respect to the Cholesky decomposition of residuals, the following ordering eliminates the price puzzle: SWAP, SELIC, IPCA, output, M1, exchange rate. In section 5, where we identify all shocks by imposing sign restrictions on the impulse response functions, we assume that prices do not increase in response to monetary policy shocks, meaning that the price puzzle is eliminated by construction.
Figure 2: IRFs with 68% probability bands, using the contemporaneous causal ordering of figure 1 to identify the SVAR (24 months ahead).
Step 2: Imposing Sign Restrictions to Identify Demand, Supply and Exchange Rate Shocks

Having identified monetary policy shocks and restricted the covariance matrix of the reduced-form residuals using the contemporaneous causal order suggested by TETRAD, now we impose sign restrictions on the remaining impulse response functions in order to identify the demand, supply, and exchange rate shocks. We impose the sign restrictions for a four months window. The sign restrictions used to identify the SVAR model are similar to those employed by Farrant and Peersman (2006) and can be justified by the short-run dynamics of a stochastic open-economy macroeconomic model, like the one presented in Appendix II. Table 1 summarizes the sign restrictions on the IRFs used to identify the demand, supply, and exchange rate shocks.

According to table 1, positive demand shocks do not decrease the SELIC rate, the price level, output, and the stock of money, and do not imply a depreciation of the real exchange rate. A positive supply shock implies that prices do not increase and output does not go down. An unexpected depreciation of the nominal exchange rate is supposed to imply changes in the same direction of the real exchange rate, and that the short-term interest rate, prices, output, and the stock of money does not go down after the exchange rate shock. The sign restrictions are supposed to hold for four months.

Table 1: Sign Restrictions Used to Identify the SVAR Model

<table>
<thead>
<tr>
<th>Type of Shock</th>
<th>SELIC</th>
<th>IPCA</th>
<th>Output</th>
<th>M1</th>
<th>Real Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Shock</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≤ 0</td>
</tr>
<tr>
<td>Supply Shock</td>
<td></td>
<td>≤ 0</td>
<td>≥ 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Rate Shock</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
</tr>
</tbody>
</table>

The IRFs that result from the imposition of sign restrictions are presented in figures 3-4, showing the median as well as the 68% probability bands for a horizon of 24 and 60 months following the shocks, respectively.

The impact of monetary policy shocks over the exchange rate is not significant, however, there is a strong response of the exchange rate to demand shocks and to shocks originating in the foreign exchange market. The fact that the exchange rate moves in response to demand shocks, suggests that the exchange rate acts as a shock absorber in order to restore the equilibrium of the economy. The fact that the exchange

---

10 When imposing sign restrictions, we assume that $\frac{\partial q_t}{\partial \epsilon_t} \leq 0$ and $\frac{\partial i_t}{\partial \epsilon_t} \geq 0$, which is consistent with the conventional view about the effects of demand shocks.

11 The (log) real exchange rate is defined as $q_t = s_t + p_t^* - p_t$, where $s_t$ is the (log of ) nominal exchange rate, $p_t$ ($p_t^*$) is the (log of) domestic (foreign) price level. We assume that the foreign price level is constant, so that a restriction on the real exchange rate translates into a restriction on $s_t - p_t$. 
rate moves in response to shocks originating in the foreign exchange market, together with the finding that exchange rate shocks have an important role in explaining short-run fluctuations on prices and output, and that the exchange rate is the only disturbance that affects the price level in the long run, allow us to conclude that the exchange rate is an independent source of shocks, in addition of being a shock absorber.

The identification of structural shocks by sign restrictions is not exact, meaning that there are multiple models satisfying the sign restrictions. Therefore, the IRFs to demand, supply, and exchange rate shocks, exhibited in figures 3-4 display not only sampling (parameter) uncertainty, but also model uncertainty. Figures 5-6 disentangle model uncertainty (the bold line) from parameter and model uncertainty (the distance between the dashed and the bold lines). In doing so, when we simulate only models, we consider as “true” the values of parameters at the peak of the posterior density function.

\[12\] In the case of monetary policy shocks, identified by short-run restrictions, there is no model uncertainty, only parameter uncertainty.
Figure 3: IRFs based on the hybrid identification (24 months ahead), with 68% probability bands.
Figure 4: IRFs based on the hybrid identification (60 months ahead), with 68% probability bands.
Figure 5: Disentangling model uncertainty from parameter uncertainty – hybrid identification (24 months ahead), with 68% probability bands.
Figure 6: Disentangling model uncertainty from parameter uncertainty – hybrid identification (60 months ahead), with 68% probability bands.
6. An Alternative Identification Strategy Using only Sign Restrictions to Identify All Shocks

We consider now an alternative identification where we impose sign restrictions on the IRFs to all shocks, including the monetary policy shock. We maintain the previous restrictions summarized in table 1, and in addition we assume that in response to a “contractionary” monetary policy shock, interest rates does not fall, and that output, prices, the stock of money, and the real exchange rate do not increase. These additional restrictions follow from the open-economy macroeconomic model presented in Appendix II. Table 2 shows the sign restrictions on the IRFs used to identify the monetary policy, demand, supply, and exchange rate shocks. We impose the sign restrictions for a four months window.

Table 2: Sign Restrictions Used to Identify the SVAR Model

<table>
<thead>
<tr>
<th>Type of shock</th>
<th>SELIC</th>
<th>IPCA</th>
<th>Output</th>
<th>M1</th>
<th>Real Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary Policy Shock</td>
<td>≥ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
</tr>
<tr>
<td>Demand Shock</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≤ 0</td>
</tr>
<tr>
<td>Supply Shock</td>
<td></td>
<td>≤ 0</td>
<td>≥ 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Rate Shock</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
<td>≥ 0</td>
</tr>
</tbody>
</table>

The IRFs based on the alternative identification that uses only sign restrictions to identify all shocks are presented on figures 7-8, showing the median as well as the 68% probability bands for a horizon of 24 and 60 months following the shocks, respectively. The main difference with respect to the IRFs based on the hybrid identification relies on the responses to monetary policy shocks. Now, monetary policy disturbances have an important role as a source of short-run fluctuations on output, prices, and the exchange rate. Monetary policy shocks are still neutral in the long run, but now it has long run effect over the price level, together with the exchange rate. Figures 9-10 disentangle model uncertainty (the bold line) from model and parameter uncertainty (the distance between the dashed and the bold lines). In doing so, when we simulate only models, we consider as “true” the values of parameters at the peak of the posterior density function.

Here we consider that innovations to the SELIC rate are monetary policy shocks, whereas the model in the Appendix assumes that monetary policy shocks are represented by money supply innovations. Just have in mind that money and interest rates are negatively related.
Figure 7: IRFs based on the sign restrictions identification (24 months ahead), with 68% probability bands.
Figure 8: IRFs based on the sign restrictions identification (60 months ahead), with 68% probability bands.
Figure 9: Disentangling model uncertainty from parameter uncertainty – sign restrictions (24 months ahead), with 68% probability bands.
Figure 10: Disentangling model uncertainty from parameter uncertainty – sign restrictions (60 months ahead), with 68% probability bands.
7. Concluding Remarks

In this article we analyzed the impacts of several (monetary policy, exchange rate, demand, and supply) exogenous disturbances on the Brazilian economy using a structural vector autoregression (SVAR) model identified by two alternative methodologies. The first methodology used sign restrictions on impulse responses of the shocks based on short-run dynamics of a stochastic open-economy macroeconomic model. The second methodology (hybrid) is a new methodology, which we developed, that combines sign restrictions with restrictions on the contemporaneous causal interrelationships among variables, derived by DAGs (Directed Acyclic Graphs). The hybrid identification strategy pursued in this article consists of two steps. In the first step, we used DAGs to select over-identifying restrictions on the contemporaneous coefficients based on the conditional independence relations between the variables. These over-identifying restrictions allow us to identify a monetary policy shock and to restrict the covariance matrix of the reduced-form residuals. In the second step, maintaining restricted the covariance matrix of reduced-form residuals, we kept the identified monetary policy shock and imposed sign restrictions on the impulse response functions of the other three shocks to identify the demand, supply, and exchange rate shocks.

The results of the hybrid identification show a delayed response of the price level to monetary policy shocks, consistent with the presence of some price rigidity. A comparison of the results of the two identification approaches shows that while the effects of exchange rate shocks are nearly the same, the effects of monetary policy shocks depend on the methodology adopted. We find a higher contribution of monetary policy shocks to output, prices, and exchange rate fluctuations when using sign restrictions only. There is a strong response of the exchange rate to demand shocks and to shocks originating in the foreign exchange market. Exchange rate shocks have an important role in explaining short-run fluctuations of prices and output. We conclude that the exchange rate is an independent source of shocks and a shock absorber.
References


Appendix I: Description of the Data

Short-term interest rate: SELIC interest rate – adjusted average rate of daily financing guaranteed by federal government securities, calculated in the Special Settlement and Custody System (SELIC) and published by the Central Bank of Brazil (BCB) – annualized rate.

Nominal exchange rate: R$ / US$ – end of period buying rate – source: BCB.

Price index: IPCA price index – source: IBGE.

Medium-term interest rate: 180 days Swap rate (PRE x CDI) – source: Brazilian Mercantile & Futures Exchange – annualized rate.

Output: the industrial production index – three month moving average – source: IBGE.

Monetary Aggregate: M1 – working days average – source: BCB.
Appendix II: A Stochastic Mundell-Fleming Model

In this appendix we present a stochastic rational expectations open economy model with sticky prices, that can be used to justify the sign restrictions employed to identify the SVAR model. Let $y^d_t$ be aggregate demand, $s_t$ be the nominal exchange rate (the domestic value of foreign currency), $p_t$ be the domestic price level, $i_t$ be the domestic nominal interest rate, $m_t$ be the nominal money stock, and $E_t(X_t)$ be the mathematical expectation of the random variable $X_t$ conditioned on date $t$ information. Foreign variables are taken as given so without loss of generality we set the foreign price level and foreign interest rate equal to zero ($p^*=0$ and $i^* = 0$). The shadow values associated with the flexible-price equilibrium are denoted with a superscript ‘flex’.

$$y^d_t = d_t + \eta q_t - \sigma[i_t - E_t(p_{t+1} - p_t)] \quad (A.1)$$

$$p_t = (1-\theta)E_{t-1}p^{\text{flex}}_t + \theta p^{\text{flex}}_t \quad (A.2)$$

$$m_t' - p_t = y_t - \lambda i_t \quad (A.3)$$

$$i_t = E_t(s_{t+1} - s_t) + c_t \quad (A.4)$$

Equation (A.1) is an open economy IS equation where demand for output ($y^d_t$) depends on a demand shock ($d_t$), is increasing in the real exchange rate ($q_t = s_t - p_t$), and decreasing in the ex ante real interest rate. The sticky-price adjustment rule (A.2) says that the price level in period $t$ is an average of the market-clearing price expected in $t-1$ to prevail in $t$ ($E_{t-1}p^{\text{flex}}_t$), and the price that would clear the market in period $t$ ($p^{\text{flex}}_t$). Prices are instantaneously perfectly flexible if $\theta = 1$ and they are completely fixed one-period in advance if $\theta = 0$. Intermediate degrees of price flexibility are characterized by $0 < \theta < 1$. Equation (A.3) is a standard LM equation where the income elasticity of money demand is assumed to be 1. Capital market equilibrium is given by equation (A.4), where $c_t$ is a term that reflects the exchange rate risk of the domestic currency.

The stochastic processes that drive the dynamics in this model – supply, demand, monetary policy, and exchange rate shocks – are given by\(^1\):

$$y^s_t = y^s_{t-1} + \epsilon^s_t \quad (A.5)$$

$$d_t = d_{t-1} + \epsilon^d_t - \gamma \epsilon^d_{t-1} \quad (A.6)$$

$$m_t = m_{t-1} + \epsilon^m_t \quad (A.7)$$

$$c_t = c_{t-1} + \epsilon^c_t \quad (A.8)$$

\(^1\) Here we consider money supply innovations as monetary policy shocks, whereas in the text we assume that innovations to the SELIC rate are monetary policy shocks. Just have in mind that money and interest rates are negatively related.
where \( \varepsilon_i^s \sim N(0, \sigma_s^2) \), \( \varepsilon_i^d \sim N(0, \sigma_d^2) \), \( \varepsilon_i^m \sim N(0, \sigma_m^2) \), \( \varepsilon_i^c \sim N(0, \sigma_c^2) \), \( 0 < \gamma < 1 \)

The long-run or the steady-state is not conveniently characterized in a stochastic environment because the economy is constantly being hit by shocks to the non-stationary exogenous state variables. Instead of a long-run equilibrium, we work with an equilibrium concept given by the solution formed under hypothetically fully flexible prices. Then as long as there is some degree of price-level stickiness that prevents complete instantaneous adjustment, the disequilibrium can be characterized by the gap between sticky-price solution and the shadow flexible-price equilibrium. We apply a two-stage procedure for solving the equilibrium system (A.1) – (A.8). In the first stage, we solve for a flexible-price equilibrium that corresponds to this system. In the second stage, we use the flex-price equilibrium to arrive at a full-fledged solution for the mixed fix-flex-price system.

It is possible to show that the flexible-price equilibrium values of the model are given by:

\[
y_t = y_{t-1} + \varepsilon_i^s
\]

(A.9)

\[
q_t^{\text{flex}} = \frac{y_t - d_t}{\eta} + \frac{\gamma \sigma}{\eta(\eta + \sigma)} \varepsilon_i^d + \frac{\sigma}{\eta} c_t
\]

(A.10)

\[
p_t^{\text{flex}} = m_t - y_t + \lambda c_t + \frac{\lambda \gamma}{(\eta + \sigma)(1 + \lambda)} \varepsilon_i^d
\]

(A.11)

\[
s_t^{\text{flex}} = m_t + \frac{(1 - \eta)}{\eta} y_t - \frac{1}{\eta} d_t + \left( \frac{\sigma}{\eta} + \frac{\lambda \gamma}{(\eta + \sigma)(1 + \lambda)} \right) \varepsilon_i^d
\]

(A.12)

\[
i_t^{\text{flex}} = \left( \frac{\gamma}{\eta + \sigma} - \frac{\lambda \gamma}{(\eta + \sigma)(1 + \lambda)} \right) \varepsilon_i^d + c_t
\]

(A.13)

We now use the flex-price equilibrium values obtained in the first stage to solve for the full-fledged equilibrium in this second stage:

\[
p_t = p_t^{\text{flex}} - (1 - \theta)(\varepsilon_i^m - \varepsilon_i^s + \alpha \varepsilon_i^d + \lambda \varepsilon_i^c)
\]

(A.14)

\[
q_t = q_t^{\text{flex}} + \frac{(1 + \lambda)(1 - \theta)}{\eta + \sigma + \lambda} (\varepsilon_i^m - \varepsilon_i^s + \alpha \varepsilon_i^d + \lambda \varepsilon_i^c)
\]

(A.15)

\[
s_t = s_t^{\text{flex}} + \frac{(1 - \eta - \sigma)}{(\eta + \sigma + \lambda)} (1 - \theta)(\varepsilon_i^m - \varepsilon_i^s + \alpha \varepsilon_i^d + \lambda \varepsilon_i^c)
\]

(A.16)

\[
y_t^d = y_t + \frac{(1 + \lambda)(1 - \theta)(\eta + \sigma)}{\eta + \sigma + \lambda} (\varepsilon_i^m - \varepsilon_i^s + \alpha \varepsilon_i^d + \lambda \varepsilon_i^c)
\]

(A.17)
\[ i_t = i_t^{\text{flex}} - \frac{(1 - \eta - \sigma)}{(\eta + \sigma + \lambda)} (1 - \theta)(\epsilon_t^m - \epsilon_t^t + \alpha \epsilon_t^d + \lambda \epsilon_t^e) \] (A.18)

where \( \alpha = \frac{\lambda \gamma}{(\eta + \sigma)(1 + \lambda)} \)

The effect of a one-unit increase in the variable \( j \) innovation at date \( t \) (\( \epsilon_t^j \)) on the value of variable \( l \) at time \( t \), holding all other innovations constant, is given by \( \frac{\partial l}{\partial \epsilon_t^j} \):

\[ \frac{\partial y_t^d}{\partial \epsilon_t^j} = 1 - \frac{(1 + \lambda)(1 - \theta)(\eta + \sigma)}{\eta + \sigma + \lambda} \geq 0 \]

\[ \frac{\partial y_t^d}{\partial \epsilon_t^d} = \frac{(1 + \lambda)(1 - \theta)(\eta + \sigma)}{\eta + \sigma + \lambda} \alpha \geq 0 \]

\[ \frac{\partial y_t^d}{\partial \epsilon_t^m} = \frac{(1 + \lambda)(1 - \theta)(\eta + \sigma)}{\eta + \sigma + \lambda} \lambda \geq 0 \]

\[ \frac{\partial y_t^d}{\partial \epsilon_t^c} = \frac{(1 + \lambda)(1 - \theta)(\eta + \sigma)}{\eta + \sigma + \lambda} \geq 0 \]

\[ \frac{\partial p_t^s}{\partial \epsilon_t^s} = -\theta \leq 0 \]

\[ \frac{\partial p_t^m}{\partial \epsilon_t^m} = \alpha \theta \geq 0 \]

\[ \frac{\partial p_t^c}{\partial \epsilon_t^c} = \lambda \theta \geq 0 \]

\[ \frac{\partial q_t^s}{\partial \epsilon_t^s} = \frac{1}{\eta} - \frac{(1 + \lambda)(1 - \theta)}{\eta + \sigma + \lambda} \]

\[ \frac{\partial q_t^d}{\partial \epsilon_t^d} = -\frac{1}{\eta} + \frac{\gamma \sigma}{\eta(\eta + \sigma)} + \frac{(1 + \lambda)(1 - \theta)}{\eta + \sigma + \lambda} \alpha \]

[The conventional view is that \( \frac{\partial q_t^d}{\partial \epsilon_t^d} \leq 0 \) ]

\[ \frac{\partial q_t^m}{\partial \epsilon_t^m} = \frac{(1 + \lambda)(1 - \theta)}{\eta + \sigma + \lambda} \geq 0 \]
\[
\frac{\partial q_{i}}{\partial \epsilon_{i}^{c}} = \frac{\lambda + (1 + \lambda)(1 - \theta)}{\eta + \sigma + \lambda} \quad \lambda \geq 0
\]

\[
\frac{\partial i_{i}}{\partial \epsilon_{i}^{c}} = \frac{(1 - \eta - \sigma)}{(\eta + \sigma + \lambda)(1 - \theta)} \quad \text{if } \eta + \sigma \leq 1
\]

\[
\frac{\partial i_{i}}{\partial \epsilon_{i}^{m}} = \left(\frac{\gamma}{\eta + \sigma} - \frac{\alpha(1 - \eta - \sigma)}{(\eta + \sigma + \lambda)}(1 - \theta)\right) \quad \text{[The conventional view is that } \frac{\partial i_{i}}{\partial \epsilon_{i}^{d}} \geq 0 \text{]}
\]

\[
\frac{\partial i_{i}}{\partial \epsilon_{i}^{m}} = -\frac{(1 - \eta - \sigma)}{(\eta + \sigma + \lambda)(1 - \theta)} \quad \text{if } \eta + \sigma \leq 1
\]

\[
\frac{\partial i_{i}}{\partial \epsilon_{i}^{e}} = 1 - \frac{(1 - \eta - \sigma)}{(\eta + \sigma + \lambda)(1 - \theta)} \quad \lambda \geq 0
\]