Growth, Distribution and Effective Demand: the supermultiplier growth model alternative

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Abstract

The paper presents an explicitly dynamic version of the Sraffian supermultiplier growth model in order to analyze its equilibrium path, local stability conditions and dynamic behavior in the neighborhood of equilibrium. This analysis is used to address the criticisms to model found in the literature and also to compare the model with the Cambridge and neo-Kaleckian growth models. The comparative inquiry confirms that the model can be considered a theoretical alternative to the Cambridge and neo-Kaleckian growth models in the analysis of the relationship between economic growth, income distribution and effective demand. The specific closure provided by the supermultiplier growth model allows it to generate a demand-led pattern of economic growth characterized by a tendency towards the normal utilization of productive capacity, while considering income distribution exogenously determined by political, historical and economic forces.

Key words: Effective Demand; Growth; Income Distribution.

JELCode: E11; E12; O41

Introduction

According to the theory of economic growth the existence of an equilibrium path requires the rate of output growth and the rate of capital accumulation (i.e. the rate of growth of the capital stock) to be equal to each other. The heterodox literature that deals with the relationship between economic growth, income distribution and effective demand, proposes two types of adjustment mechanisms to achieve such equality. The first type consists of appropriate and concomitant movements of the investment share of output and of the saving ratio, while the second one involves required changes in the actual rate of capacity utilization. Cambridge growth models generally make use of the first type of adjustment mechanism, whereas neo-Kaleckian growth models mainly adopt the second type. In Cambridge growth models the investment share of output needed to sustain the equilibrium rates of output growth and of capital accumulation determines the saving ratio through the endogenous determination of income distribution. Thus the endogeneity of income distribution allows the existence of the equilibrium path and, therefore, such endogeneity provides the theoretical closure to bring about the adjustment between the rate of output growth and the rate of capital accumulation in these models. On the other hand, in neo-Kaleckian growth models income distribution and the saving ratio are exogenous variables. The saving ratio determines the investment share of output and, hence, the latter variable cannot be changed according to the requirements of the pace of capital accumulation. It follows that, in the latter models, the equalization of rates of output growth and capital accumulation depends upon the endogenous determination of a required (long run) rate of capacity utilization, the latter being, therefore, the theoretical closure furnished by the neo-Kaleckian growth models.

In this paper we explore an alternative closure for the problem of the reconciliation of rates of output growth and capital accumulation. Such alternative closure is found in the Sraffian supermultiplier growth model developed by Serrano (1995 and 1996).[1] Like in Cambridge growth models, in the supermultiplier model the investment share of output and the saving ratio are endogenous variables and the investment share determines the saving ratio. But the adjustment mechanism of the saving ratio to the investment share of output is different from the one present in the Cambridge models. In the supermultiplier model income distribution is determined from outside the model and, therefore, it cannot be part of the adjustment mechanism in the model. However, in its simplified version for a closed economy without government, the model assumes the existence of an autonomous component in the consumption function, which implies that the marginal propensity to save is greater than the average propensity to save (i.e. the saving ratio). So although the marginal propensity to save is exogenously fixed by the level of income distribution and by consumption habits, the saving ratio can change endogenously through a change in the ratio between autonomous consumption and aggregate investment which, for its turn, leads to a change in the ratio of the average to the marginal propensities to save. In the supermultiplier growth model, the variability of the investment share of output and of the saving ratio allows the existence of a demand(consumption)-led growth path characterized by a fully adjusted equilibrium; that is, an equilibrium in which we have, at the same time, the equality between aggregate output and demand and the equality between the actual and normal rates of capacity utilization. Since the normal rate of capacity utilization is an exogenous variable in the supermultiplier growth model, then the tendency for the adjustment of the actual rate of capacity utilization to its normal level implies that the equilibrium rate of capacity utilization is exogenously determined. Therefore, the closure offered by the supermultiplier growth model involves neither an endogenous determination of a required level of income distribution nor the endogenous determination of a required equilibrium rate of capacity utilization.

However, the original contributions by Serrano presented the supermultiplier growth model without a discussion of the dynamic stability of its equilibrium path and, therefore, without a detailed account of the dynamic adjustment process of productive capacity to aggregate demand. The lack of such dynamic analysis opened space for criticisms related to alleged inability of the model in providing plausible explanations for economic growth as demand-led process and for the tendency towards normal utilization of productive capacity. The purpose of this paper is to provide an explicit dynamic version of the Sraffian supermultiplier growth model to analyze the process of adjustment of productive capacity to aggregate demand and to use this characterization for a discussion of the formal dynamic stability properties of the model. In doing so, we

[1] The same type of closure, with minor differences, was proposed independently by Bortis (1979 and 1997).

aim to address the main criticisms of the supermultiplier growth model found so far in the literature. We also expect that the more formal characterization of the model here presented will facilitate the comparability of the supermultiplier growth model with the Cambridge and neo-Kaleckian growth models. Such comparative analysis will enable us to confirm that the supermultiplier model is a true theoretical alternative to the Cambridge and neo-Kaleckian growth models in the investigation of the relationship between economic growth, income distribution and effective demand. With this last objective in mind, besides the formal analysis of the supermultiplier model, the paper undertakes a brief comparison of the supermultiplier, Cambridge and neo-Kaleckian growth models in order to establish their similarities and, more importantly, their main differences.

To accomplish these objectives the rest of the paper is organized as follows. Section 1 presents the supermultiplier growth model with a discussion of its assumptions and their roles in the operation of the model. Section 2 proceeds to the formal analysis of the equilibrium path of the model. Section 3 discusses the local stability conditions and the dynamic behavior of the model in the neighborhood of the equilibrium. Section 4 is dedicated to the discussion of the criticisms of the supermultiplier model. Section 5 compares and contrasts the supermultiplier model with the Cambridge and neo-Kaleckian growth models. Finally, we conclude the paper with some brief final remarks.

1. The Supermultiplier Growth Model

We will present the supermultiplier growth model in its simplest form in order to facilitate the formal analysis of the dynamic behavior of the model and the comparison with alternative heterodox growth models. Hence let us assume a closed capitalist economy without government. Aggregate income is distributed in the form of wages and profits, and we suppose that income distribution is exogenously determined.[2] The single method of production in use requires a fixed combination of homogeneous labor input with homogeneous fixed capital to produce a single output. Natural resources are supposed to be abundant and constant returns to scale and no technological progress are also assumed, implying that the method of production in use does not change.[3] Finally, it is also assumed that growth is not constrained by labor scarcity.

In this very simple analytical context, the level of capacity output[4] of the economy depends on the existing level of capital stock available and on the technical capital-output ratio according to the following expression:

$$Y_{Kt} = \left(\frac{1}{v}\right) K_t \tag{1}$$

where Y_{Kt} is the level of the capacity output of the economy, K_t is the level of capital stock installed in the economy and $v > 0$ is the technical capital-output ratio. Since v is given, then the growth of capacity output is equal to the rate of capital accumulation

$$g_{Kt} = \left(\frac{I_t/Y_t}{v}\right) u_t - \delta \tag{2}$$

where g_{Kt} is rate of capital accumulation, $u_t = Y_t/Y_{Kt}$ is the actual rate of capacity utilization defined as the ratio of the current level of aggregate output (Y_t) to the current level of capacity output, I_t/Y_t is the investment share of aggregate output defined as the ratio of gross aggregate investment (I_t) to the level of

[2] In this respect, the supermultiplier growth model is compatible with various theories of income distribution available within the surplus approach to political economy, including the Kaleckian income distribution theory. In these theories, income distribution is determined by political, historical and economic forces not directly and necessarily *a priori* related to the process of economic growth. This latter feature makes them suitable for a model of economic growth that treats income distribution as an exogenous variable. On the other hand, the lack of such feature implies that Cambridge distribution theory is not compatible with the supermultiplier growth model.

[3] For an analysis of technological change based on the supermultiplier growth model see Cesaratto, Serrano & Stirati, (2003)

[4] All variables are measured in real terms. Moreover, output, income, profits, investment and savings are presented in gross terms.

aggregate gross output, and $\delta > 0$ is the capital depreciation ratio which is exogenously determined in the present model.[5]

According to equation (2) the value of the rate of capital accumulation depends on the behavior of the rate of capacity utilization and of the investment share of output. The dynamic behavior of the investment share of output will be specified later on. For its turn, given the technical capital-output ratio, the evolution of the rate of capacity utilization through time is specified by the following differential equation

$$\dot{u} = u_t(g_t - g_{Kt}) \quad (3)$$

where g_t is the rate of growth of aggregate output.

On the demand side of the model, real aggregate demand is composed of real aggregate consumption and gross aggregate investment. We assume that capitalist firms realize all investment expenditures in the economy.[6] Aggregate consumption is composed of an autonomous component and an induced component. So we have

$$D_t = C_{wt} + Z_t + I_t \quad (4)$$

Where $D_t$is aggregate demand, C_{wt} is aggregate induced consumption, and Z_t is aggregate autonomous consumption.

We suppose that induced consumption is related to the purchasing power introduced in the economy by the production decisions of capitalist firms. More specifically, to simplify the model, we assume that induced consumption is equal to the economy's wage bill and, therefore, that the marginal propensity to consume out of wages is equal to one. Given the method of production in use and the real wage, the wage share is also given. Thus induced consumption is positively related to the level of output resulting from the production decisions of capitalist firms. These assumptions are represented by the following equation

$$C_{wt} = \omega Y_t \quad (5)$$

where ω is the wage share of output. Note that, from equation (5), the wage share can be interpreted as the marginal propensity to consume out of aggregate income. For a capitalist economy the wage share is lower than one (in fact we suppose that $0< \omega<1$). So the marginal propensity to consume out of aggregate income has a value lower than one. In this case, an increase in aggregate output causes a less than proportional expansion of induced consumption. Consequently, the expansion of aggregate demand that results from the increase in induced consumption caused by the increase in the level of aggregate output cannot absorb the whole expansion of aggregate output. On the other hand, we assume that autonomous consumption Z_t is the component of aggregate consumption that is not financed by the purchasing power introduced in the economy by capitalists' production decisions. The purchasing power used to finance autonomous consumption comes from the monetization of accumulated wealth and/or the access to new credit finance.[7]

From the hypotheses adopted to explain the behavior of induced and autonomous consumption components, it follows that total aggregate consumption is represented by the equation bellow

$$C_t = Z_t + \omega Y_t \quad (6)$$

where C_t is the level of aggregate consumption. With this specification for the consumption function and for any level of aggregate investment, the level of aggregate demand is given by

$$D_t = Z_t + \omega Y_t + I_t$$

In equilibrium between aggregate output and aggregate demand it follows that the equilibrium level of output is determined as follows:

$$Y_t = \left(\frac{1}{1-\omega}\right)(Z_t + I_t) = \frac{Z_t + I_t}{s}$$

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[5] The equation of capital accumulation is derived from the equation $I_t = \dot{K} + \delta K_t$ that defines the level of aggregate gross fixed investment. Indeed, dividing both sides of the equation by K_t we obtain $I_t/K_t = g_{Kt} + \delta$. Solving this last equation for the rate of capital accumulation gives us $g_{Kt} = (I_t/K_t) - \delta = (I_t/Y_t)(Y_t/Y_{Kt})(Y_{Kt}/K_t) - \delta = ((I_t/Y_t)/v)u_t - \delta$.

[6] Thus, there is no residential investment in our simplified model.

[7] In this respect see Steindl (1982).

where $s = 1 - \omega$ is the marginal propensity to save, equal to the profit share of output.[8] The term $1/(1-\omega) = 1/s$ is the usual Kaleckian multiplier. Since we have 0< ω<1, then the multiplier has a positive and higher than one value, which allows the existence of a positive level of equilibrium output. Note that from the last equation it follows that

$$S_t = sY_t - Z_t = I_t$$

That is, in equilibrium the level of aggregate investment determines the level of aggregate savings since the level of output adjusts to the level of aggregate demand. Moreover, dividing both sides of the last equation by the level of output, we obtain the economy's saving ratio (or average propensity to save)

$$\frac{S_t}{Y_t} = s - z_t = sf_t = \frac{I_t}{Y_t} \qquad (7)$$

where $z_t = Z_t/Y_t$ is the ratio of autonomous consumption to aggregate output and $f_t = I_t/(I_t + Z_t) = 1/\big(1 + (Z_t/I_t)\big)$ is what Serrano (1996) called "fraction", the ratio between the average and the marginal propensities to save $f_t = (S_t/Y_t)/s$. Observe that according to (7) if there were no autonomous consumption (i.e., $Z_t = 0$) then we would have $z_t = 0$, $f_t = 1$ and $S_t/Y_t = s$. So the marginal propensity to save is the maximum saving ratio of the economy corresponding to a given wage share. Note also that, in this case, if income distribution is exogenously given, then the marginal propensity to save determines the saving ratio and the investment share of output. However, with a positive level of autonomous consumption (i.e., $Z_t > 0$), it follows that $z_t > 0$, $f_t < 1$ and $S_t/Y_t < s$. In this case, the saving ratio depends not only on the marginal propensity to save but also on the proportion between autonomous consumption and investment. Thus, an increase (decrease) in aggregate investment in relation to autonomous consumption leads to an increase (decrease) the saving ratio.[9] As a result, the existence of a positive level of autonomous consumption is sufficient to make the saving ratio an endogenous variable even when income distribution is exogenously determined.

Let us now suppose in addition to the existence of a positive level of autonomous consumption that the level of aggregate investment is an induced expenditure according to the following expression

$$I_t = hY_t$$

where h (with $0 \leq h < 1$) is the marginal propensity to invest of capitalist firms, which we assume, *provisionally*, to be determined exogenously. With these assumptions the level of aggregate demand is now given by

$$D_t = Z_t + \omega Y_t + hY_t = (\omega + h)Y_t + Z_t$$

In the second equation above the term within the parentheses (i.e, $\omega + h$) can be interpreted as the economy's marginal propensity to spend. If the marginal propensity to spend has a positive value lower than one, then an expansion (a reduction) in aggregate output resulting from the production decisions of capitalist firms induces a less than proportional increase (decrease) in aggregate demand.

In equilibrium between aggregate demand and output, the level of output is given by the following equation

$$Y_t = \left(\frac{1}{s-h}\right) Z_t$$

where the term within the parenthesis is the supermultiplier, so named because it captures the inducement effects associated with both, consumption and investment.[10] We can verify that if the marginal propensity to

[8] This simplifying assumption implies that the marginal propensity to consume out of current profits is equal to zero and, therefore, that the marginal propensity to save out of current profits is equal to one.

[9] From equation (7) we can verify that $z_t = s(1 - f_t)$. Hence, the increase (decrease) in aggregate investment in relation to autonomous consumption leads to an increase (decrease) in the fraction f_t and, accordingly, to a decrease (increase) in z_t. Since $S_t/Y_t = s - z_t$, the decrease (increase) in z_t, given s (i.e., given income distribution), implies an increase (decrease) in the saving ratio.

[10] The term supermultiplier was first used by Hicks (1950) to designate the combination of the multiplier and accelerator effects. Hicks (1950) utilized the supermultiplier as a central element in his model of economic fluctuations. He used an explosive specification of the accelerator investment function combined with the constraints associated with the full employment ceiling and with the zero level of gross fixed investment floor in order to able to generate persistent and regular fluctuations in the level of output. Besides this, Hicks supposes that the growth rate of autonomous investment is the main determinant of the trend rate of output growth, while, at the same time, he arbitrarily assumes that autonomous investment grows at the same rate as the full

spend is positive and lower than one, then we have that $0 < s - h = 1 - \omega - h < 1$, which implies that the supermultiplier is positive and has a value greater than one.[11] Further, considering that the marginal propensity to spend has a positive and lower than one value, we can see that the existence of a positive equilibrium level of aggregate output requires that autonomous consumption has a positive value. Indeed, if $Z_t = 0$ then according to the equation above the equilibrium level of output would be equal to zero. This is so because, if $\omega + h < 1$ and $Z_t = 0$ then any positive level of output would be associated with a situation of excess aggregate supply and could not be sustained. The adjustment process induced by capitalist competition would lead the economy towards an equilibrium in which aggregate output and demand would be equal to zero. This result shows that if the marginal propensity to spend has a positive and lower than one value, then the existence of a positive level of autonomous expenditure (in our case here an autonomous consumption) is required for the economy to be able to sustain a positive level of output.

Next, observe that if the marginal propensity to spend has a value lower than one, then, starting from the equilibrium position, an expansion (a reduction) in aggregate output would lead the economy to a situation of excess aggregate supply (demand). However, the situation of excess supply (demand) would not be sustainable, because the competitive process would induce the revision of the production decisions of capitalist firms to promote the reduction (expansion) in the level of aggregate output and produce a tendency for the adjustment of aggregate output to aggregate demand. For this reason the condition that the marginal propensity to spend has a positive value lower than one has been considered a stability condition for the equilibrium between aggregate output and demand in models based on the principle of effective demand.

Finally, note that $z = s - h = s\big(1 - (h/s)\big) = s(1 - f)$ and, therefore, $S_t/Y_t = s - z = sf = h$. So, for a given level of income distribution and, hence, a given marginal propensity to save, the marginal propensity to invest (equal to the investment share of output) determines the ratio of autonomous consumption to aggregate output (z), and, accordingly, it also determines the saving ratio of the economy. Thus, an exogenous increase (decrease) in the investment share of output in relation to the marginal propensity to save would raise (reduce) the fraction f, and, therefore, would cause a decrease (an increase) in the ratio of autonomous consumption to output z and an increase (a decrease) in the saving ratio.

Now let us suppose that autonomous consumption grows at an exogenously determined rate $g_Z > 0$. Since the marginal propensities to consume and to invest are given exogenously, the supermultiplier is also exogenous and constant. Therefore, *ceteris paribus*, aggregate output, induced consumption and investment grow at the same rate as autonomous consumption. The capital stock also tends to grow at this same rate since its pace of expansion is governed by the growth rate of investment.[12] It follows that capacity output tends to grow at the same rate as autonomous consumption. Moreover, the rate of capacity utilization tends to a constant value as can be verified from equation (3) and its level is determined using equation (2) as follows

$$u^* = \frac{v(g_Z + \delta)}{h} \tag{8}$$

The last equation reveals that given the propensity to invest and, accordingly, the investment share of output, a higher (lower) autonomous consumption growth rate implies a higher (lower) rate of capacity utilization. Therefore, under the hypotheses adopted so far, the tendency for a normal utilization of the productive capacity available in the economy requires the propensity to invest to be an endogenous variable capable of being properly changed according to the pace of capital accumulation. In fact, it can be verified that

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employment ceiling (i.e. the natural rate of growth). In contrast, our supermultiplier growth model aims to analyze economic growth as a stable demand-led process where the main determinant of the trend rate of output growth is the pace of expansion of the non-capacity generating autonomous demand component (in this paper the rate of growth of autonomous consumption). In order to accomplish the latter objective, the model makes use of a non-explosive specification of the accelerator investment function as we will see below. We also believe that the notion of autonomous capitalist investment expenditure is problematic when it is utilized in the context of long run economic growth. In this connection, we refer the reader to the critical assessment of this concept found in Kaldor (1951), Duesenberry (1956) and Cesaratto, Serrano and Stirati (2003).

[11] Observe that since $Z_t > 0$, if $\omega + h \geq 1$ then we would have a situation of permanent excess aggregate demand that would necessarily lead the economy to operate in a situation of full capacity utilization.

[12] This last result can be explained as follows. From the definitional equation $I_t = \dot{K} + \delta K_t$ we can obtain the differential equation $\dot{g}_{Kt} = (g_{Kt} + \delta)(g_{It} - g_{Kt})$ relating the investment rate of growth to the rate capital accumulation. From this differential equation it can be verified that, if initially the rate of capital accumulation is different from the rate of investment growth, the rate of capital accumulation would change towards the investment growth rate and the rate of capital accumulation would only be constant when it is equal to the investment growth rate.

according to the equation above, given the growth rate of autonomous consumption, a higher (lower) propensity to invest (i.e., investment share of output) implies a lower (higher) capacity utilization rate. So the endogenous determination of the investment share of output would in principle allow the adjustment of actual capacity utilization ratio to its normal level.

Thus, we shall now present our hypotheses concerning the behavior of aggregate investment[13] according to the supermultiplier growth model, in which, as we will verify, the endogeneity of the marginal propensity to invest (i.e. the investment share of output) plays an important role. The distinctive characteristic of investment expenditure is its dual nature. On the one side, investment is an aggregate demand component and, on the other, it is responsible for the creation of productive or supply capacity in the economy. Capitalist production is directed to the market with the objective of obtaining profits. Thus the primary function of the capitalist investment process is the construction of the productive capacity required to meet market demand at a price that cover the production expenses and allows, at least, the obtainment of a minimum required profitability rate. As occurs with the demand for other inputs, the demand for capital goods is fundamentally of the nature of a derived demand with the objective of profitably meeting the requirements of the production process. But the market demand for capitalist production does not expand steadily. The fluctuation of market demand is another important fact that characterizes the process of economic growth in capitalist economies. These fluctuations exert an important influence on the behavior of the fixed capital formation. Indeed, for various reasons, capacity output cannot be immediately adjusted to the requirements of production and demand, and sometimes such an adjustment involves relatively long periods of time.[14] Now, since capitalist firms don't want to lose their market shares for not being able to supply peak levels of demand, they usually operate with margins of planned spare productive capacity.[15] So market demand fluctuations imply the occurrence of corresponding fluctuations, in the same direction, of the actual rate of capacity utilization. Further, we assume the existence of a normal rate of capacity utilization, which is compatible with the corresponding margins of spare capacity desired by capitalist firms and, for the purpose of the present analysis, is supposed to be strictly positive and determined exogenously to the model. Therefore, our main hypothesis concerning the behavior of aggregate investment is that the inter-capitalist competition process would imply a tendency for the growth rate of aggregate investment to be higher than the rate of growth of output/demand whenever the actual rate of capacity utilization is above its normal level and *vice-versa*. This behavior of aggregate investment, for its turn, would lead to changes in the investment share of output (i.e. in the marginal propensity to invest) that would eventually produce, under conditions to be discussed below, a tendency for the adjustment of productive capacity to aggregate demand and, accordingly, a tendency towards the normal utilization of productive capacity.

The above discussion of the capitalist investment process is compatible with the capital stock adjustment principle associated with the *flexible* accelerator investment model. Thus, in this paper we use a version of the flexible accelerator investment function for modeling the aggregate investment process.[16] Our investment function is specified as follows[17]

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[13] Note that in our discussion of the investment function we do not claim to provide an explanation for individual capitalist investment decisions. Our investment function only tries to explain the behavior of aggregate (at the level of the economy as whole or at the industry level) investment. With this procedure we want to avoid the problems associated with representative agent type of argument in the discussion of aggregate investment. These problems are related to the fact that the latter type of argument tends to ignore the interdependence between individual capitalist firms decisions associated with the influence of capitalist competition process. In this sense, we think that individual capitalist investment decisions cannot be thought to be indefinitely insensible to changes in the market shares of capitalist firms implied by the competition process.

[14]Among the various reasons we can draw attention to: the relatively large lag between the investment decisions and the entrance of operation of the corresponding productive capacity due to technical and economic factors; the durability of fixed capital assets combined with the lack of organized secondary markets for used capital goods; and the fact that the expansion of productive capacity normally requires the acquisition of bundles of capital goods which involves the mobilization of relatively large amounts of financial resources and the existence of important technical indivisibilities.

[15] Besides the ratio of peek to average level of demand, the existence of margins of planned excess capacity are also explained: by the presence of important indivisibilities in the investment process in the context of a growing economy and/or industry; by precautionary reasons; and by the high operation costs associated with production activities at high rates of capacity utilization. Concerning these issues, we refer the reader to Steindl (1952) for a seminal discussion of the existence and role of planned spare capacity in the operation of capitalist firms. See also Ciccone (1990), Kurz (1990) and Garegnani (1992) for a discussion of the issue from a sraffian point of view.

[16] On the flexible accelerator investment function see Goodwin (1951), Chenery (1952), Koyck (1954), and Mattews (1959).

[17] Other specifications of a flexible accelerator induced investment function have been explored by Cesaratto, Serrano &Stirati (2003), Serrano & Wilcox (2000) and Freitas & Dweck (2010) in a multi-sectoral context. The particular function used here was

$$I_t = h_t Y_t \tag{9}$$

and

$$\dot{h} = h_t \gamma(u_t - \mu) \tag{10}$$

where $0 \leq h_t < 1$ is the marginal propensity to invest of capitalist firms,[18] $\mu > 0$ is the normal rate of capacity utilization discussed above, and $\gamma > 0$ is a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of u_t from μ.

From equations (9) and (10) we can see that investment growth is given by the following expression:

$$g_{It} = g_t + \gamma(u_t - \mu) \tag{11}$$

where g_{It} is the rate of growth of aggregate gross investment. According to the last equation we have the relations bellow:

$$g_{It} \gtreqless g_t \text{ as } u_t \gtreqless \mu$$

Thus for $u_t > \mu$ the margin of spare capacity is below its desired level, putting in risk the capacity of some firms to meet peak levels of demand and eventually compromising the maintenance of their market shares. So the capitalist competition process would exert a pressure for the increase in the margins of spare capacity, which is equivalent to a decrease of the divergence of the actual rate of capacity utilization from its normal level. Since investment creates productive capacity and, consequently, the pace of investment growth drives the growth rates of capital stock and of capacity output, the increase in the margin of spare capacity requires that investment grows at a higher rate than output (i.e, $g_{It} > g_t$). In the opposite situation ($u_t < \mu$) the existence of unplanned spare capacity indicates that capitalist firms can normally meet the peak levels of demand. However the low rate of capacity utilization in relation to its normal level implies that realized rate of profit over installed capacity is lower than it could be with a normal utilization of capacity.[19] Then the competitive process would exert a pressure for the increase in the capacity utilization since that would raise the realized rate of profit without compromising the objective of meeting the peak levels of demand, at least until the actual rate of capacity utilization reaches its normal level (i.e, until $u_t = \mu$). As a consequence, in this case, the competitive process would induce a tendency for investment to grow at a lower rate than aggregate output (i.e, $g_{It} < g_t$).

Nevertheless, we must not forget that investment is also an aggregate demand component. Thus, although the adjustment process triggered by capitalist competition leads the economy in the right direction, the intensity of the process may prevent the actual rate of capacity utilization to converge to its normal level. The stability analysis of the model will reveal that such a convergence of the rate of capacity utilization depends fundamentally on the value of the reaction parameter γ. For relatively high values of γ, changes in the actual rate of capacity utilization have a relatively high impact on investment growth and, therefore, on aggregate demand growth, which causes instability. Conversely, for relatively low values of γ, changes in the actual rate of capacity utilization have a relatively small impact on investment growth and, therefore, on aggregate demand growth, which is conducive to stability.

Let us now use the specified consumption and investment functions (equations (6) e (9) respectively) to obtain an equation for the level of aggregate demand. Hence, we have the following expressions

$$D_t = \omega Y_t + Z_t + h_t Y_t = (\omega + h_t) Y_t + Z_t \tag{12}.$$

In the foregoing analysis of the supermultiplier growth model we are not interested in pursuing an analysis of the short term adjustment process between aggregate demand and aggregate output. Thus we assume way the

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chosen as the simplest for the purpose of the presentation of the discussion of adjustment of capacity to demand. Note however that the nature of the stability conditions is basically the same for these variants of the investment function.

[18] We have $h_t \geq 0$ because, for the economy as whole, gross investment cannot have a negative value, so that the rate of capital accumulation has a floor value equal to minus the rate of capital depreciation (i.e. $g_{Kt} \geq -\delta$).

[19] To understand this result better, note that the realized rate of profit r_t is equal to the ratio of total profits (Π_t) to the existing capital stock (K_t), that is $r_t = \Pi_t / K_t$. This later expression can be decomposed as follows $r_t = (\Pi_t/Y_t)(Y_t/Y_{Kt})(Y_{Kt}/K_t) = (1 - \omega)u_t(1/v)$. As the wage share and technical capital-output ratio are both given, changes in the rate of capacity utilization are the only source of variability of the realized rate of profit. In particular, it can be verified that if $u_t < \mu$ then the realized rate of profit would be lower than the rate of profit associated with the normal rate of capacity utilization.

existence of such short term disequilibria (i.e., we have $Y_t = D_t$ permanently) in order to concentrate our efforts on the investigation of the adjustment process of capacity to demand. Thus in equilibrium we have

$$Y_t = (\omega + h_t)Y_t + Z_t \qquad (13).$$

Therefore, if we suppose that $s - h_t = 1 - \omega - h_t > 0$, we can solve the last equation for the level of output obtaining:

$$Y_t = \left(\frac{1}{s - h_t}\right) Z_t \qquad (14).$$

Next, from equation (13) we can obtain the following equation for the real aggregate output and demand growth rate

$$g_t = g_Z + \frac{h_t \gamma (u_t - \mu)}{s - h_t} \qquad (15).$$ [20]

Equation (15) shows us that, when the actual and normal rates of capacity utilization are different, the rate of growth of output and demand is determined by the rate of expansion of autonomous consumption plus the rate of growth of the supermultiplier given by the second term on the RHS of the above equation. Observe also that equation (15) implies the following relations

$$g_{It} \gtreqless g_t \gtreqless g_Z \text{ as } u_t \gtreqless \mu$$

Hence, according to the supermultiplier growth model, if the actual rate of capacity utilization is above (below) the normal one, capitalist competition would induce an increase (decrease) in the marginal propensity to invest and, therefore, of the investment share of output. On the other hand, the increase (decrease) in the investment share would be accompanied by a corresponding increase (decrease) in the saving ratio. For, given the marginal propensity to save (i.e. given income distribution), the increase (decrease) in the propensity to invest implies an increase (decrease) in the "fraction" ($f_t = h_t/s$) and a decline (rise) in the ratio of autonomous consumption to output ($z_t = s(1 - f_t)$). Thus, for values of the propensity to invest strictly below the marginal propensity to save (i.e. for $h_t < s$), the behavior of the saving ratio is completely determined by the behavior of the propensity to invest (i.e., of the investment share of output),.[21]

To conclude our presentation of the supermultiplier growth model, we can summarize it by a system containing equations (2), (3), (10) and (15), which, for convenience of exposition, will be repeated below with their respective numbers for further reference.

$$g_t = g_Z + \frac{h_t \gamma (u_t - \mu)}{s - h_t} \qquad (15)$$

$$g_{Kt} = \left(\frac{h_t}{v}\right) u_t - \delta \qquad (2)$$

$$\dot{h} = h_t \gamma (u_t - \mu) \qquad (10)$$

$$\dot{u} = u_t (g_t - g_{Kt}) \qquad (3)$$

Let us now substitute equations (15) and (2) into equation (3). Then we obtain a system of two first order nonlinear differential equations in two variables, h and u, which we present below:

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[20] The equation is deduced as follows. Taking the time derivatives of the endogenous variables involved in expression (13) and dividing both sides of the resulting equation by the level of aggregate output, lead us to equation $g_t = \omega g_t + h_t g_t + \dot{h} + z_t g_Z$. If $z_t = s - h_t > 0$, then we can solve the last equation for the rate of growth of aggregate output and demand, obtaining $g_t = g_Z + \dot{h}/(s - h_t)$. Finally, we can substitute the right hand side (RHS) of equation (10) in the second term on the RHS of the last equation, which gives us equation (15) in the text.

[21] Note that the propensity to invest would determine the saving ratio even when we have $h_t = s$ if we admit the endogenous determination of income distribution as in the Cambridge growth models (see section 2 below) in such situation.

$$\dot{h} = h_t \gamma (u_t - \mu) \qquad (10)$$

$$\dot{u} = u_t \left(g_Z + \frac{h_t \gamma (u_t - \mu)}{s - h_t} - \left(\frac{h_t}{v}\right) u_t + \delta \right) \qquad (16)$$

This is the system that we will use to develop the analysis of the dynamic behavior of the supermultiplier growth model.

2. – Equilibrium Analysis

The model is in equilibrium if $\dot{h} = \dot{u} = 0$. Imposing this condition on the system comprised by equations (10) and (16) yields a system of equations whose solutions are the equilibrium values of the rate of capacity utilization and of the investment share of output. It can be easily verified that from a purely mathematical point of view the referred system admits two solutions and, therefore, the model has in principle two possible equilibrium points. One equilibrium occurs when the investment share of output and the rate of capacity utilization are both equal to zero (i.e. $h^{\ddagger} = u^{\ddagger} = 0$). This specific equilibrium, nevertheless, is of little interest, because it represents a completely unrealistic situation from the economic point of view.[22] For its turn, the other possible equilibrium has an economic meaning since the values of the investment share and of the capacity utilization rate in equilibrium are both strictly positive. That is, we have $u^*, h^* > 0$, where h^* and u^* are the equilibrium values for the investment share and the capacity utilization rate, respectively. In what follows, we shall analyze only this latter equilibrium.

Let us start our investigation by the analysis of the characteristics of the model's equilibrium path. Indeed, from equation (10), if $\dot{h} = 0$ and $h^* > 0$, then we obtain

$$u^* = \mu \qquad (17).$$

In the equilibrium path, the actual rate of capacity utilization is equal to the normal capacity utilization rate. This result shows that the supermultiplier growth model generates a fully adjusted *equilibrium* path. That is, a path characterized by the equilibrium between aggregate output and demand, *and* also by the normal utilization of productive capacity. Therefore, the *equilibrium* rate of capacity utilization is not an endogenous variable in the supermultiplier growth model, in the sense that the equilibrium value of the rate of capacity utilization is determined by the normal rate of capacity utilization, which is an exogenous variable in the model.

Next, using the equilibrium condition (17) in equations (11) and (15) we obtain the equilibrium value of aggregate output (and aggregate demand) and aggregate investment growth rates

$$g^* = g_I^* = g_Z$$

The equilibrium aggregate output (and demand) and investment growth rates are determined by the pace of expansion of autonomous consumption expenditure. Thus the simplified version of the supermultiplier

[22] However, if such equilibrium were stable, then at least it would reveal that the model had been somehow poorly specified. But this is not the case, for we can show that the equilibrium in question exhibits a saddlepoint behavior. In fact, the elements of the Jacobian matrix of the system evaluated at the equilibrium point $h^{\ddagger} = u^{\ddagger} = 0$ are given by $[\partial \dot{h}/\partial h]_{h^{\ddagger},u^{\ddagger}} = -\gamma\mu$, $[\partial \dot{h}/\partial u]_{h^{\ddagger},u^{\ddagger}} = 0$, $[\partial \dot{u}/\partial h]_{h^{\ddagger},u^{\ddagger}} = 0$ and $[\partial \dot{u}/\partial u]_{h^{\ddagger},u^{\ddagger}} = g_Z + \delta$. Since $\gamma, \mu, \delta, g_Z > 0$, then the determinant of $\mathbf{J}^{\ddagger}$is strictly negative (i.e. $\mathbf{Det}[\mathbf{J}^{\ddagger}] = -\gamma\mu(g_Z + \delta) < 0$). This last result implies that $h^{\ddagger} = u^{\ddagger} = 0$ is a saddlepoint equilibrium. So the stationary point with $h^{\ddagger} = u^{\ddagger} = 0$ is unstable because for any possible pair of initial values h_0 and u_0 such as $h_0 \geq 0$ and $u_0 > 0$ the model would not generate a convergence to this equilibrium. The only possibility of convergence occurs if, by luck, we have $u_0 = 0$. In this very particular case, equation (10) shows that for any possible initial value of h_0 such as $h_0 \geq 0$, the investment share converges to the equilibrium with $h^{\ddagger} = u^{\ddagger} = 0$. The set of the pairs of possible initial values such as $u_0 = 0$ and $h_0 \geq 0$ defines the stable manifold (or arm) of the saddlepoint equilibrium. But if we suppose that, at $t = 0$, autonomous consumption has always a positive value (i.e., $Z_0 > 0$) and the marginal propensity to spend has a positive and lower than one value (i.e., $0 < \omega + h_0 < 1$), then, from equation (14), output would also has a positive value (i.e., $Y_0 = [1/(s - h_0)]Z_0 > 0$). Besides, if we always have also a positive and finite initial value of the economy's capital stock (i.e., $K_0 > 0$), so capacity output would also have a positive and finite initial value (i.e., $Y_{K0} = (1/v)K_0 > 0$). Therefore, these hypotheses imply that the initial value for the actual rate of capacity utilization would always be positive (i.e., $u_0 = Y_0/Y_{K0} > 0$), which would exclude the existence of the equilibrium with $h^{\ddagger} = u^{\ddagger} = 0$.

growth model under discussion generates a consumption-led pattern of economic growth.[23] Further, if $\dot{u} = 0$ and $u^* > 0$, then from equation (3) we have that $g_K = g$ and, therefore, since $g^* = g_Z$, we obtain:

$$g_I^* = g_K^* = g^* = g_Z \qquad (18).$$

So the rate of capital accumulation is also driven by the autonomous consumption growth rate. This result shows that the equilibrium growth trajectory is sustainable, because the growth of the autonomous expenditure component of aggregate demand is able to induce the expansion of the economy's productive capacity required to maintain the demand-led growth path of aggregate output with a normal utilization of the productive capacity.

The adjustments of the rate of capital accumulation to the equilibrium output/demand growth rate and of the rate of capacity utilization to its normal level require the determination of an appropriate level of the investment share of output. The latter is determined as follows. Since along the equilibrium path $u^* = \mu$ and $g_K^* = g_Z$, then we can solve equation (2) for the equilibrium level of the investment share of output (i.e., the capitalist propensity to invest, h^*) obtaining the result below

$$h^* = \frac{v}{\mu}(g_Z + \delta) \qquad (19).$$

The last equation shows the determinants of the value of the investment share required to sustain the demand led growth path according to the supermultiplier growth model: the rate of growth of autonomous consumption, the technical capital-output ratio and the normal rate of capacity utilization and the depreciation rate. Observe that the endogeneity of the investment share of output is an important feature of the supermultiplier growth model because it allows that a high (low) output growth trend rate can be sustained by a high (low) pace of capital accumulation without the need of the maintenance of a rate of capacity utilization above (below) its normal level. This result is possible because the supermultiplier growth model supposes that investment and autonomous consumption can growth at different rates and that capitalist competition induces the adjustment of productive capacity to aggregate demand. Moreover, equation (19) shows that the supermultiplier growth model implies the existence of a theoretically necessary and positive relationship between the equilibrium level of the investment share of output and the equilibrium level of the output growth rate (equal to the growth rate of autonomous consumption).[24]

Note, however, that changes in the investment share of output require corresponding and appropriate modifications in the saving ratio in order to maintain the equilibrium between aggregate output and aggregate demand. We saw above that the endogeneity of the saving ratio in the supermultiplier growth model is a consequence of the hypothesis of the existence of positive level of autonomous consumption. Actually, the latter makes it possible for the fraction $f_t = h_t/s$ to change its value according to the modifications of the investment share of output. As a result, recalling that $z_t = 1 - f_t$, the ratio of autonomous consumption to aggregate output can change, making the saving ratio an endogenous variable and allowing its adjustment to the investment share of output. In fact, in the equilibrium path of the model, once the investment share is determined we can obtain the equilibrium values of the fraction, of the aggregate autonomous consumption to output ratio and, accordingly, of the equilibrium value of the saving ratio (i.e., the average propensity to save) as follows

$$f^* = \frac{h^*}{s} = \frac{\frac{v}{\mu}(g_Z + \delta)}{1 - \omega} \qquad (20)$$

$$z^* = s(1 - f^*) = s - h^* = s - \frac{v}{\mu}(g_Z + \delta) \qquad (21)$$

and

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[23]However, in a more general setting, the model also admits other patterns of economic growth such as an export led growth pattern or a pattern of economic growth led by the expansion of public expenditures.

[24] Such relationship is theoretically necessary in the sense that the model requires it in order to allow the existence an equilibrium path with a normal rate of capacity utilization. However, it is important to note that, besides being theoretically necessary, this last result of the model is also compatible with one of the most robust findings of the empirical literature on economic growth, the existence of a positive correlation between GDP growth rates and fixed investment to GDP ratio. See DeLong & Summers (1991), Blomstrom, Lipsey & Zejan (1996) and Sala-i-Martin (1997) for analyzes of the empirical regularity in question.

$$\left(\frac{S_t}{Y_t}\right)^* = s - z^* = sf^* = h^* = \frac{v}{\mu}(g_Z + \delta) \quad (22).$$

We saw that the equilibrium investment share of output is positively related to the equilibrium output growth rate. Thus, given income distribution (and, therefore, the marginal propensity to save), according to equations (20), (21) and (22), a higher (lower) equilibrium rate of economic growth entails, on the one hand, higher (lower) equilibrium values of the fraction and of the saving ratio, and, on the other, a lower (higher) equilibrium value of the autonomous consumption to output ratio.[25]

Now, as we just verified, in the supermultiplier growth model the adjustment mechanism that leads to the tendency for the rate of output/demand growth and the rate capital accumulation to be equal to each other involves appropriate and concomitant changes in the investment share of output and in the saving ratio. It is known that Cambridge growth models also use this type of adjustment mechanism, but the distinctive feature of the theoretical closure provided by the supermultiplier growth model is that the functioning of such adjustment mechanism does not require any change in the level of income distribution. In the supermultiplier model, the saving ratio adjusts to investment share of output through changes in the value of the fraction, without resorting to changes in income distribution. This feature of the model can be clearly visualized in equation (20) which contains the essential information about the closure provided by the model. It shows the value that the fraction must assume (i.e. f^*) in order to be compatible with the values of the exogenous variables and the parameters of the model (among them, income distribution) and, therefore, to allow the existence of an equilibrium path characterized by a demand-led growth pattern and by the normal utilization of productive capacity. So according to the simple supermultiplier growth model under discussion, the existence of an autonomous component in consumption by allowing the required variability of the fraction and of the saving ratio is a *necessary condition* for the adjustment of capacity to demand when income distribution is exogenously determined.[26]

Equations (17) to (22) give us the equilibrium levels of the endogenous variables of the supermultiplier growth model that are stationary in the equilibrium path. We represent the equilibrium situation described by these equations in Figure 1. In that figure the lines g and g_K represent equations (15) and (2) respectively. The vertical dotted line represents equilibrium condition (17) and the horizontal dotted line represents equilibrium condition (18). These four lines cross at point *E,* when the rate of capacity utilization is at its normal level, characterizing the equilibrium situation. Note that, since the depreciation rate is given, the g_K line can only cross the equilibrium if its slope is properly determined. This occurs when the equilibrium levels of the investment share of output and the saving ratio are endogenously determined. So the slope of the g_K line represents the endogenous determination of these variables as presented above in equations (20), (21) and (22). Observe also that with a given income distribution the marginal propensity to save is given from outside the model. Then the endogenous obtainment of the equilibrium saving ratio requires the appropriate determination of the fraction f^* such as the slope of the g_K line is exactly the one that is compatible with the simultaneous validity of the equilibrium conditions (17) and (18), which is represented by the equilibrium point E where the two dotted lines intercept each other. This gives us a graphical illustration of the closure provided by the supermultplier growth model.

Figure 1 – about here

[25] The last result is important for the empirical analysis of the economic growth process. For, according to the supermultiplier growth model, an increase in the trend rate of output growth caused by an increase in the autonomous consumption growth rate, would raise the investment share of output and reduce the autonomous consumption to output ratio. An analyst observing this pattern of change in reality could interpret this result as an evidence of an investment-led growth process, while, if the supermultiplier growth model is correct, the pattern of economic growth would be a consumption-led one. Of course the same type of interpretative problem can occur in a more general setting with patterns of economic growth led by exports or government expenditures. See Medeiros & Serrano (2001) and Freitas & Dweck (2013), for empirical analyses based on the supermulplier growth model that deal with this interpretative problem.

[26] It is not, however, a sufficient condition because, as we saw in section 1, if the investment share of output (i.e. the marginal propensity to invest) is given exogenously, then the equilibrium rate of capacity utilization would be an endogenous variable and its level would generally be different from the normal level of capacity utilization.

Besides, we can also discuss the behavior of the endogenous variables of the model that are not stationary in the equilibrium path. So introducing in equation (14) the equilibrium level of the investment share we obtain

$$Y_t^* = \left(\frac{1}{s - \frac{v}{\mu}(g_Z + \delta)} \right) Z_t = \varphi^* Z_t \qquad (23).$$

At each moment t in the equilibrium path of the model the level of autonomous consumption at t and the equilibrium level of the supermultiplier (i.e., the term in parenthesis on the RHS of equation (23) and also denoted φ^*) determine the equilibrium level of output at t (denoted Y_t^*).[27]

On the other hand, from equations (5) and (23) we have the equation that describes the behavior of the endogenous consumption in the equilibrium path

$$C_{wt}^* = \omega Y_t^* = \omega \varphi^* Z_t \qquad (24).$$

Equation (24) shows that in the equilibrium path of the model the level of autonomous consumption at t, the equilibrium level of the supermultiplier and the wage share of output determine the equilibrium level of endogenous consumption at t (denoted C_{wt}^*).

Next, from equations (9), (19) and (23) we obtain the equation for the behavior of the level of investment in the equilibrium trajectory

$$I_t^* = h^* Y_t^* = h^* \varphi^* Z_t \qquad (25).$$

Equation (25) means that in the equilibrium path of the model the level of autonomous consumption at t, the equilibrium level of the supermultiplier and equilibrium investment share of output determine the equilibrium level of investment at t (denoted I_t^*).[28]

To derive the equation for the level of output capacity in the equilibrium path we use the fact that from the definition of the rate of capacity utilization and since in equilibrium $u^* = \mu$ then we have that $Y_{Kt}^* = Y_t^* / \mu$. Thus, we use the last result and equation (23) to obtain the desired expression for the equilibrium level of output capacity

$$Y_{Kt}^* = \frac{1}{\mu} Y_t^* = \frac{1}{\mu} \varphi^* Z_t \qquad (26).$$

Now from equation (1), we can see that in equilibrium we have that $K_t^* = v Y_{Kt}^*$ and, therefore, using the last result in equation (26) we have

$$K_t^* = \frac{v}{\mu} Y_t^* = \frac{v}{\mu} \varphi^* Z_t \qquad (27).$$

Equations (26) and (27) show that in the fully adjusted equilibrium path of the model not only aggregate output adjusts to the level of aggregate demand but also that capacity output and the capital stock adjust to the level of aggregate demand. Therefore, according to the supermultiplier growth model, aggregate demand determines the levels of capacity output and capital stock. In this sense, according to the model, aggregate demand determines the level of potential output, which is therefore an endogenous variable in the long run.

We shall now use the characterization of the model's equilibrium path based on (17) to (27) to discuss the role of income distribution in the simplified version of the supermultiplier growth model presented in this paper. So from equation (18) we can verify that a change in income distribution (i.e. in the wage share) does not have a permanent effect on the pace of economic growth. Hence, in the supermultiplier model there is no relationship between income distribution and the trend growth rate of output and demand. On the other hand, a change in income distribution also does not affect the equilibrium value of the capacity utilization rate as can be verified from equation (17). Next, since, as we just saw, a change in income

[27]Note that here we have the fully adjusted equilibrium level of output in contrast with the equilibrium level of output present in equation (14). The former level of output equilibrium occurs when both aggregate output and demand are equal, and the productive capacity is normally utilized, while the latter type of equilibrium requires only the adjustment of aggregate output to aggregate demand.

[28]Since the level of investment determines the level of savings the equation presenting the behavior of the last variable in the equilibrium trajectory of the model is the same as the one describing the behavior of the level of investment.

distribution does not have a permanent growth effect, then equation (19) shows that a variation in income distribution does not have a permanent effect on the equilibrium value of the investment share of output. On the other hand, we know that, according to the supermultplier growth model, the investment share of output determines the saving ratio. Thus, there is no effect of a modification in income distribution on the saving ratio also. Note, however, that a change in income distribution does affect the marginal propensity to save. In fact, since the latter variable is equal to the profit share (i.e. $s = 1 - \omega$), an increase (decrease) in the wage share would reduce (raise) the marginal propensity to save (i.e. $ds = -d\omega$). But making use of equations (21) and (22) we can verify how this latter result is reconciled with the invariability of the saving ratio in relation to an income distribution change. From equation (21) we can see that, given the value of the investment share of output, a change in the marginal propensity to save leads to a change in the same direction and by the same amount of the ratio of autonomous consumption to output (i.e. $dz^* = d(s - h^*) = ds$). According to equation (22), this explains why the saving ratio is unaffected by change in income distribution, since $d(S_t/Y_t)^* = d(s - z^*) = ds - dz^* = 0$.

Nevertheless, although a change in income distribution does not have a permanent growth effect, such a change does have a *level effect* over the equilibrium values of all non-stationary variables of the simplified supermultiplier growth model here presented. This is so because a change in income distribution affects, through its influence over the marginal propensity to save, the equilibrium value of the supermultiplier and, hence, the equilibrium value of aggregate output. So a change in the wage share has wage led output level effect. Indeed, as can be easily verified, an increase (decrease) in the wage share reduces (raises) the marginal propensity to save, which yields an increase (a decrease) in the equilibrium value of the supermultiplier (i.e. we have $\partial\varphi^*/\partial\omega > 0$) and, accordingly, an increase (a decrease) in the equilibrium level of output.[29] Thus, from equations (23) to (27), we can see that an increase (decrease) in the wage share, through its influence over the equilibrium level of output, causes a positive (negative) level effect over the equilibrium values of induced consumption, investment, output capacity and capital stock. Moreover, in the case of the equilibrium level of induced consumption, additionally to the influence of the change of the equilibrium value of output, the wage share has a direct and positive contribution to the change of the equilibrium value of the variable (since $C^*_{wt} = \omega Y^*_t$).

3. Local Stability and Dynamic Behavior Analyzes

Let us now analyze the stability of the equilibrium. More precisely, we will analyze the dynamic stability conditions of the linearized version of the model in the neighborhood of the equilibrium point (i.e., a local stability analysis).[30] Thus, from the system defined by equations (10) and (16) we can obtain the corresponding Jacobian matrix evaluated at the equilibrium point with $u^* = \mu$ and $h^* = \frac{v}{\mu}(g_Z + \delta)$

$$\mathbf{J}^* = \begin{bmatrix} \left[\frac{\partial \dot{h}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{h}}{\partial u}\right]_{h^*,u^*} \\ \left[\frac{\partial \dot{u}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{u}}{\partial u}\right]_{h^*,u^*} \end{bmatrix} = \begin{bmatrix} 0 & \gamma \frac{v}{\mu}(g_Z + \delta) \\ -\frac{\mu^2}{v} & \frac{s\gamma\mu}{s - \frac{v}{\mu}(g_Z + \delta)} - \gamma\mu - g_Z - \delta \end{bmatrix}$$

The trace $\mathbf{Tr}[\mathbf{J}^*]$and the determinant $\mathbf{Det}[\mathbf{J}^*]$of the above Jacobian matrix are given by the following expressions:

[29] Note that, in this sense, the Supermultiplier growth model allows the extension of the validity of the paradox of thrift concerning the level of output to the context of analysis of the economic growth process.

[30] The legitimacy of an analysis of the local behavior of the nonlinear system using a linearized version of it in the neighborhood of the equilibrium requires that the linearized system is a uniformly good approximation of the original nonlinear system near the equilibrium point and that the Jacobian is a matrix of constants. It is known that if the nonlinear system is autonomous, then these two properties are assured. As can be verified, the nonlinear system described by equations (10) and (16) is autonomous since the variable "time" (t) is not an explicit argument of the two functional equations. Therefore, the system under analysis meets the requirements stated above. Nevertheless, it should be noted that, even when these requirements are met, the stability of the equilibrium point may not be determined by the analysis of the linearized system if all the eigenvalues of the Jacobian matrix have non positive real parts and at least one eigenvalue has a zero real part. For the discussion of the relevant mathematical theorems and for mathematical references concerning the issues discussed in this footnote see Gandolfo (1997, chap. 21, pp. 361-3).

$$\mathbf{Tr}[\mathbf{J}^*] = \frac{s\gamma\mu}{s - \frac{v}{\mu}(g_Z + \delta)} - \gamma\mu - g_Z - \delta \quad (28).$$

and

$$\mathbf{Det}[\mathbf{J}^*] = \gamma\mu(g_Z + \delta) \quad (29).$$

The mathematical literature (e.g., for a reference see Gandolfo (1997)) establishes that the necessary and sufficient stability conditions for a 2x2 system of differential equations are the following:

$$\mathbf{Tr}[\mathbf{J}^*] = \frac{s\gamma\mu}{s - \frac{v}{\mu}(g_Z + \delta)} - \gamma\mu - g_Z - \delta < 0 \quad (30)$$

and

$$\mathbf{Det}[\mathbf{J}^*] = \gamma\mu(g_Z + \delta) > 0 \quad (31).$$[31]

The determinant is necessarily positive since we suppose that $\gamma, \mu, \delta, g_Z > 0$. Hence, the local stability of the system depends completely on the sign of the trace of the Jacobian matrix evaluated at the equilibrium point. Recalling that $s = 1 - \omega$, inequality (30) implies the following stability condition for the supermultiplier growth model

$$\omega + h^* + \gamma v < 1 \quad (32).$$

We can interpret (32) as an expanded marginal propensity to spend that besides the equilibrium propensity to spend $(\omega + h^*)$ includes also a term (i.e., γv) related to the behavior of induced investment in disequilibrium. The extra adjustment term captures the fact that the investment function of the model under discussion is inspired in the capital stock adjustment principle (i.e, a type of flexible accelerator investment function) and that outside the fully adjusted trend path there must be room not only for the induced gross investment necessary for the economy to grow at its equilibrium rate g_Z, but also for the extra disequilibrium induced investment responsible for adjusting capacity to demand. Thus, *ceteris paribus*, for a sufficiently low value of the reaction parameter γ the equilibrium described above is stable.[32]

Moreover, we can further investigate the economic meaning of the stability condition transforming equation (32) in the following way. From equation (32) we have that

$$\omega + h^* + \gamma v < 1 \Rightarrow \gamma v < s - h^* \Rightarrow \frac{\gamma}{s - h^*} < \frac{1}{v} \Rightarrow \frac{h^*\gamma}{s - h^*} < \frac{h^*}{v}$$

Next, from equations (15) and (2) the partial derivatives of g and g_K with respect to u evaluated at the equilibrium point are given by

[31] Note that in the general case of a n x n system the stability conditions involving the signs of the trace and determinant of the Jacobian matrix are only necessary conditions (Gandolfo, 1997, chap. 18, p. 254). The combination of the stated conditions is only sufficient for stability in the case of a 2 x 2 system. This is so because the stability of the equilibrium requires that the eigenvalues of matrix $\mathbf{J}^*$ have negative real parts. Further, it is known that the trace of $\mathbf{J}^*$ is equal to the sum of the eigenvalues of the matrix and that the determinant of $\mathbf{J}^*$ is equal to the product of the eigenvalues of $\mathbf{J}^*$. Thus, $\mathbf{Det}[\mathbf{J}^*] > 0$ implies, in the 2 x 2 case, that the two eigenvalues have both either negative or positive real parts. Therefore, if $\mathbf{Det}[\mathbf{J}^*] > 0$ and $\mathbf{Tr}[\mathbf{J}^*] < 0$ then the two eigenvalues must have negative real parts (Ferguson & Lim, 1998, pp. 83-4). Moreover, since in this case the eigenvalues have *strictly* negative real parts, then the equilibrium point is locally asymptotically stable. Finally, observe that in the case of an equilibrium characterized by eigenvalues with *strictly* negative real parts we can perform the local stability analysis of such an equilibrium using the linearized version of the original nonlinear model, since in this case we don't have any eigenvalue with a zero real part (see the previous footnote).

[32] By a sufficiently low value of γ we mean a value lower than the critical level $\gamma_c = (1 - \omega - h^*)/v$.

$$\left[\frac{\partial g}{\partial u}\right]_{h^*,u^*} = \frac{h^*\gamma}{s-h^*}$$

and

$$\left[\frac{\partial g_K}{\partial u}\right]_{h^*,u^*} = \frac{h^*}{v}$$

Therefore, the stability condition implies that at the equilibrium point we have the following inequality:

$$\left[\frac{\partial g}{\partial u}\right]_{h^*,u^*} = \frac{h^*\gamma}{s-h^*} < \frac{h^*}{v} = \left[\frac{\partial g_K}{\partial u}\right]_{h^*,u^*} \qquad (33).$$

This last inequality shows that the required stability condition means that a change in the rate of capacity utilization should have a greater impact on the growth of capacity output than on the growth of aggregate demand. This condition would guarantee that an increase (or decrease) in the rate of capacity utilization will not be self-sustaining. To check this, note that in equilibrium we have $g^* = g_K^*$ and, therefore, $u^* = \mu$ is constant. Now, if in the neighborhood of the equilibrium point we have $u > u^* = \mu$ ($u < u^* = \mu$), then according to inequality (33) we would have $g < g_K$ ($g > g_K$) and, from equation (3), $\dot{u} < 0$ ($\dot{u} > 0$), implying a decrease (increase) in the capacity utilization rate. Thus, the stability condition implies that movements of the capacity utilization rate way from its equilibrium level are not self-sustaining.

We can visualize the last result in Figure 1. Thus, at the RHS (LHS) of the equilibrium point *E*, we have that $u_t > \mu$ ($u_t < \mu$). On the other hand, we can see that the slope of the equation for the growth rate of output/demand evaluated at the equilibrium point $h^*\gamma/(s-h^*)$ is lower than the slope of the capital accumulation equation evaluated at the equilibrium point h^*/v. Hence, at the RHS (LHS) of the equilibrium *E* we have that $g < g_K (g > g_K)$ and, according to equation (3), $\dot{u} < 0$ ($\dot{u} > 0$), indicating that the disequilibrium is not self-sustaining. In fact, in this sense, Figure 1 represents a situation of a stable equilibrium.[33]

From the stability condition we can also derive a condition concerning the viability of a demand-led growth regime for given levels of income distribution and technical conditions of production. Since we know that $h^* = (v/\mu)(g_Z + \delta)$, we can put (32) in the following form:

$$g_Z < g_{max} = \frac{s}{v}\mu - \delta - \gamma\mu \qquad (34).$$

This means that, given s and v, in order for a demand led growth regime to be viable, its equilibrium growth rate must be smaller than an maximum growth rate given by $(s/v)\mu - \delta$ minus an extra term (i.e., $\gamma\mu$) associated with the disequilibrium induced investment required for the adjustment of capacity to demand.[34] This maximum growth rate defines a ceiling for the expansion rate of autonomous demand compatible with a dynamically stable demand-led growth trajectory according to the supermultiplier growth model. As can be verified, the ceiling in question, is lower (higher), *ceteris paribus*, the more (less) sensitive investment is to changes in the actual rate of capacity utilization (i.e. the higher (lower) is the reaction parameter γ).

Let us now investigate the dynamic behavior of the model. To do so we will perform a qualitative analysis of the phase portrait generated by the linearized version of the model in the neighborhood of the equilibrium point. First, we have to derive the nullclines for u and h. Regarding the nullcline for the investment share, given $h > 0$ then the condition $\dot{h} = 0$ implies that $u_t = \mu$. In Figure 2 this result is represented by the horizontal line corresponding to the equilibrium level of the rate of capacity utilization. On the other hand, the slope of the nullcline for the rate of capacity utilization in the neighborhood of the equilibrium point is given by the following expression

[33] A similar Figure could be used to represent a situation of unstable equilibrium. The only difference would be that the slope of the equation for the growth rate of output/demand would be higher than the slope of the capital accumulation equation, both evaluated at the equilibrium point.

[34] Originally, Serrano (1995 and 1996) presented the maximum growth rate as given by $g_{max} = \frac{s}{v}\mu - \delta$. The present stability analysis shows that maximum growth rate has a lower value for given values of the marginal propensity to save (i.e. income distribution) and of the capital-output ratio during the adjustment process of capacity to demand.

$$\left[\frac{du}{dh}\right]_{\dot{u}=0,h^*,u^*} = -\left(\frac{\left[\frac{\partial \dot{u}}{\partial h}\right]_{h^*,u^*}}{\left[\frac{\partial \dot{u}}{\partial u}\right]_{h^*,u^*}}\right) = -\left(\frac{-\frac{\mu^2}{v}}{\frac{s\gamma\mu}{s-\frac{v}{\mu}(g_Z+\delta)} - \gamma\mu - g_Z - \delta}\right)$$

Since by the specification of the model we have that $[\partial\dot{u}/\partial h]_{h^*,u^*} = -(\mu^2/v) < 0$ and, in the case of a stable equilibrium, we have that $[\partial\dot{u}/\partial u]_{h^*,u^*} = (s\gamma\mu/[s-(v/\mu)(g_Z+\delta)]) - \gamma\mu - g_Z - \delta < 0$, hence the nullcline $\dot{u} = 0$ has a negative slope in the neighborhood of the equilibrium point (i.e., $[du/dh]_{\dot{u}=0,h^*,u^*} < 0$) on the phase plane (h, u), as represented in Figure 2.[35] Point E, where the two nullclines intersect each other, is the representation of the equilibrium point of the model in the phase plane.

Figure 2 – about here

We can easily verify that, according to equation (10), when $u_t > \mu$ ($u_t < \mu$) then $\dot{h} > 0$ ($\dot{h} < 0$). So, above (below) the nullcline $\dot{h} = 0$ the investment share increases (decreases), a fact indicated in Figure 2 by the horizontal arrows pointing rightwards (leftwards). On the other hand, every point over the nullcline $\dot{h} = 0$ on the RHS (LHS) of the equilibrium investment share is associated with a decreasing (an increasing) rate of capacity utilization. To understand why, observe that if $u_t = \mu$, then equation (16) can be expressed as follows

$$\dot{u} = \mu\left(g_Z - \left(\frac{h_t}{v}\right)\mu + \delta\right) = \frac{\mu^2}{v}\left(\frac{v}{\mu}(g_Z+\delta) - h_t\right) = \frac{\mu^2}{v}(h^* - h_t)$$

Thus, if $u_t = \mu$ and $h^* < h_t$ ($h^* > h_t$), then $\dot{u} < 0$ ($\dot{u} > 0$). Accordingly, on the RHS (LHS) of the nullcline $\dot{u} = 0$ the rate of capacity utilization decreases (increases), which is indicated in Figure 2 by the vertical arrows pointing downward (upward).[36]

Therefore, we can define four regions on the phase space denoted in Figure 2 by the Roman numerals I, II, III and IV.[37] In region I (above the nullcline $\dot{h} = 0$ and on the RHS of the nullcline $\dot{u} = 0$) we have $\dot{u} < 0$ and $\dot{h} > 0$. Next, in region II (below the nullcline $\dot{h} = 0$ and on the RHS of the nullcline $\dot{u} = 0$) we have $\dot{u} < 0$ and $\dot{h} < 0$. For its turn, in region III (below the nullcline $\dot{h} = 0$ and on the LHS of the nullcline $\dot{u} = 0$) we have $\dot{u} > 0$ and $\dot{h} < 0$. Finally, in region IV (above the nullcline $\dot{h} = 0$ and on the LHS of the nullcline $\dot{u} = 0$) we have $\dot{u} > 0$ and $\dot{h} > 0$.

The analysis of the phase portrait allows us to determine the dynamic behavior of the supermultiplier growth model in the neighborhood of the equilibrium point. Indeed, it can be verified that the movement around the equilibrium has a cyclical and convergent pattern. Thus, for instance, let us suppose that at $t = t_0$ the economy is at point D_{t_0} in Figure 2 (see also Figure 3 to visualize the behavior of the main variables of the model over time) on the $\dot{h} = 0$ nullcline on the LHS of the equilibrium level of the investment share h^* (i.e., we have $u_{t_0} = \mu$ and $h_{t_0} < h^*$). Since at D_{t_0} we have that $u_{t_0} = \mu$, then, from equations (11) and (15), it follows that the growth rates of investment and of output/demand are equal to the autonomous consumption growth rate (i.e., $g_{It_0} = g_{t_0} = g_Z$) and, accordingly, the fraction, the autonomous consumption to output ratio and the saving ratio are constant (i.e.,$\dot{f} = 0$, $\dot{z} = 0$ and $(\dot{S/Y}) = \dot{h} = 0$). But since $h_{t_0} < h^*$ the rate of capital accumulation would be lower than the pace of expansion of output (i.e. $g_{Kt_0} = (h_{t_0}/v)\mu - \delta < g_{t_0} = g_Z$) and, according to equation (3), in D_{t_0} the rate of capacity utilization increases. Therefore, the phase trajectory starting at point D_{t_0} would enter region IV in Figure 2. In this last region, the rate of capacity utilization and the investment share increase (i.e., $\dot{u} > 0$and $\dot{h} > 0$), leading to a movement

[35]Conversely, in the case of an unstable equilibrium the nullcline for the rate of capacity utilization would have a positive slope in the neighborhood of the equilibrium point.
[36] We can obtain the same result by doing a horizontal or a vertical displacement from other points of the nullcline $\dot{u} = 0$
[37]It is important to note that the analysis of such regions is restricted to the strictly positive values of u and h.

in the northeast direction on the phase space. Furthermore, since in region IV we have that $u_t > \mu$, then $g_{It} > g_t > g_Z$, $\dot{f} > 0$, $\dot{z} < 0$ and $(\dot{S/Y}) > 0$. However, the increases in the investment share and of the rate of capacity utilization cause an increase in the pace of capital accumulation along the phase trajectory in region IV. These movements lead the economy to the situation described by point D_{t_1} on the $\dot{u} = 0$ nullcline. At D_{t_1} we have that the growth rates of output/demand and capital accumulation are equal (i,e, $g_{Kt_1} = (h_{t_1}/v)u_{t_1} - \delta = g_{t_1}$). Nonetheless, since at D_{t_1} we also have that $u_{t_1} > \mu$, then $g_{It_1} > g_{t_1} > g_Z$, $\dot{f} > 0$, $\dot{z} < 0$, $(\dot{S/Y}) > 0$, and, according to equation (10), the investment share continues to increase. (i.e. $\dot{h} > 0$). The increase in the investment share departing from D_{t_1} pushes the economy to region I in Figure 2. In region I, while the investment share continues to increase, the rate of capacity utilization declines, because the rate of capital accumulation is higher than the output/demand growth rate (i.e. $g_{Kt} > g_t$). So at D_{t_1}we have an upper turning point for the path of the rate of capacity utilization (i.e., u_t increases from D_{t_0} until D_{t_1} and afterwards it decreases). Note that, similarly to what occurs in region IV, in region I the rate of capacity utilization is above its normal level, thus we also have that $g_{It} > g_t > g_Z$ and,$\dot{f} > 0$, $\dot{z} < 0$ and $(\dot{S/Y}) > 0$. Since in region I the investment share increases while the rate of capacity utilization decreases, then the phase trajectory is moving in the southeast direction. Eventually, it achieves point D_{t_2} on the nullcline $\dot{h} = 0$. At this point, once more, the productive capacity is normally utilized (i.e., $u_{t_2} = \mu$), and, thus, $g_{It_2} = g_{t_2} = g_Z$, $\dot{f} = 0$, $\dot{z} = 0$ and $(\dot{S/Y}) = \dot{h} = 0$. But now the level of the investment share is above its equilibrium value (i.e., $h_{t_2} > h^*$). As a result, the rate of capital accumulation is higher than the output/demand growth rate (i.e., $g_{Kt_2} = (h_{t_2}/v)\mu - \delta > g_{t_2} = g_Z$). Hence, the capacity utilization rate continues to decline, pushing the phase trajectory to region II. Since in this last region the rate of capacity utilization is below its normal level, then the investment share decreases ($\dot{h} < 0$) and, hence, at D_{t_2} we have an upper turning point for the investment share. Also, observe that at t_0 and t_2 we have that $u_{t_0} = u_{t_2} = \mu$ and, accordingly, that $g_{It_0} = g_{t_0} = g_{It_2} = g_{t_2} = g_Z$, while for $t_0 < t < t_2$ we have that $g_{It} > g_t > g_Z$. Therefore, between t_0 and t_2 we must have an upper turning point for investment and output/demand growth rates. On the other hand, since in region II we have that $u_t < \mu$, then $g_{It} < g_t < g_Z$, $\dot{f} < 0$, $\dot{z} > 0$, and $(\dot{S/Y}) < 0$. Moreover, in region II, the capacity utilization rate and the investment share decrease (i.e., $\dot{u} < 0$and $\dot{h} < 0$), which leads to a decline of the rate capital accumulation. Eventually the decrease of the rate of capital accumulation will be sufficient to make it equal to the growth rate of output/demand again. In Figure 2 this occurs at $t = t_3$, when the phase trajectory reaches point D_{t_3}, where $\dot{u} = u_{t_3}(g_{t_3} - g_{Kt_3}) = 0$. But, as can be seen, since $u_t < \mu$, then, according to equation (10), the investment share continues to decrease. This movement pushes the economy to region III, where the rate of capacity utilization starts to increase. Therefore, at point D_{t_3} we have a lower turning point for the capacity utilization rate. For its turn, in region III while the rate of capacity utilization increases, the investment share continues to decline because the rate of capacity utilization is still below its normal level. This latter fact also implies that in region III we have that $g_{It} < g_t < g_Z$, $\dot{f} < 0$, $\dot{z} > 0$, and $(\dot{S/Y}) < 0$. Finally, at $t = t_4$ the phase trajectory reaches point D_{t_4} on the $\dot{h} = 0$ nullcline. As we can verify, at D_{t_4} we are in a similar situation compared to the one we started with. Productive capacity is normally utilized (i.e., $u_{t_4} = \mu$) and, therefore, $g_{It_4} = g_{t_4} = g_Z$, $\dot{f} = 0$, $\dot{z} = 0$ and $(\dot{S/Y}) = \dot{h} = 0$. So at D_{t_4} we have that $u_{t_4} = u_{t_2} = \mu$ and, accordingly, that $g_{It_4} = g_{t_4} = g_{It_2} = g_{t_2} = g_Z$, while for $t_2 < t < t_4$ we have that $g_{It} < g_t < g_Z$. Hence, we have a lower turning point for the rates of growth of investment and of output/demand between $t_2$and t_4. Moreover, at D_{t_4} the investment share is lower than its equilibrium level and thus the pace of capital accumulation is insufficient to sustain the equilibrium rate of growth of output/demand (i.e., $g_{Kt_4} = (h_{t_4}/v)\mu - \delta < g_{t_4} = g_Z$). As a result, the rate of capacity utilization continues to increase and pushes the phase trajectory to region IV again. From then on, the phase path repeats the same cyclical pattern described before. Nevertheless, if the equilibrium is stable, then the phase trajectory will approach asymptotically the equilibrium point E, as shown in Figure 2. Therefore, qualitative analysis shows that the equilibrium under analysis is a *stable focus* according to the nomenclature adopted in the mathematical literature on differential equations.

In Figure 3 we can visualize the behavior of the main variables of the model over time corresponding to each situation described in Figure 2. Thus, corresponding to point D_{t_0}, at $t = t_0$ we have that $u_{t_0} = \mu$, $\dot{h} = 0$, $g_{It_0} = g_{t_0} = g_Z$,$h_{t_0} < h^*$, $g_{Kt_0} < g_{t_0}$ and $\dot{u} > 0$. Beetweent_0 and t_1 we can see the situation corresponding to region IV, where $u_t > \mu$, $\dot{h} > 0$, $g_{It} > g_t > g_Z$, $g_{Kt} < g_t$, $\dot{u} > 0$, and $\dot{g}_K > 0$. Next, similarly to point D_{t_1}, at $t = t_1$ we have that $g_{Kt_1} = g_{t_1}$, $\dot{u} = 0$, $u_{t_1} > \mu$, $g_{It_1} > g_{t_1} > g_Z$, $\dot{h} > 0$ and that there is an upper turning point for capacity utilization rate. Between $t_1$and t_2, we visualize the situation described in region I, with $g_{Kt} > g_t$, $\dot{u} < 0$, $u_t > \mu$, $\dot{h} > 0$, and $g_{It} > g_t > g_Z$. Corresponding to point D_{t_2}, at $t = t_2$ we have that $u_{t_2} = \mu$, $\dot{h} = 0$, $g_{It_2} = g_{t_2} = g_Z$, $h_{t_2} > h^*$, $g_{Kt_2} > g_{t_2}$, $\dot{u} < 0$ and that there is an upper turning point for the investment share. As we pointed out above, there exist upper turning points for the growth rates of output/demand and investment between t_0 and t_2. Note also that the turning point for investment growth is important, because the resulting decline in this variable eventually implies a reversion of the acceleration trend of the rate of capital accumulation, which explains the existence of an upper turning point for the rate of capital accumulation. In fact, note that this last turning point occurs between t_1 and t_2 exactly when the investment growth rate is equal to the rate of capital accumulation.[38] Between t_2 and t_3 we have the situation associated to region II, with $u_t < \mu$, $\dot{h} < 0$, $g_{It} < g_t < g_Z$, $g_{Kt} > g_t$, $\dot{u} < 0$, and $\dot{g}_K < 0$. Now, at $t = t_3$ we have the situation described in point D_{t_3}, with $g_{Kt_3} = g_{t_3}$, $\dot{u} = 0$, $u_{t_3} < \mu$, $g_{It_3} < g_{t_3} < g_Z$, $\dot{h} < 0$ and the occurrence of a lower turning point for the capacity utilization rate. Between t_3 and t_4 we can visualize the situation described in region III, with $g_{Kt} < g_t$, $\dot{u} > 0$, $u_t < \mu$, $\dot{h} < 0$, and $g_{It} < g_t < g_Z$. At $t = t_4$ we have the circumstances corresponding to point D_{t_4}, where we have $u_{t_4} = \mu$, $\dot{h} = 0$, $g_{It_4} = g_{t_4} = g_Z$,$h_{t_4} < h^*$, $g_{Kt_4} < g_{t_4}$, $\dot{u} > 0$ and where there is a lower turning point for the investment share. Moreover, observe that between t_2 and t_4 occurs the lower turning points for the output/demand, investment and capital stock growth rates. In particular, note that similarly to what happened in the case of an upper turning point, the lower turning point for the rate of capital accumulation occurs precisely when this rate is equal to the investment growth rate between t_3 and t_4. After $t = t_4$ the economy repeats the fluctuation pattern just described and asymptotically converges to the equilibrium point.

Figure 3 about here

4. The Criticisms

In this section we shall make use of the previous analysis to address the main criticisms so far directed against the supermultiplier growth model. According to these criticisms, the supermultiplier model would be unable to provide plausible explanations for the economic growth as demand-led process and for the tendency towards normal capacity utilization. Here we shall argue that such evaluation is based either on a misleading representation and interpretation of the supermultiplier growth model or on a misplaced requirement of precision imposed to the model by the critics.

In his early presentation of the supermultiplier growth model Serrano (1995 and 1996) did not attempted to provide a detailed analysis of the dynamic process of adjustment of capacity to demand, mainly confining his efforts to the presentation of the model's main hypothesis and equilibrium results, and to show its ability to provide an alternative heterodox closure for the analysis of the economic growth process.[39] However, the lack of a dynamic analysis leaved open space for the criticism to the model's capability to offer a plausible explanation of economic growth as demand-led process. Such criticism is based on a representation of the supermultiplier model suggested by Serrano as a model that tries to reconcile the determining role of aggregate demand in the analysis of the economic growth process with a condition of *continuous* normal utilization of productive capacity. Here, however, we shall argue that such representation of the model is misleading.

[38] This happens because, as we know, $\dot{g}_{Kt} = (g_{Kt} + \delta)(g_{It} - g_{Kt})$. So if $g_{Kt} + \delta > 0$, then $\dot{g}_{Kt} = 0$ when $g_{It} = g_{Kt}$.

[39] Note, however, that in Cesaratto, Serrano and Stirati (2003) the supermutltiplier model has been explicitly supplied with an investment function of the flexible accelerator type and the corresponding argument for explaining the adjustment of capacity to demand.

In order to show that, let us suppose that at $t = 0$ the economy is characterized by the normal utilization of productive capacity and that the level of capacity output determines the normal output level, so that we have

$$Y_0 = \frac{\mu}{v} K_0$$.[40]

Further, given the level of income distribution and given the value of autonomous consumption at $t = 0$, the initial level of consumption is also determined at $t = 0$, such as $C_0 = Z_0 + \omega Y_0$. Thus, in order to have an equilibrium between aggregate demand and aggregate output, aggregate investment at $t = 0$ *must* assume the following required value

$$I_0 = sY_0 - Z_0 = Y_0 - C_0 = S_0$$

So, at $t = 0$, Say's law prevails, aggregate demand is determined by the level of normal output and normal capacity savings determines investment. Now let us *suppose* we are interested to know the rate of expansion of aggregate demand that would be required to continuously maintain the equilibrium between aggregate demand and output and the normal utilization of productive capacity through time (i.e., the fully adjusted equilibrium). In order to maintain the normal utilization of productive capacity, aggregate output must grow at the same rate as capacity output, and then aggregate demand would also have to grow at this same rate. Under the present circumstances, the rate of growth of capacity output would be given by

$$g_t = g_{Kt} = \frac{(I_t/Y_t)}{v}\mu = \frac{(s - Z_t/Y_t)}{v}\mu$$

Thus aggregate output and demand would be required to grow at the above rate in order to continuously maintain the economy in a fully adjusted equilibrium. As can be easily verified such growth rate is equivalent to the warranted rate of growth suggested by Harrod (1939 and 1948) when we take into account the existence of an autonomous component in aggregate consumption.

Nonetheless, the existence of this autonomous component growing at an independent growth rate implies that the warranted rate growth would, in general, change through time. Actually, the dynamic behavior of the warranted growth rate incorporating the autonomous consumption component can be expressed by the following differential equation

$$\dot{g} = \frac{\mu}{v}(g_t - g_Z)\frac{Z_t}{Y_t} = (g_t - g_Z)\left(\frac{s}{v}\mu - g_t\right)$$

As can be seen from the equation above, the model has two stationary points, $g_t^* = g_Z$ and $g_t^{**} = (s/v)\mu$. Now, let us assume that the values of s, v, μ and g_Z are such that $(s/v)\mu > g_Z$. Thus, we have three possible outcomes depending on the initial situation regarding the relationship between the warranted and autonomous consumption growth rates. That is at $t = 0$ we may have three possible relations between these two rates

$$g_0 = \frac{(s - Z_0/Y_0)}{v}\mu \gtreqless g_Z$$

For the sake of future discussion, it is important to observe that these initial conditions can be alternatively represented by the corresponding relations bellow

$$Y_0 \gtreqless \frac{Z_0}{s - \frac{v}{\mu} g_Z}$$

Note that the latter relations are in fact deduced from the former relations (i.e. $g_0 \gtreqless g_Z \Rightarrow Y_0 \gtreqless Z_0/(s - (v/\mu)g_Z)$. Thus, they should only be interpreted as an alternative way to present the relations between the warranted and autonomous consumption growth rates. In particular, it is not legitimate to interpret the situation $Y_0 = Z_0/(s - (v/\mu)g_Z)$ as representing the equilibrium level of output determined by the supermultiplier growth model. Although it seems to be so (see equation (23) above), in fact, as we already asserted, the expression is only an alternative way to present the situation according to which the warranted and the autonomous consumption growth rates happens to be equal to each other at $t = 0$. The

[40] For the sake of the discussion in this section, let us suppose here that $\delta = 0$, which implies that net and gross figures for output, income, profits, investment and savings are equal to each other.

level of (normal) output at $t = 0$ is determined by the level of capacity output and, therefore, does not depend on the level of aggregate demand and it is not determined by the RHS of the equation above. In fact, as our argument above shows, aggregate output and demand are equal to each other throughout time, including $t = 0$, in the three situations previously defined.

Now we can analyze the behavior of the warranted rate through time depending on the initial relations between that rate and the rate of growth of autonomous consumption previously described. Thus if we have initially $g_0 > g_Z$ then, according to the equation above, $\dot{g} = (\mu/v)(g_0 - g_Z)(Z_0/Y_0) > 0$ at $t = 0$. It follows that $g_t > g_Z$ for all $t > 0$ and, therefore, we obtain $Z_t/Y_t \to 0$ and $g \to (s/v)\mu$ as $t \to \infty$. That is, in this case, the warranted rate asymptotically converges to $(s/v)\mu$, the value of the warranted rate without autonomous consumption.[41] On the other hand, if initially $g_0 < g_Z$ then $\dot{g} = (\mu/v)(g_0 - g_Z)(Z_0/Y_0) < 0$. As a result we have $g_t < g_Z$ and $\dot{g} < 0$ for an interval of time immediately after $t = 0$. In this case, eventually we must have $g_t < 0$ and, therefore, after some time with the economy growing at a negative rate, we would achieve a zero level of aggregate output. So in these circumstances, the economy collapses until it achieves a floor where the level of output is equal to zero. Finally, if we have initially $g_0 = g_Z$ then, from the equation representing the dynamics of the warranted growth rate, we would obtain that the warranted growth rate would be constant through time (i.e. we would have $g_t = g_0 = g_Z$ for all t). So, if the warranted growth rate happens to be equal to the growth rate of autonomous consumption at $t = 0$ then such equality would be maintained through time. It is important to be remarked, however, that the latter situation should not be interpreted as an equilibrium state where the rate of growth of autonomous consumption determines the rate of growth of output. Actually, the latter situation is only the result of a coincidence. Accordingly, the divergence of the warranted growth rate from the rate of growth of autonomous consumption obtained in the other two situations previously analyzed cannot also be interpreted as an evidence of the instability of a supposed equilibrium growth path according to which output would grow at a rate determined by the pace of expansion of autonomous consumption. [42] More generally, the assumption that the growth rates of aggregate demand and actual output are always the ones required to continuously maintain the normal utilization of productive capacity implies that the specific model here investigated is in fact a formalization of the economic growth process based on the validity of Say's law. Thus the model under analysis specifically shows the characteristics of the growth path based on Say's law when one component of aggregate demand grows at rate independent from the rate of growth of aggregate output and demand.

Hence, although Serrano's (1995 and 1996) declared intention was to provide an alternative heterodox growth model based on "the validity of the Keynesian-Kaleckian principle of effective demand in the long run",[43] his supermultiplier growth model has been represented by the model described above, that is, as we saw, a growth model based on Say's law. This is the source of the misleading interpretation of Serrano's supermultiplier model found in Trezzini (1998), Schefold (2000) and Barbosa-Filho (2000).[44]

To show that this is the case, from the equation of the warranted rate it follows that

$$I_t = \left(\frac{v}{\mu}\right) g_t Y_t = sY_t - Z_t \quad .$$

The equation above determines the level of investment required to maintain the normal utilization of productive capacity through time. Now if, besides assuming normal utilization of productive capacity at $t = 0$, we also suppose that capitalist firms perfectly foresee the behavior of the warranted rate of growth

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[41] Observe that while the warranted rate asymptotically converges to $(s/v)\mu$, the growth trajectory presents an implausible pattern according to which the investment share of output increases throughout the whole path converging to the value of the marginal propensity to save (i.e. $I_t/Y_t \to s$)

[42] Note that notwithstanding the fact that the model is based on the notion of a warranted growth rate, it does not deal with the problem of instability of the warranted growth rate pointed out by Harrod. Actually, in the model it is supposed that the actual growth rate of output is always equal to the warranted rate throughout the growth paths generated by the model.

[43] According to Serrano (1995 and 1996) the trend level and rate of growth of autonomous demand is supposed to determine the trend level and rate of growth of capacity output.

[44] Park (2000) also represents the supermultiplier model along these lines, but the focus of his paper is on the dependence of the warranted rate of growth on the rate of growth of autonomous demand. From the existence of this dependence the author concludes that effective demand is an important element in the process of economic growth. Our analysis, however, shows that the model under analysis (but not the supermultiplier model) is based on Say's Law. Thus, such model is not compatible with the notion of effective demand as a main determinant of the level of aggregate output.

according to the model presented above,[45] it follows that the expression $I_t = (v/\mu)g_tY_t$ can be interpreted as an unlagged version of the accelerator investment function. In this case, the investment level determined by the accelerator investment function would be able to maintain the normal utilization of productive capacity throughout the entire growth path. But note that, under the stringent conditions suggested, the investment level determined by the unlagged accelerator investment function is nothing more than the level of required investment determined by normal capacity savings. Thus the level of investment is not independent from the level of normal capacity savings, and we are still under the rule of Say's Law.

Trezzini (1998), Schefold (2000) and Barbosa-Filho (2000) used this specific version of the accelerator investment function, combined with the assumptions of continuous equilibrium between aggregate demand and output and of initial normal utilization of productive capacity, in their representation and interpretation of Serrano's Supermultiplier.[46] In fact, substituting such investment function into the equilibrium condition between aggregate demand and output, we obtain the following first order ordinary differential equation

$$\left(\frac{v}{\mu}\right)\dot{Y} - sY_t + Z_0e^{g_Zt} = 0$$

Apart from differences in notation, this is essentially the same differential equation used by Schefold (2000, p. 345) and Barbosa-Filho (2000, p. 31) to analyze the dynamic behavior of the supermultiplier model. Following Schefold (2000, p. 345),[47,48] the general solution of the above equation is given by

$$Y_t = \left(\frac{Z_0}{s - \left(\frac{v}{\mu}\right)g_Z}\right)e^{g_Zt} + \left(Y_0 - \frac{Z_0}{s - \left(\frac{v}{\mu}\right)g_Z}\right)e^{\left(\frac{s}{v}\mu\right)t}$$

Recalling that we are supposing that $(s/v)\mu > g_Z$, the above equation provides us with the same solutions obtained previously depending on the three possible initial conditions. Thus if $Y_0 > Z_0/(s - (v/\mu)g_Z)$ (i.e., as we saw, when $g_0 > g_Z$) then the economy would converge asymptotically to an equilibrium path where $g = (s/v)\mu$. On the other hand, if $Y_0 < Z_0/(s - (v/\mu)g_Z)$ (i.e., as we saw, when $g_0 < g_Z$) then the economy would collapse until it reaches the floor associated with a zero level of economic activity. Finally, when we have initially a situation according to which $Y_0 = Z_0/(s - (v/\mu)g_Z)$ (i.e., as we saw, when $g_0 = g_Z$) the economy would be, by a fluke, in an equilibrium path where $g = g_Z$ for all $t \geq 0$. Thus the model used by Trezzini, Schefold and Barbosa-Filho describes the dynamics of the rate of growth of output

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[45] It should be remarked that what we have here is an assumption that is stronger than the assumption of perfect foresight on the part of capitalist firms. In fact, we are supposing that, in their investment decisions, capitalist firms are able to behave consistently with the model's assumption according to which aggregate demand exactly grows at a rate required to maintain normal capacity utilization throughout the whole growth path. In this sense, capitalist firms would have to be interested in predicting the behavior of warranted rate of growth, and not the behavior of aggregate demand growth rate as such. There is no reason to believe that this would be the kind of behavior induced by capitalist competition.

[46] Trezzini (1995 and 1998) claims that the model under discussion also represents Hicks' supermultiplier model (Hicks, 1950). However, Hicks uses a lagged version of the accelerator investment function to study the cyclical behavior associated with the multiplier/accelerator interaction under the constraints of a full employment ceiling and zero gross investment floor. In this sense, it is important to note that Hicks' supermultiplier model generates fluctuations of the rate of capacity utilization and not a fully adjusted growth path. Thus, the model with continuous normal utilization of productive capacity under analysis does not adequately represent Hicks' supermultiplier as well. Nevertheless, it confirms and extends to the case of an economy with an autonomous demand component Garegnani's (Garegnani, 1992) argument according to which the combination of continuous normal utilization with a given level of income distribution is incompatible with the application of the principle of effective demand to the analysis of the economic growth process.

[47] Schefold (2000, p. 345) refers to the solution for the above differential equation that can be found in Allen (1968, p. 343). In this connection, it is interesting to note that Allen calls the model under discussion "Equilibrium accelerator-multiplier model". This designation tries to capture the fact that with the use of an unlagged accelerator investment function the model can only discuss the dynamics of the equilibrium growth rate (the modified warranted growth rate). Thus, such model would be unable to deal with the (harrodian) disequilibrium problems associated with the divergence of the actual output/demand growth rate from the equilibrium growth rate. In fact, Allen (1968, p. 346) makes use of a disequilibrium accelerator-multiplier model, with a lagged accelerator investment function, in order to deal with the possible disequilibrium process between actual and equilibrium growth rates in the presence of a growing autonomous expenditure. In this sense, we think that is not legitimate to refer to the harrodian knife edge instability process in the context of the model under discussion as Schefold (2000, p. 345) does.

[48] Barbosa-Filho (2000, p. 31), discusses the dynamic behavior of the model under analysis based on the nonlinear first order differential equation of the Riccati type deduced from the differential equation for the level of output presented above. The equation he uses is essentially the same as the one we used above to discuss the behavior of the model. That is, in our notation, he analyzes the behavior of the model according to the equation $\dot{g} = (g_t - g_Z)((s/v)\mu - g_t) = -g_t^2 + (g_Z + (s/v)\mu)g_t - (s/v)\mu g_Z$.

that allows the continuous maintenance of the equilibrium between aggregate output and demand and the continuous normal utilization of productive capacity (i.e. the harrodian warranted growth rate). That is, the model under analysis would always generate a *continuous* fully adjusted growth path. Therefore, it is not surprising that if such model could be considered a reasonable representation of the supermultiplier growth model, then the only possible conclusion would be that the latter model would be unable to provide a plausible demand-led explanation for the process of economic growth. However our analysis in the present paper shows that such conclusion is based on a misleading representation of the supermultiplier growth model. The model used to represent the supermultiplier growth model is, in reality, a model based on the Say's Law, a model according to which the level of investment is determined by level of normal capacity savings.

Contrary to the analysis based on the misleading representation of the supermultiplier growth model provided by the model discussed above, in our presentation of the supermultiplier model we showed that the level of aggregate demand determines the level of aggregate output. Further, our previous analysis of the dynamic behavior of the model clearly has shown that it generates trajectories that are *not* continuously fully adjusted. As can be verified in Figures 2 and 3 and the corresponding discussion above, the model produces growth paths characterized by deviations of the actual rate of capacity utilization from its normal level. In fact, according to the model, it is exactly the influence of such deviations on the investment share of output (i.e. the marginal propensity to invest) that makes it possible for the model to explain the eventual convergence of the actual growth path of output to the trend expansion path of autonomous demand and also the tendency towards normal utilization of productive capacity. In this sense, the divergences between actual and normal rates of capacity utilization are an *essential* element in the theoretical account of the adjustment process of capacity to aggregate demand provided by the supermultiplier growth model. Hence, according to the latter, the reaction of the rate of capital accumulation to changes in the trend rate of autonomous demand growth in a capitalist economy relies on both, changes in the rate of capacity utilization and changes in the investment share of output. Actually, it should be noted, the model shows that these changes are complementary to each other. For, in the case of an unexpected increase in the trend rate of autonomous demand growth, the first reaction of the economy is an increase in the rate of capacity utilization and, accordingly, a relatively fast increase in the rate of capital accumulation, which are only possible because the system operates with margins of planned spare capacity. On the other hand, it is the tendency for an increase in the investment share of output that ultimately explains the long run adjustment of the rate of capital accumulation to the trend rate of autonomous demand growth; while, at the same time, such tendency restores the margins of planned spare capacity and, hence, the ability of the economy to react to further changes in the trend rate of autonomous demand growth. Therefore, the combined changes in the rate of capacity utilization and in the investment share of output subjacent to the process of adjustment of capacity to aggregate demand associated with the supermultiplier growth model offer a plausible demand-led explanation for the long run process of economic growth. In this sense, the supermultiplier growth model proves to be able to extend the applicability of the Keynesian-Kaleckian principle of effective demand to the context of analysis of long run economic growth and that this extension is fully compatible with the tendency towards the normal utilization of productive capacity.

Observe also that the assumption of continuous normal utilization adopted by Trezzini, Schefold and Barbosa-Filho in their representation of the supermultiplier growth model implies that the actual growth rates of output and capital stock are always equal to the warranted rate of growth. In contrast, according to our analysis of the supermultiplier growth model, these rates are only equal to each other in the equilibrium path of the model, where they are all determined by rate of growth of autonomous consumption (see equation (18) above). Outside the equilibrium path, these rates are, in general, different to each other. The warranted rate of growth is equal to the notional rate of capital accumulation that would be obtained if capacity utilization were normally utilized. That is, from equation (2) and recalling that we are supposing in this section that δ=0, we would have in this case

$$g_t^w = \left(\frac{h_t}{v}\right)\mu = \left(\frac{s - z_t}{v}\right)\mu$$

It follows that the supermultiplier growth model determines the warranted growth rate endogenously. The latter depends, *ceteris paribus*, on the value of the investment share of output and, thus, it varies according to changes in the investment share of output along the process of adjustment of capacity to demand. Moreover, as a comparison with equation (2) demonstrates, the warranted rate is related to the actual rate of capital accumulation as follows

$$g_{Kt} \gtreqless g_t^w \text{ as } u_t \gtreqless \mu$$

We can verify that the actual rate of capital accumulation exhibits more variability than the warranted rate of growth since, although both rates positively depend on the investment share of output, the former also depends positively on the actual rate of capacity utilization. This confirms the importance of the variability of the actual rate of capacity utilization in the process of adjustment of capacity to demand. The variability of the actual rate of capacity utilization allowed by the existence of margins of spare capacity increases the responsiveness of the pace of capital accumulation and, thus, of the expansion of productive capacity to the stimulus provided by the demand-led growth process. Further, the two rates are only equal to each other when the rate of capacity utilization happens to be equal to its normal level, a situation that is not sustainable outside the equilibrium path. But when the latter path is stable, the values of the three rates of growth tend to converge to the rate of growth of autonomous consumption. Figure 4 illustrates the relationship between the warranted rate of growth, the rate of growth of output/demand and the actual pace of capital accumulation discussed above. Finally, it should be remarked that, in the context of the supermultiplier growth model, the warranted rate of growth loses much of its significance. For, on the one hand, outside the equilibrium path nothing grows at the warranted rate and the latter does not exert any influence on the dynamic behavior of model. On the other hand, in the equilibrium path of the model, the rate of growth of autonomous consumption determines the warranted rate of growth and, therefore, the latter does not have a determining role in the equilibrium path also.

Figure 4 about here

The ability of the supermultiplier growth model to provide a plausible explanation for the economic growth process from a demand-led point of view has also been questioned by Palumbo & Trezzini (2003) on the ground that the model supposedly requires the equality between the time average value of the actual rate of capacity utilization and its normal level. Here, however, we shall argue that this requirement is simply too strong and is not necessary for the validity of the model's conclusions. As argued by Vianello (1989) in relation to the analysis of the gravitation process of market to normal prices, requiring the equality between the time average of market prices and normal prices implies the validity of the implausible hypothesis of reciprocal compensation of the divergences between them. Since, as convincingly argued by Ciccone (1990), the gravitation of market to normal prices would be faster than the adjustment of capacity to demand, then it seems to be reasonable to believe that the requirement under discussion is even more stringent when applied to the investigation of the latter adjustment process. Such would be the case because the relation between average and normal rates of capacity utilization is almost always sensitive to the initial conditions present in the dynamic analysis, to the arbitrary time period used to calculate the time average of actual values and to the prevalence of any non-random pattern of change in exogenous variables. Actually, the restrictive and arbitrary nature of the requirement under discussion is evident from the fact that it fails to distinguish between an unstable and a stable adjustment process of capacity to demand, both of which would not be able to meet such requirement in normal conditions. Thus, in general, we are only able to assert the existence of a *tendency* for the actual rate of capacity utilization to converge to its normal (or equilibrium) level[49] which is exactly what the stability analysis pursued above allows us to assert. Requiring more than that would impose a level of precision to the analysis that is not achievable in general, being a kind of misplaced exigency of precision.

Finally, Palumbo & Trezzini (2003) also questions whether the supermultiplier model would be able to plausibly guarantee the existence of a tendency for the actual rate of capacity utilization to converge to its normal (or equilibrium) level. First, they argue that such tendency would require the very strong assumptions of perfect foresight and perfect knowledge on the part of capitalist firms in their investment decisions. In contrast, our analysis of the supermultiplier model shows that these hyper-rational behavioral assumptions are not needed. Actually, our analysis of the investment process is not intended to deal with the behavior of individual capitalist firms in their particular or individual process of capital accumulation. The investment function adopted in this paper is meant to capture the influence of capitalist competition process on the

[49] Of course, when more specific knowledge is available from empirical inquiries, we may also be able to infer something about the time profile and intensity characterizing such tendency.

behavior of aggregate investment.[50]. So we suppose that if the actual rate of capacity utilization is above the normal level (i.e. if the margins of spare capacity are on average below their planned levels), aggregate capitalist investment would expand at a faster rate than aggregate demand (and output) because capitalist firms would try restore the levels of planned spare capacity in order to avoid the possibility of losing market share for not being able to meet peak levels of demand. On the other hand, if the actual rate of capacity utilization is below the normal level (i.e. if the margins of spare capacity are on average above their planned levels), we assume that capitalist investment would expand at a lower rate than aggregate demand (and output) since, in this situation, capitalist firms would be able to increase their profitability without compromising their market shares. As can be verified, our reasoning does not involve any of the hyper-rational assumptions suggested by Palumbo and Trezzini, and yet we are able to show that, under the conditions stated in the last section, the model produces a tendency towards normal utilization of productive capacity.[51] Secondly, Palumbo and Trezzini argue that the tendency towards normal utilization of productive capacity can be countervailed by other forces associated with the very process of capitalist competition that would trigger such tendency. In particular, according to them, the adjustment of capacity to demand affects aggregate demand through its effect over aggregate investment, which, according to the authors, would be able to frustrate the latter adjustment process. Note, however that this is exactly the type of interaction associated with the multiplier/accelerator mechanism that is built in the supermultiplier growth model. Thus, we are already taking care of such countervailing force when we discuss the dynamic behavior and the stability of the equilibrium path of the model. Indeed, we have seen that, although the process of adjustment of capacity to demand unleashed by capitalist competition leads the economy in the right direction, the intensity of the process may prevent the actual rate of capacity utilization to converge to its normal level. Nonetheless, we also pointed out above that the latter convergence is assured if the stability condition is met, which would be the case if we have a sufficiently low value of the parameter γ. Therefore, in this case, the tendency towards normal utilization of productive capacity is not prevented by the multiplier/accelerator interaction and we can overcome this last objection by Palumbo and Trezzini.[52]

5. A Comparison with alternative models[53]

In this section we shall compare the main results obtained in our discussion of the supermultiplier growth model with alternative heterodox growth models that also deal with the relationship between economic growth, income distribution and effective demand. More specifically, we will compare the supermultiplier, Cambridge and neo-Kaleckian growth models trying to point out their similarities and, more importantly, their main differences. The comparison will be made within the same simplified analytical context used in the discussion of the supermultiplier model. Thus, the following assumptions will be maintained: we have a closed capitalist economy without government; aggregate income is distributed in the form of wages and profits; the method of production in use requires a fixed combination of homogeneous labor input with homogeneous fixed capital to produce a single output; there is constant returns to scale and no technological progress; and there exists a permanent labor surplus. Moreover, as occurred in the case of

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[50] With this procedure we try to avoid the problematic use of the representative agent type of reasoning on aggregate investment theory. The problem with this approach follows from the fact that, although aggregate capitalist investment is the result of a decentralized decision process by individual capitalist firms, their individual investment decisions are intrinsically connected by the functioning of the process of inter-capitalist competition and the existence of these interconnections exerts an important influence on individual capitalist decisions.

[51] This part of the argument seems to rely on the misleading representation of the supermultiplier model proposed by Trezzini (1998) that was discussed above.

[52] Besides the latter objection, Palumbo and Trezzini (2003) also argue that capitalist competition induces the process of technical change and the latter could hamper the tendency towards normal capacity utilization. In the present paper, we are not dealing with the connections between the process of technical change and of adjustment of capacity to demand. In fact, in order to simplify our analysis, we assumed away the existence of technical change from the start. Nevertheless, it is our position that the technical change process does not impair the adjustment of capacity to demand when we are thinking in terms of the economy as whole or in terms of whole industries, contrasted with the situation of individual capitalist firms. In this connection, we can refer the reader to the work by Cesaratto, Serrano and Stirati (2003) which specifically analyzes the relationship between technical change and economic growth from the point of view of the supermultiplier growth model.

[53]This section is based on and confirms the main findings contained in the more general comparative analysis presented in Serrano (1996, chapter 3). See also Serrano and Freitas (2007), for an early discussion of the main differences between the supermultiplier, Cambridge and neo-Kaleckian growth models containing similar results to the ones here obtained.

the supermultipier model, the alternative growth models will be presented in their simplest form in order to facilitate the comparisons and to draw our attention to the different theoretical closures provided by each model.

Let us start with Cambridge growth models. Maintaining the hypothesis of permanent labor surplus, the version of the Cambridge growth model presented here[54] supposes that the level of aggregate output is determined by the level of capacity output. So, contrary to the supermultiplier growth model, the equilibrium level of aggregate output is not determined by effective demand, but by the supply constraint associated with full utilization of productive capacity. That is, we have

$$Y_t = Y_{Kt} = \frac{1}{v} K_t$$

and thus

$$u^* = 1$$

Also differently from the supermultiplier growth model, there is no autonomous consumption and, additionally to the consumption induced by the wage bill, there is also a component of aggregate consumption induced by total current profits. We retain the assumption that the propensity to consume out of wages c_w is equal to one (i.e., $c_w = 1$) and suppose that the propensity to consume out of profits c_π is a positive constant and has a value lower than one (i.e., $0 < c_\pi < 1$). Thus the consumption function is given by the following expression

$$C_t = \big(\omega_t + c_\pi(1 - \omega_t)\big)Y_t$$

where $\omega_t + c_\pi(1 - \omega_t)$ is the marginal propensity to consume, which is equal to the average propensity to consume since there is no autonomous component in the consumption function. Moreover, in contrast to the supermultiplier growth model, aggregate investment is an autonomous expenditure in this version of the Cambridge model, and we suppose, for the sake of simplicity, that investment expands at an exogenously determined rate g_I>0. From these hypotheses we obtain the aggregate demand equation of the Cambridge model

$$D_t = \big(\omega_t + c_\pi(1 - \omega_t)\big)Y_t + I_t$$

Since aggregate output is determined by capacity output, the equilibrium between aggregate demand and output requires that the former adjusts to the latter. In the Cambridge model such an adjustment involves a change in the marginal propensity to consume through the modification of income distribution (i.e., of the wage share). Thus, according to the model, a situation of excess aggregate demand (supply) raises (reduces) the general price level and, with a relatively rigid nominal wage, it causes a decrease (increase) in real wages. So, given labor productivity, the excess aggregate demand (supply) causes a reduction (an increase) in the wage share ω and, since $0 < c_\pi < c_w = 1$, it causes a decline (rise) in the marginal propensity to consume. Therefore, the adjustment implies a tendency for the establishment of an equilibrium between aggregate demand and supply at the level of output capacity (i.e. the level of potential output), which in equilibrium determines the level of aggregate demand.[55] At the same time, in equilibrium between aggregate demand and output capacity the model endogenously determines the income distribution between wages and profits. So, in the Cambridge model, the determination of a required level of income distribution allows the adjustment of aggregate demand to potential output, while in the supermultiplier growth model it is the appropriate change in the level of aggregate output that explains the adjustment of aggregate output to the level of aggregate demand.

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[54] See Robinson (1962). For similar formalizations of the Cambridge growth model see Dutt (1990 and 2011) and Lavoie (1992).

[55] Note however that the adjustment mechanism based on endogenous modifications of income distribution only guarantees the adjustment of aggregate demand to the level of potential output and not necessarily the adjustment of the rate of capacity utilization to its full capacity level. As Kaldor (1955-6) argued in his seminal discussion of such adjustment mechanism, the latter can be viewed as an alternative to the usual Keynesian adjustment based on variations of the level of aggregate output leading to the equilibrium between aggregate output and demand. In the present version of the Cambridge growth model, the maintenance of a full utilization of capacity resulting from the operation of the adjustment mechanism involving changes in income distributions is a consequence of the assumption that the binding supply constraint in the economy is the availability of capital. If the operative supply constraint were the full employment of the labor force, then the adjustment of aggregate demand to potential output based on endogenous changes in income distribution would not guarantee the full (or the normal) utilization of the available capital stock. In Kaldor's full employment growth models (c.f. Kaldor, 1957, 1958 and 1962) the investment function is responsible for the adjustment of the rate of capacity utilization. For an analysis of this role of the investment functions in Kaldor's full employment growth models see Freitas (2002, chapter 2; and 2009) and Palumbo (2009, pp. 343-345).

Now, in equilibrium between aggregate output and aggregate demand, we have $Y_t = \big(\omega^* + c_\pi(1-\omega^*)\big)Y_t + I_t$ and, thus

$$S_t^* = s^* Y_t = s_\pi(1-\omega^*)Y_t = I_t$$

where $s_\pi = 1 - c_\pi$ is the marginal propensity to save out of total profits. The last equation shows that, according to the Cambridge model, aggregate investment determines aggregate (capacity) savings, although, as we saw above, the level of potential output determines the level of real aggregate demand. Furthermore, dividing both sides of the last equation by the level of aggregate output, we have an expression relating the saving ratio to the investment share of output as follows

$$\left(\frac{S_t}{Y_t}\right)^* = s^* = s_\pi(1-\omega^*) = \frac{I_t}{Y_t}$$

From the above equation we can see that in the Cambridge model the investment-output ratio determines the saving ratio $s^* = s_\pi(1-\omega^*)$, a result shared with the supermultiplier growth model. But, since in the Cambridge model there is no autonomous consumption component, the marginal and average propensities to save are equal to each other. Hence, the burden of the adjustment of the saving ratio to the investment share relies on required modifications in the *marginal* propensity to save and, therefore, on appropriate changes in income distribution.

Let us now discuss the determination of the equilibrium level of the investment share of output. From the assumption of full capacity utilization, the growth rate of output is given by the rate of capital accumulation (i.e., $g_t = g_{Kt}$). Thus, if we have initially $g_{Kt} < g_I$ ($g_{Kt} > g_I$), then we also have $g_t < g_I$ ($g_t > g_I$). It follows that the investment share of output would increase (decrease) and, according to equation (2) and with the rate of capacity utilization constant, the rate of capital accumulation would increase (decrease). Eventually this type of adjustment leads to the convergence of the rate capital accumulation to the investment growth rate.[56] Therefore in the equilibrium path of the model we have

$$g^* = g_K^* = g_I$$

Substituting this last result in equation (2), solving for the investment share of output and recalling that $u^* = 1$, we have the equilibrium value of the investment share given by

$$\left(\frac{I_t}{Y_t}\right)^* = v(g_I + \delta)$$

Thus, according to the Cambridge growth model, a higher (lower) rate of growth of investment implies higher (lower) equilibrium growth rates of capacity output and output. Moreover, with a constant capacity utilization rate, this result is possible because the equilibrium level of the investment share is positively related to the growth rate of investment and, therefore to the equilibrium growth rates of output and capacity output. This last result is shared by the supermultiplier growth model although the process by which it is achieved is different from the process featured in the Cambridge growth model.

It is important to note, however, that the growth determining role of investment in the Cambridge model follows from its effect on productive capacity and *not* from its influence on aggregate demand. As we saw above, in the Cambridge growth model, the level of capacity output determines the level of aggregate demand. Thus, a higher (lower) growth rate of investment only causes a higher (lower) rate of output growth because it raises (reduces) the level of the investment share of output and, through this way, the pace of capital accumulation and the growth of capacity (or potential) output. Therefore, the Cambridge model displays, in fact, a supply (capacity) constrained pattern of economic growth, and hence, it is *not* a demand-led growth model.

Finally, we can substitute the expression for the equilibrium level of the investment share in the equation relating the investment share and the saving ratio. Doing so we can obtain the following result

[56] The adjustment process between the rate of capital accumulation and the expansion rate of investment can also be explained as follows. As we saw before, $\dot{g}_{Kt} = (g_{Kt} + \delta)(g_{It} - g_{Kt})$, so, since $g_{It} = g_I$, if initially $-\delta < g_{K0} \neq g_I$, then the capital accumulation rate would converge to the investment growth rate because from the last differential equation we can verify that $\dot{g}_{Kt} \gtreqless 0$ according to $g_{It} \gtreqless g_{Kt}$.

$$(1-\omega^*) = \frac{v(g_I+\delta)}{s_\pi} \quad (35).$$

The last equation shows us the determinants of the required level of income distribution in the Cambridge model. In particular, we can see that, given v, δ and s_π, a higher (lower) growth rate yields a higher (lower) profit share of output. Thus, according to the Cambridge growth model and in contrast with the supermultiplier model, there exists a theoretically necessary *a priori* relationship between income distribution and economic growth, the profit share of output (the wage share) is positively (negatively) related to the rate of economic growth. In the Cambridge model, such relationship is necessary for obtaining a growth path characterized by the equilibrium between aggregate demand and aggregate output with a constant rate of capacity utilization. Therefore, the closure provided by the Cambridge model requires the endogenous determination of an appropriate level income distribution, which must be compatible with various combinations of the values of the model's exogenous variables and parameters.[57]

The main features of the Cambridge growth model's equilibrium path discussed above can be represented in Figure 5, which may be compared with Figure 1 of the supermultiplier growth model. In Figure 5, the vertical dotted line represents the hypothesis that output capacity determines the level aggregate output, implying the full utilization of productive capacity (i.e. $u^* = 1$). The horizontal dotted line represents the equilibrium condition according to which the exogenous growth rate of investment determines the growth rates of output and of the capital stock. The point where these two lines intercept each other defines the equilibrium situation envisaged by the model. Observe that the existence of such equilibrium requires the endogenous determination of the equilibrium investment share of output and of the saving ratio as represented by the slope of the solid line corresponding to the rate of capital accumulation. Further, note the slope of the g_K line depends on the level of income distribution and, therefore, the existence of the equilibrium path relies on the endogenous determination of the required profit share (i.e. $1-\omega^*$) that allows the adjustment of the saving ratio to the investment share of output.

Figure 5 about here

We shall now compare the supermultiplier and the neo-Kaleckian growth models.[58] Like in the supermultiplier model, in the neo-Kaleckian growth model the level of aggregate demand determines the equilibrium level of aggregate output. Also, as occurs with the supermultiplier model, income distribution is exogenously determined and, therefore, cannot be part of the adjustment mechanism that allows the existence of the equilibrium path as in the Cambridge model. However, contrarily to the supermultiplier model, aggregate investment is autonomous and grows at an exogenously determined growth rate $g_I > 0$,[59] and also there is no autonomous component in aggregate consumption. In fact, in our representation of the basic neo-Kaleckian model we utilize the same specification for the consumption function used in the Cambridge model above. The only, but important, difference being that in the neo-Kaleckian model the wage share is exogenously determined and, accordingly, the marginal propensity to consume (equal to the average propensity to consume) is also an exogenous variable.

With these hypotheses, aggregate demand is given by the following expression

$$D_t = \big(\omega + c_\pi(1-\omega)\big)Y_t + I_t$$

and in equilibrium between aggregate demand and output we have

[57] Contrast equation (35) with equation (20) that represents the closure provided by the supermultiplier growth model.

[58] These models descend from Kalecki (1971) and Steindl (1952 and 1979) original contributions. The modern neo-Kaleckian model was presented originally by Dutt (1984) and Rowthorn (1981). See also Dutt (1990) and Lavoie (1992) for a formalization and comparison of the neo Kaleckian model with alternative growth models. For a detailed survey of the literature, see Blecker (2002).

[59] Note that our presentation of the neo-Kaleckian model has some minor differences in relation to the usual presentation of these models. The main difference is that we specify the investment function in terms of the determinants of the investment growth rate, whilst the usual neo-Kaleckian specification is in terms of the determinants of the desired rate of capital accumulation. Observe that this difference doesn't affect the equilibrium values of the model's endogenous variables because, since by definition $g_{It} = g_{Kt} + \dot{g}_K/(g_{Kt}+\delta)$ and since, in equilibrium, we have $\dot{g}_K = 0$, then in equilibrium we obtain $g_I^* = g_K^*$. Moreover, as we shall see shortly, our specification of the investment function doesn't affect also the equilibrium stability condition. Therefore, we claim that nothing essential is altered by our specification of the investment function of the neo-Kaleckian model.

$$Y_t^* = \left(\frac{1}{s}\right) I_t = \left(\frac{1}{s_\pi(1-\omega)}\right) I_t \qquad (36).$$

According to the last equation aggregate investment is the main determinant of the equilibrium level of output. Further, given income distribution (i.e. the wage share), the value of the multiplier $1/s = 1/\left(s_\pi(1-\omega)\right)$ is constant. Thus, in the neo-Kaleckian model, as can be verified from the last equation, the pace investment expansion determines the equilibrium output growth rate of the economy for a given level of income distribution. That is, we have

$$g^* = g_I$$

So like the supermultiplier growth model, the neo-Kaleckian model produces a demand-led growth pattern. But while in the supermultiplier model we have a consumption-led growth pattern, the neo-Kaleckian model generates an investment-led growth pattern.

From equation (36) we can also verify that

$$S_t^* = sY_t^* = s_\pi(1-\omega)Y_t^* = I_t$$

and

$$\frac{I_t}{Y_t^*} = \frac{S_t^*}{Y_t^*} = s = s_\pi(1-\omega)$$

Thus, according to the first of the two equations above, the level aggregate savings adjusts to the level of aggregate investment through the variation of the level of aggregate output, the only endogenous variable in the equation. Note however that, as we can verify from the second equation above, given s_π and ω, the saving ratio, equal to the marginal propensity to save s, is exogenously determined and thus it determines the investment share of output in the neo-Kaleckian model. This feature of the model contrasts with the related result obtained from the Cambridge and supermultiplier growth models. Indeed, as we pointed out before, in these latter models the investment share of output determines the saving ratio. In the Cambridge model this result follows from changes in income distribution and in the marginal propensity to save, whilst in the supermultiplier growth model the same result follows from the existence of an autonomous component in aggregate consumption which makes the saving ratio endogenous even though income distribution and the marginal propensity to save are given exogenously. In contrast, the neo-Kaleckian model assumes that income distribution (and thus the marginal propensity to save) is exogenously determined and that there is no autonomous consumption component, which implies, in combination with the other assumptions of the model, the exogeneity of the saving ratio.

Now, since the saving ratio is an exogenous variable and it determines the investment share of output, then the latter variable cannot be changed according to the requirements of the pace of economic growth. So, in contrast with the supermultiplier growth model, according to the neo-Kaleckian growth model, given income distribution, a change in the investment and output growth rates does not have any effect on the equilibrium value of the investment share of output. More importantly, from equation (2) we can verify that in the neo-Kaleckian model the rate of capital accumulation can only be reconciled with the output/demand growth rate if the rate of capacity utilization is properly adjusted. Indeed, since in the neo-Kaleckian model the saving ratio determines the investment share of output, then, according to equation (2), the rate of capital accumulation is given by

$$g_{Kt} = \left(\frac{s_\pi(1-\omega)}{v}\right) u_t - \delta$$

On the other hand, we saw that in the neo-Kaleckian model the growth rate of investment determines the equilibrium growth rate of output, so we have that $g^* = g_I$. Thus, using these results in equation (3), we obtain the following differential equation for the dynamic adjustment of the rate of capacity utilization

$$\dot{u} = u_t \left(g_I - \left(\frac{s_\pi(1-\omega)}{v}\right) u_t + \delta\right) \qquad (37)$$

The last equation shows that if the investment growth rate is higher (lower) than the rate of capital accumulation, then the rate of capacity utilization increases (declines) and this raises (reduces) the pace of capital accumulation. As a result, the capital accumulation rate converges to the investment growth rate through changes in the rate of capacity utilization. Therefore, in the equilibrium path of the neo-Kaleckian model we have

$$g_K^* = g_I = \left(\frac{s_\pi(1-\omega)}{v}\right)u^* - \delta$$

Now, solving the last equation for the equilibrium rate of capacity utilization we obtain

$$u^* = \frac{v(g_I + \delta)}{s_\pi(1-\omega)} \tag{38}$$

Equation (38) shows the determinants of the required rate of capacity utilization u^* in the simplified version of the neo-Kaleckian model here presented. This latter rate is the one that reconciles the rate of capital accumulation with the pace of economic growth and, therefore, allows the existence of an equilibrium growth path in the model. Observe that, in its role as an adjusting variable, the equilibrium rate of capacity utilization has to able to assume any value between zero and one, however implausible it may be. Therefore the neo-Kaleckian model is not compatible with the related notions of planned spare capacity and normal (or desired) capacity utilization rate. Indeed, if we suppose the existence of a normal rate of capacity utilization, the closure provided by the model implies that it would be possible to have large and persistent deviations of the equilibrium rate of capacity utilization from its normal level and also that such divergence would not have any repercussion on capitalist investment decisions.[60] It is important to be remarked that the required long run endogeneity of the equilibrium rate of capacity utilization does not depend on the particular specification for the investment function adopted here, being in fact valid for all the usual specifications of the investment function in neo-Kaleckian models.[61] Actually, the necessity concerning the variability of the equilibrium rate of capacity utilization follows from the specification of the consumption function and *not* from the particular formulation of the investment function adopted in the model. As we argued above, it is the rigidity of the investment share of output implied by the exogeneity of the saving ratio that leads to the requirement of the long run variability of the equilibrium capacity utilization rate.

Further, note that admitting the possibility of an adjustment of the actual rate of capacity utilization to the normal one in the context of a neo-Kaleckian model only leads to an instability process of the Harrodian type.[62] Indeed, suppose, following Skott (2008), that in trying to adjust the actual rate of capacity utilization to its normal level, capitalist firms change the investment growth rate according to $\dot{g}_I = \eta(u_t - \mu)$, $\eta > 0$. Thus, in the equilibrium path of this particular model (i.e. with $\dot{g}_I = \dot{u} = 0$), we would have $u^* = \mu$ and $g_I^* = (s_\pi(1-\omega)/v)\mu - \delta$. Now, if initially we have $u_0 = \mu$ and $g_{I0} \gtreqless g_I^*$, then, according to equation (37), we would have $\dot{u} \gtreqless 0$, which implies that thereafter we would have $u_t \gtreqless \mu$ and, hence, $\dot{g}_I \gtreqless 0$ and $g_{It} \gtreqless g_I^*$. So the equilibrium rate of growth would be unstable. Thus, according to the neo-Kaleckian growth model, we would have a dilemma: either, on the one hand, we assume way the possibility of an adjustment of the actual rate of capacity utilization towards its normal level and admit the possibility of obtaining an equilibrium path with an implausibly high or low equilibrium rate of capacity utilization or, on the other, we allow an adjustment of the actual to the normal rate of capacity utilization and obtain an unstable growth trajectory as we just saw. Observe, however, that the dilemma exists only if we restrict ourselves to the set of assumptions of the neo-Kaleckian model. In fact, once we admit the existence of an autonomous component in aggregate consumption the saving ratio becomes an endogenous variable and the

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[60]Compare equation (38) with the theoretical closure associated with the supermultiplier growth model as represented by equation (20) above.

[61]For instance, the result under discussion is valid for the investment function given by equation (39) (below in the text). Indeed, the endogenous character of the equilibrium rate of capacity utilization and its implications are maintained as can be verified from equation (40) (below in the text) which is the equation that shows the determinants of the equilibrium rate of capacity utilization corresponding to the investment function represented by equation (39). The same point is valid for other investment functions that frequently appear in the neo-Keleckian growth literature and, in particular, it is valid in the case of the investment function given by $g_{It} = \alpha + \beta(u_t - \mu)$ with $\alpha, \beta > 0$ and μ exogenous, Note that in the latter investment function the normal rate of capacity utilization appears as an argument. Nonetheless, the endogenous character of the equilibrium rate of capacity utilization is also maintained in this case and the corresponding value of the equilibrium rate is given by $u^* = \big(v(\alpha + \delta - \beta\mu)\big)/(s_\pi(1-\omega) - \beta v)$.

[62] In this connection, see Hein, Lavoie and van Treeck (2012) for a survey on Harrodian instability and the tendency for the normal capacity utilization rate in neo-Kaleckian growth models.

investment share of output can change allowing the adjustment of the actual rate of capacity utilization to its normal level, as we have in the supermultiplier growth model.[63]

Now, similarly to what we have done in the cases of the supermultiplier and Cambridge growth models (in Figures 1 and 5 respectively), we can represent the main characteristics of the equilibrium path of the neo-Kaleckian growth model in Figure 6. In this Figure the slope of the g_K line is given since, according to the neo-Kaleckian growth model, income distribution is exogenously determined and, hence, the saving ratio is also exogenous to the model. So the horizontal dotted line represents the equilibrium condition according to which the growth rate of investment determines the growth rates of aggregate output and of the capital stock. The intercept point between this horizontal dotted line and the g_K line with a given slope determines endogenously the equilibrium rate of capacity utilization that clearly need not be compatible with a predetermined rate of capacity utilization corresponding to either the normal or the full utilization of productive capacity. Thus Figure 6 illustrates the role of the equilibrium rate of capacity utilization in the theoretical closure offered by the neo-Kaleckian growth model for the reconciliation of output and capital stock growth rates.

Figure 6 about here

Finally, we shall analyze the role of income distribution in the neo-Kaleckian growth model. Thus, in the very simple version of the model presented here there is no relationship between the pace of economic growth and the level of functional income distribution. Since the investment growth rate is supposed to be an exogenous variable in the model, the equilibrium rate of output growth does not affect and is not affected by the level of the wage share of output. Nevertheless, a change in the wage share has a level effect over the equilibrium value of output according to the simple neo-Kaleckian model under analysis. In fact, an increase (decrease) in the wage share, raises (reduces) the value of the multiplier $1/\big(s_\pi(1-\omega)\big)$ and, through it, such a change has a positive (negative) level effect on equilibrium output. These two latter results are shared with the supermultiplier growth model. On the other hand, in contrast with the latter model, from equation (38) we can see that, in the neo-Kaleckian model, an increase (decrease) in the wage share leads to an increase (a reduction) in the equilibrium rate of capacity utilization. A model that presents this type of result is classified in the neo-Kaleckian literature as a "stagnationist" or "wage led aggregate demand" model.[64]

We must say, however, that the independence between the pace of economic growth and income distribution in the simple version of the neo-Kaleckian model presented above is a direct consequence of the specific investment function adopted, which, as we saw, assumes the rate of investment growth to be completely exogenous. Indeed, if we consider the more usual formulations of the investment function in the neo-Kaleckian models, then we can obtain a causal relationship running from income distribution to the pace of economic expansion. So let us consider, for instance, a linear version of the investment function suggested by Marglin & Bhaduri (1990) and Bhaduri & Marglin (1990)

$$g_{It} = \alpha + \beta u_t + \rho(1-\omega) \qquad (39)$$

where $\alpha > 0$ is an autonomous component of the investment function, $\beta > 0$ is a parameter measuring the sensibility of the growth rate of investment to the capacity utilization rate, and $\rho > 0$ is a parameter measuring the sensibility of the investment growth rate with respect to the profit share (i.e. $(1-\omega)$). The introduction of an induced component βu_t in the investment function turns the investment growth rate into an endogenous variable of the model that positively depends on the rate of capacity utilization. With such specification for the investment function, the equilibrium value of the rate of capacity utilization is given by

$$u^* = \frac{v\big(\alpha + \delta + \rho(1-\omega)\big)}{s_\pi(1-\omega) - \beta v} \qquad (40)$$

[63] See Allain (2013) for a growth model that comes from the Kaleckian tradition pointing in the direction of a closure similar to the one provided by the supermultiplier growth model presented in this paper.

[64] See Blecker (2002) for discussion of the neo-Kaleckian models based on this type of classification.

From the equation above we can see that an equilibrium with a positive value for the rate of capacity utilization requires that $\beta < s_{\Pi}(1-\omega)/v$.[65] Also, it can be shown that, *ceteris paribus*, an increase (decrease) in the wage share raises (reduces) the equilibrium rate of capacity utilization.[66] Thus, the model still is classified as "stagnationist" or "wage led aggregate demand". Now, substituting this last result in the investment function, we obtain the equilibrium level of the investment growth rate as follows

$$g_I^* = \alpha + \beta\left(\frac{v\big(\alpha+\delta+\rho(1-\omega)\big)}{s_\pi(1-\omega)-\beta v}\right) + \rho(1-\omega)$$

Thus, since u^* is positively related to the level of the wage share, then the effect of a change in the wage share on the investment growth rate can be either positive or negative according to the value of the parameter ρ. As can be seen from the equation above, a higher value of the latter parameter reduces the positive (and indirect) effect of a change of the wage share exerted through the equilibrium rate of capacity utilization and increases the direct contribution of a change in the wage share through the third term on the RHS of the equation above. Hence, for a sufficiently *low* value of ρ the positive effect of a modification in the wage share on the investment growth rate through the capacity utilization rate dominates the direct negative effect related to the term $\rho(1-\omega)$. In this case, according to the neo-Kaleckian literature, the model would produce a wage led growth pattern. On the other hand, for a sufficiently *high* value of ρ we would have the opposite situation and the model would generate a profit led pattern of economic growth.[67] In both cases, a change in income distribution has a permanent growth effect. The existence of a relationship between economic growth and income distribution featured in the last version of the neo-Kaleckian model, also characterizes the Cambridge growth model as we saw. In the latter model there is an inverse relationship between the wage share and the rate of output growth, whereas the last specification of the neo-Kaleckian model admits either, a positive (in the wage led growth case) or a negative (in the profit led growth case) relationship between the two variables. In contrast with these results, the absence of any permanent relationship between income distribution and economic growth is an important feature of the supermultiplier growth model.

Table 1 summarizes the principal results obtained from the above comparative analysis. The main conclusion that emerges from this analysis is that the supermultiplier growth model can be considered a true heterodox alternative to the Cambridge and neo-Kaleckian growth models in the analysis of the relationship between economic growth, income distribution and effective demand. In this sense, first of all, the supermultiplier growth model shows how is it possible for a heterodox growth model to obtain a tendency towards the normal utilization of productive capacity without relying on the endogenous determination of the level of income distribution as in Cambridge growth model. Thus, the supermultiplier model does not impose the existence of any necessary *a priori* relationship between income distribution and economic growth in the interpretation of economic reality and, therefore, leave open the space for the determination of income distribution from outside the model by political, historical and economic factors not directly and necessarily related to the process of economic expansion. Secondly, the model also shows that the existence of a stable process of demand-led growth does not require the endogenous determination of an equilibrium rate of capacity utilization as in the neo-Kaleckian growth model. Actually, the supermultiplier model shows that

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[65] The inequality $\beta < s_{\Pi}(1-\omega)/v$ is also necessary for the stability of the model. To see this, note that, in the case of the present version of the model, the equation for the dynamic adjustment of the rate of capacity utilization (equation (37)) would be given by $\dot{u} = u_t(\alpha + \beta u_t + \rho(1-\omega) - (s_\pi(1-\omega)/v)u_t + \delta) = u_t\big(\alpha + \rho(1-\omega) - \big((s_\pi(1-\omega)/v) - \beta\big)u_t + \delta\big)$. Observe that now a change in the capacity utilization rate affects both the investment growth rate and pace of capital accumulation in the same direction. Thus, if initially the investment growth rate were higher (lower) than the capital accumulation rate, then the capacity utilization rate would increase (decline). For its turn, this latter change would produce the adjustment between investment growth and capital accumulation rates only if the impact of a change in the capacity utilization rate over the capital accumulation rate is greater than the impact of such a change on the pace of investment growth. That is, only if $\partial g_{Kt}/\partial u_t = s_\pi(1-\omega)/v > \beta = \partial g_{It}/\partial u_t$, which is the inequality mentioned above.

[66] Thus taking the partial derivative of u^* with respect to ω we obtain that $\partial u^*/\partial\omega = (v[\rho\beta v + s_\pi(\alpha+\delta)])/(s_\pi(1-\omega)-\beta v)^2 > 0$, which justifies the "stagnationist" or "wage led aggregate demand" classification attributed to the model. Observe, however, that if a nonlinear specification of the investment function were adopted then it would be possible, according to neo-Kaleckian literature, to obtain a negative relationship between $u^*$and ω. In this case, following the suggestion of Marglin and Bhaduri (1990), the model would be classified as "exhilarationist" (or "profit led aggregate demand"). See Blecker (2002) for a detailed discussion of these topics.

[67] For sufficiently low or high value of ρ we mean a value of ρ respectively lower or higher than a critical value $\rho_c = \big(\beta v(\alpha+\beta)\big)/\big((s_\pi(1-\omega)-2\beta v)(1-\omega)\big)$. Therefore, we would have a wage led (profit led) growth pattern as $\rho < \rho_c (\rho > \rho_c)$. That is we have $\partial g_I^*/\partial\omega \gtreqless 0$as $\rho \lesseqgtr \rho_c$.

the existence of a demand-led pattern of economic growth is fully compatible with the tendency towards the normal utilization of productive capacity. Therefore, it shows that allowing the possible prevalence of arbitrarily high or low rates of capacity utilization and permanent deviations of the actual rate of capacity utilization from its normal level are not necessary requirements for the existence of a demand-led pattern of economic growth.

Table 1

	Cambridge Growth Model	**neo-Kaleckian Growth Model**	**Supermultiplier Growth Model**
Output capacity, aggregate output and aggregate demand	*Capacity (potential) output determines the levels of aggregate output and aggregate demand*	*Aggregate demand determines the level of aggregate output*	*Aggregate Demand determines the levels of aggregate output and capacity (potential) output*
Income distribution	*Endogenous*	*Exogenous*	*Exogenous*
Investment share of output and saving ratio	*The investment share of output determines the saving ratio*	*The exogenous saving ratio determines the investment share of output*	*The investment share of output determines the saving ratio*
Pattern of economic growth	*Supply (capacity) constrained growth*	*Demand (investment) led growth*	*Demand (consumption) led growth*
Rate of capacity utilization	*Tends to full capacity utilization rate*	*Tends to an endogenous equilibrium value*	*Tends to the normal rate of capacity utilization*
Investment share of output and the trend rate of economic growth	*There is a theoretically necessary and positive relationship between the two variables*	*No theoretically necessary relationship between the two variables*	*There is a theoretically necessary and positive relationship between the two variables*
Wage share of output (income distribution) and the rate of economic growth	*There is a theoretically necessary and negative relationship between the two variables*	*There is a positive relationship between the two variables in the "wage led growth" case and a negative one in the "profit led growth" case*	*No relationship between the two variables. There is a positive wage led output level effect*
Theoretical closure	*Endogenous determination of a required level of income distribution*	*Endogenous determination of a required equilibrium rate of capacity utilization*	*Endogenous determination of a required value of the fraction (i.e. the ratio between the average and marginal propensities to save)*

Concluding Remarks

The purpose of this paper has been to provide a formal analysis of an explicit dynamic version of the Sraffian supermultiplier growth model in order to characterize the adjustment process of productive capacity to aggregate demand according to the model and in order to investigate the dynamic stability properties of such process. We started by presenting the model's main assumptions and by discussing their specific roles in the functioning of the model. Then we analyzed the economic properties of the model's equilibrium path showing that it generates a demand(consumption)-led growth trajectory which, in equilibrium, is characterized by the normal utilization of productive capacity. We argued that the obtainment of such fully adjusted equilibrium path implies the need for the endogenous determination of the investment share of output and of the saving ratio, according to which the investment share of output is positively related to the trend rate of economic growth and there is an adjustment of the saving ratio to the investment share of output. We also argued that, in the supermultiplier growth model, the latter adjustment does not require the endogenous determination of income distribution. In fact, we demonstrated that, according to the model, the possibility of such adjustment follows from the assumption about the existence of an autonomous consumption component which allows the endogenous determination of the saving ratio (i.e. of the average propensity to save). As has been remarked, this last result is the distinguishing feature of the theoretical closure provided by the supermultiplier growth model. Finally, our analysis of the equilibrium path of the model also showed that an exogenous change in income distribution (i.e. in the wage share of output) does not have a permanent output growth effect, but only a wage led output level effect.

In the sequence, we undertook a formal analysis of the dynamic behavior of the model in the neighborhood of its equilibrium point. First, we investigated the local dynamic stability properties of the equilibrium path of the model. From such inquiry we obtained a local dynamic stability condition according to which the model's equilibrium is asymptotically stable if the expanded marginal propensity to spend has a value strictly lower than one in the neighborhood of the equilibrium point. We verified that if this stability condition is met then a change in the rate of capacity utilization has a greater impact on the pace of capital accumulation than on the rates of aggregate demand and output growth in the neighborhood of the equilibrium point. Further, we established that from the stability condition it also follows that there exists a maximum rate of economic growth below which a stable demand led growth process is viable. After the investigation of the model's stability, we undertook a qualitative analysis of its dynamic behavior in the neighborhood of the equilibrium point. From such investigation we obtained a characterization of the disequilibrium trajectories of the model's stationary endogenous variables. This investigation revealed that, if the stability condition is met then these trajectories display a cyclical and convergent pattern (i.e. the equilibrium is a stable focus).

In the sequence, we used the results obtained in our analysis of supermultiplier growth model to address the criticisms advanced against it. We argued that much of the criticisms are based on a misleading representation and interpretation of the model according to which the supermultiplier model would be required to generate continuously fully adjusted growth paths. Clearly, in this case, the model would not be a plausible explanation for economic growth as demand-led process. However, we pointed out that the model used to represent the supermultiplier model is in fact a formalization of the economic growth process based on Say's Law. Thus, this model could not be a good representation of a model whose main objective is to extend the applicability of the principle of effective demand to the analysis of the long run economic growth process. In contrast, our analysis of the of the supermultiplier model demonstrated that the model produces growth trajectories in which aggregate demand determines aggregate output. It showed, as well, that the model does not generate continuously fully adjusted paths, in the sense that, outside the equilibrium path, the growth trajectories produced by the model are featured by the divergence of the actual rate of capacity utilization from its normal level. Actually, in this respect, we argued that such divergence is an essential element in the characterization of the adjustment process of productive capacity to aggregate demand according to the supermultiplier growth model. Thus, against the critics' opinion, our analysis has demonstrated that in the latter model the expansion of aggregate demand has a determining role in the process of economic growth and that this determining role is fully compatible with the tendency towards the normal utilization of productive capacity. Further, we also argued that the validity of model's conclusions does not require the exact equality between the time average of the actual rates of capacity utilization and the normal rate of capacity utilization. We asserted that such requirement is excessively strong and not needed, and, therefore, that the imposition of such requirement to the supermultiplier model is a kind of misplaced exigency of precision on the part of some of the model's critics. Finally, we addressed the criticism according to which the existence of the tendency towards normal capacity utilization would require hyper-rational behavioral assumptions in individual capitalist investment decisions and that it would be impaired by the influence of the process of adjustment of capacity to demand on aggregate investment as a component of aggregate demand. In this connection, we argued that no such hyper-rational behavioral assumptions are needed according to our analysis of the adjustment process of capacity to demand based on the supermultiplier growth model. We also showed that the effect of the adjustment process of capacity to demand on aggregate investment as a component of aggregate demand is already considered in our analysis of the stability of the model's equilibrium path. Therefore, we argued that if the stability condition presented in this paper is met the latter effect is not an obstacle to the prevalence of the tendency towards normal capacity utilization.

In the last section of the paper we compared the supermultiplier, Cambridge and neo-Kaleckian growth models pointing out their main similarities and differences as summarized in Table 1. The most important conclusion obtained from the comparative analysis is that the supermultiplier growth model provides an alternative heterodox explanation for the relationship between economic growth, income distribution and effective demand. Such explanation does not involve, as in the case of the neo-Kaleckian growth models, the endogenous determination of an equilibrium (long run) rate of capacity utilization that must be able to assume very high or low values and, hence, can permanently diverge from its normal (or planned) level. Moreover, such explanation does not depend on an endogenous determination of income distribution that implies the existence of a theoretically necessary *a priori* relationship between income distribution and economic growth, as we can find in growth models based on the Cambridge theory of income distribution. In fact, the alternative theoretical closure provided by the supermultiplier growth model enables a demand-led interpretation of the growth process that is compatible with a tendency towards normal utilization of productive capacity and with a determination of income distribution from outside the model by political, historical and economic forces. Therefore, the supermultiplier growth model provides a true alternative to the Cambridge and neo-Kaleckian growth models in the analysis of the relationship between economic growth, income distribution and effective demand. An alternative that has not yet been much explored as the Cambridge and neo-Kaleckian ones, but that, we suggest, establishes a sound foundation for a fruitful research program in heterodox economic growth theory

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Figure 1 – The equilibrium point in the (g, u) plane

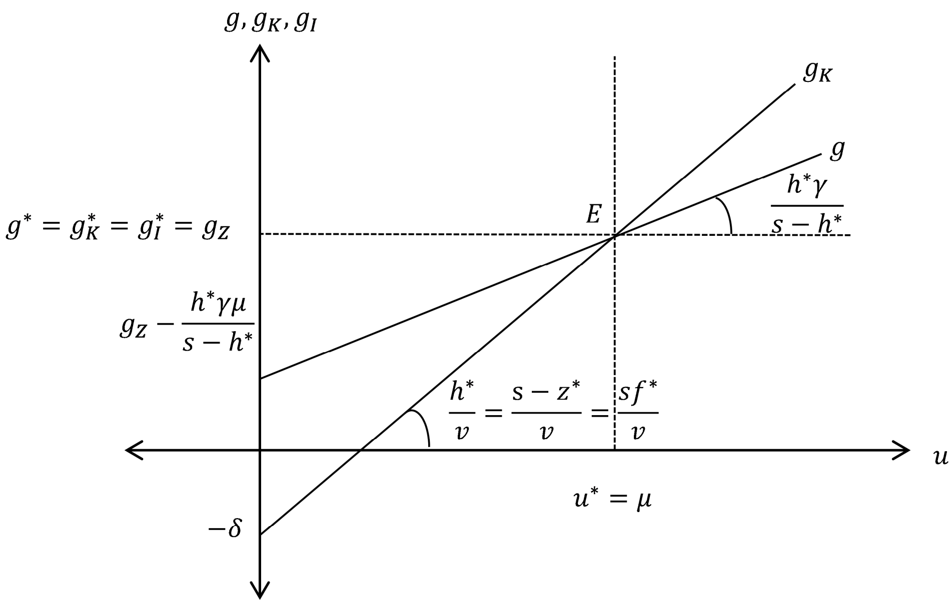


Figure 2 – Phase Plane

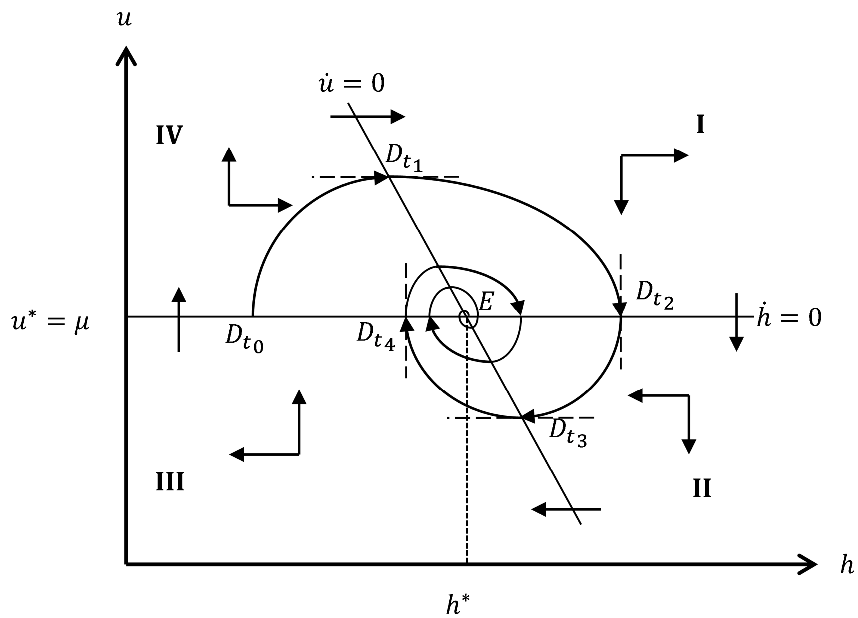


Figure 3

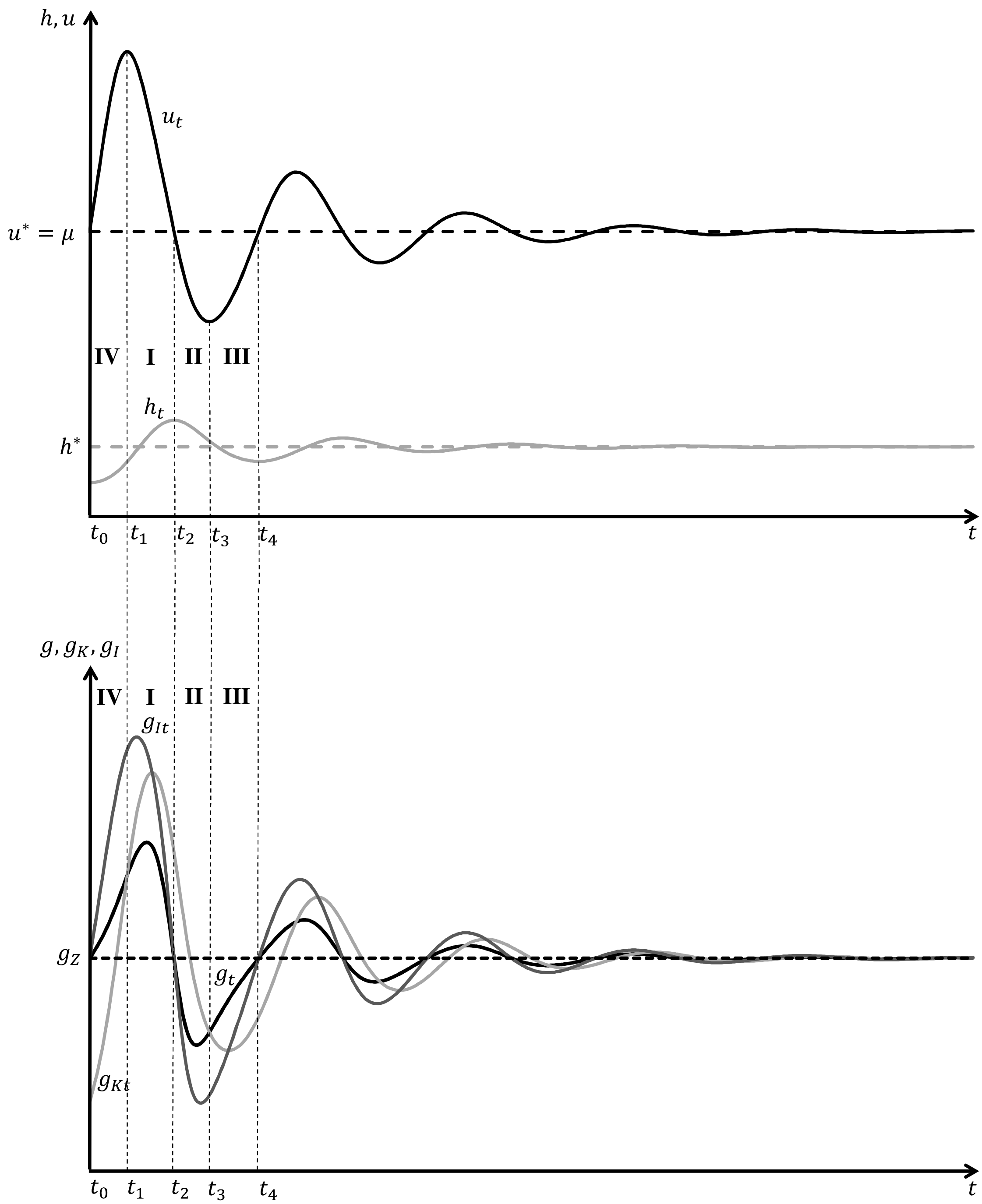
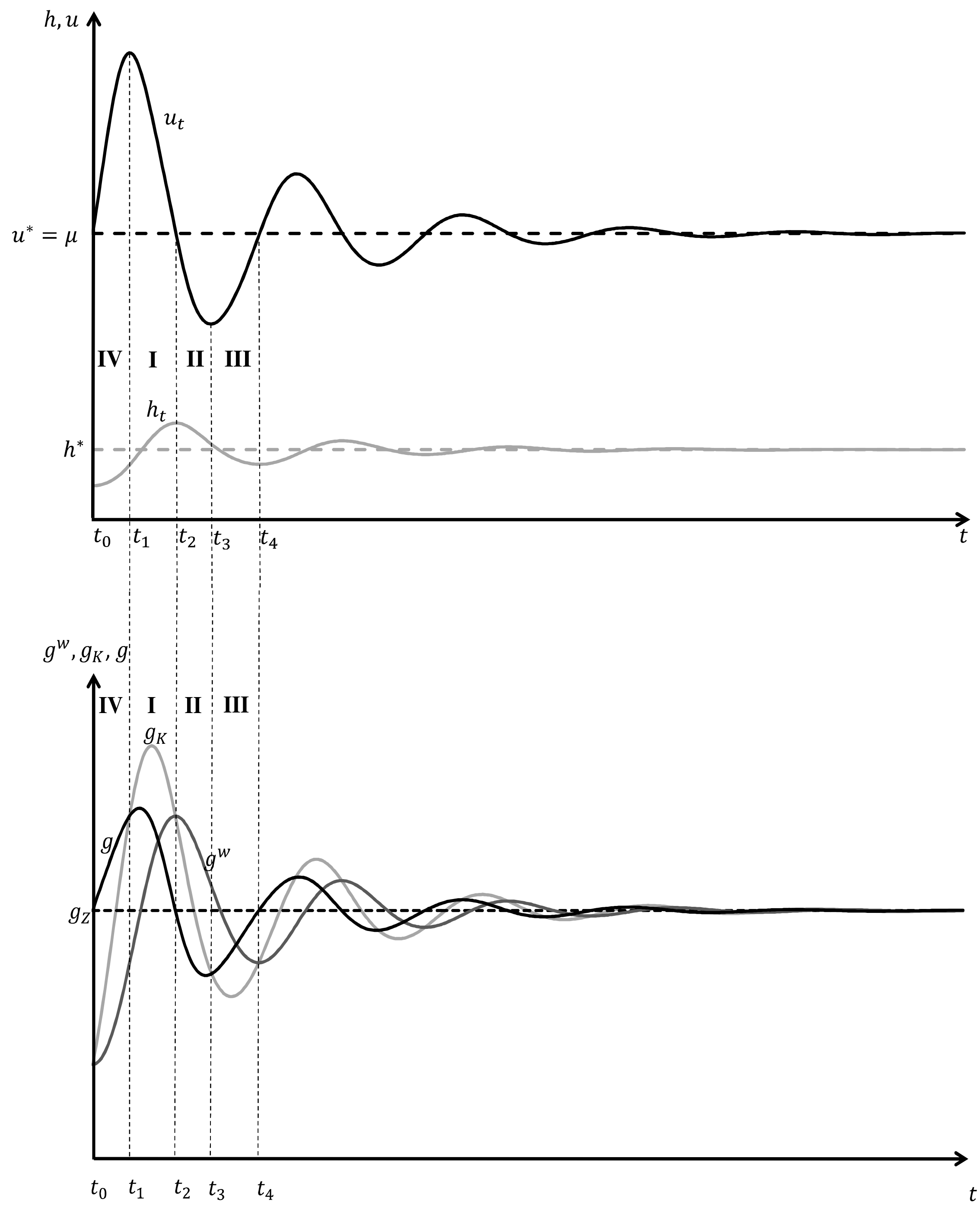

h, u
u_t
$u^* = \mu$
IV I II III
h_t
h^*
t_0 t_1 t_2 t_3 t_4
t
g, g_K, g_I
IV I II III
g_{It}
g_Z
g_t
g_{Kt}
t_0 t_1 t_2 t_3 t_4
t

Figure 4



h, u
u_t
$u^* = \mu$
IV
I
II
III
h_t
h^*
t_0
t_1
t_2
t_3
t_4
t
g^w, g_K, g
IV
I
II
III
g_K
g
g^w
g_Z
t_0
t_1
t_2
t_3
t_4
t

Figure 5

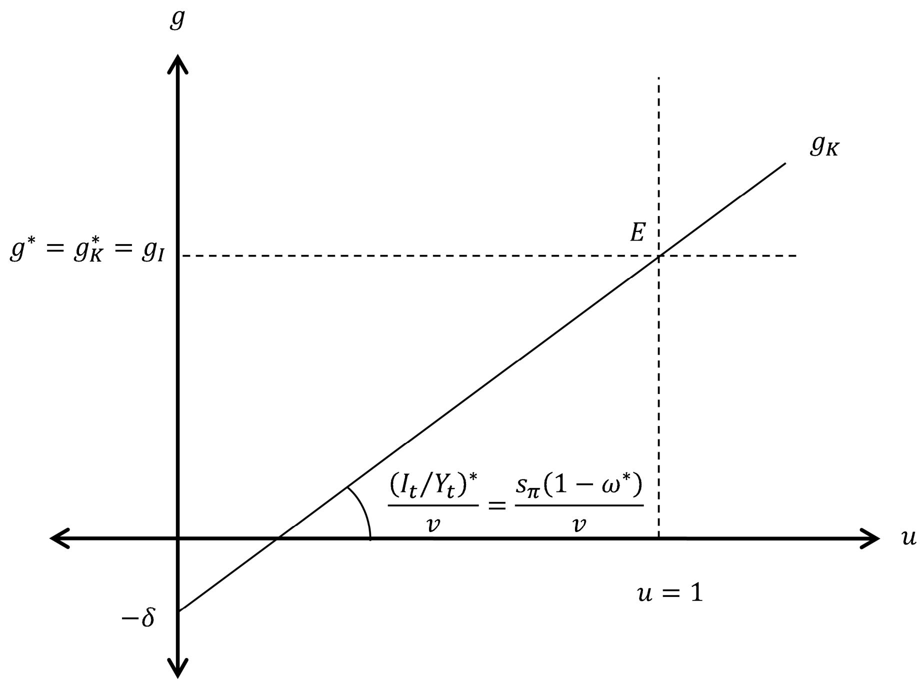
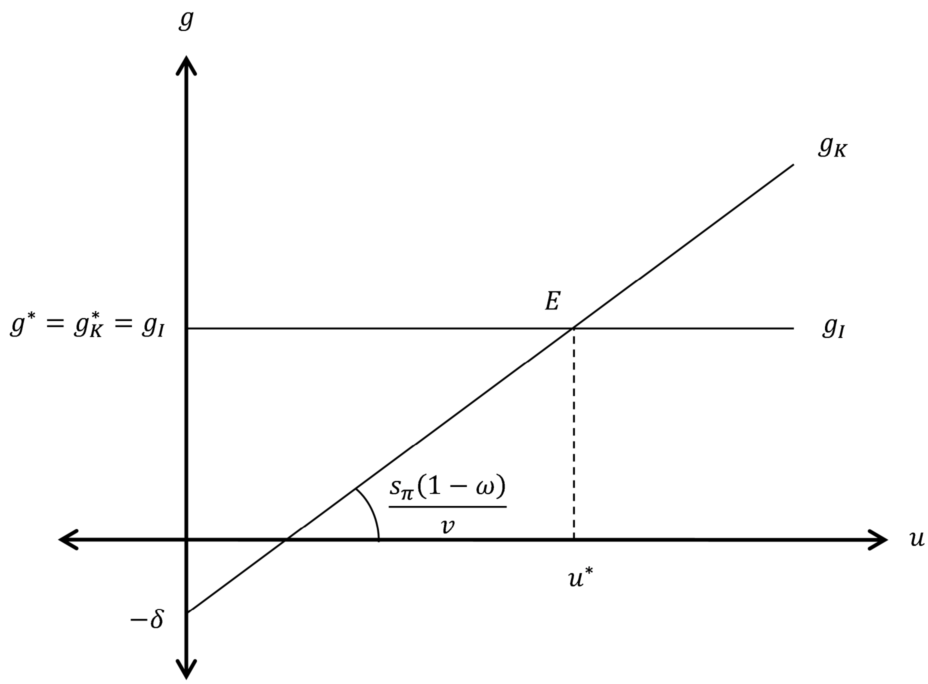


Figure 6



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