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The simple analytics of the Sraffian Supermultiplier

Franklin Serrano
Institute of Economics, Federal University of Rio de Janeiro

Gustavo Bhering
Institute of Economics, Federal University of Rio de Janeiro

Ricardo Summa
Institute of Economics, Federal University of Rio de Janeiro

Fabio Freitas
Institute of Economics, Federal University of Rio de Janeiro

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Franklin Serrano
Institute of Economics, Federal University of Rio de Janeiro

Gustavo Bhering
Institute of Economics, Federal University of Rio de Janeiro

Ricardo Summa
Institute of Economics, Federal University of Rio de Janeiro

Fabio Freitas
Institute of Economics, Federal University of Rio de Janeiro

Abstract

This chapter presents a very simple introduction to some of the main features of the Sraffian supermultiplier(Serrano1995a, 1995b) demand-led growth model and its particular analytical closure through the endogenous changes in the ratio between capacity generating business investment and the autonomous non capacity generating component of aggregate demand ("the fraction"). In the Sraffian supermultiplier, distribution is given exogenously, capacity generating business investment is induced by demand following the capital stock adjustment principle (flexible accelerator) and it is the growth of autonomous expenditures that do not create capacity for the business sector of the economy that drives the rate of demand-led growth. We show that these two key assumptions allow us both to understand better and overcome the well-known problem of Harrod´s "Fundamental Instability". Moreover, these same assumptions also allow us to overcome what we call the Oxbridge Dilemma of the two traditional closures of post-Keynesian growth models, which consists of having to choose between exogenous distribution and a tendency towards a normal degree of capacity utilization, as productive capacity adjusts to the trend of demand.

[1] Draft prepared for the Handbook of Alternative Theories of Economic Growth, Second Edition, edited by Mark Setterfield

1 Introduction

The purpose of this chapter is to present a very simple introduction to some of the main features of the Sraffian supermultiplier (Serrano1995a, 1995b) demand-led growth model[2] and its particular analytical closure through the endogenous changes in the ratio between capacity generating business investment and the autonomous non capacity generating component of aggregate demand ("the fraction"). The Sraffian supermultiplier was developed as a contribution to the theory of long period levels of output and of the longer run process of growth and capital accumulation driven by effective demand, in the context of the revival of the Classical Surplus approach proposed by Sraffa, a project initiated by Garegnani (Eatwell & Milgate,1983; Garegnani, 2015[1962]). [3]

Accordingly, in the Sraffian supermultiplier model, distribution is given exogenously, capacity generating business investment is induced by demand, following the capital stock adjustment principle (flexible accelerator), and it is the growth of autonomous expenditures that do not create capacity for the business sector of the economy that drives the rate of demand-led growth. Here we will show that these two key assumptions of the Sraffian supermultiplier allow us both to understand better and overcome the well-known problem of Harrod´s "Fundamental Instability". Moreover, we will also show how these same assumptions also allow us to overcome what we call the "Oxbridge dilemma" of the two traditional closures of post-Keynesian growth models. These models, in which all expenditures that do not create capacity for the business sector are induced by income, share the need to assume that business investment has an autonomous component, even in the very long run, in order to avoid Fundamental Harrodian instability. The latter assumption evidently prevents productive capacity from adjusting to the trend of demand. And this in turn leads inevitably to a dilemma. On one hand, a tendency towards a normal degree of capacity utilization may be retained , but at the very high cost of having to

[2] For detailed expositions of the model, see the textbooks by Lavoie (2022), Blecker and Setterfield (2019) and Hein (2023).

[3] The model (though with some different specific assumptions) was originally proposed independently by Bortis (1979). A curious and even earlier use of the supermultiplier for demand-led growth was found in an econometric model for Italy by Ackley in 1963 by Cesaratto (2020).

assume that distribution endogenously adjusts aggregate demand to aggregate supply, thus making the model supply constrained instead of demand led in the long run, as in the closure based on the Cambridge theory of distribution (Robinson, 1962; Kaldor, [1956] 1970; Khan, 1959). Alternatively, the much more plausible assumption of exogenous distribution and demand determined levels and rates of growth of output can be made, but at the cost of assuming that the actual degree of capacity utilization may become permanently very different from its normal or desired degree, while business investment implausibly continues to grow at a steady pace. The latter result follow from the closure based on an endogenous actual degree of capacity utilization, introducing by Kalecki and Steindl (and Harrod) that for the sake of symmetry we call the "Oxford" closure (Kalecki,1971; Steindl,1952, 1979). We will show that the Sraffian supermultiplier, with an autonomous non capacity generating component in aggregate demand and its associated closure through the fraction allows us to overcome the Oxbridge dilemma of having to choose between exogenous distribution and a tendency towards a normal degree of capacity utilization, thus providing a better foundation for models of demand led growth.

The rest of this chapter is organized as follows. Section 2 briefly discusses the Harrodian Principle of Fundamental Instability. Section 3 then summarizes the two Oxbridge closures and their dilemma. Section 4 introduces the Sraffian supermultiplier and the role of the autonomous expenditures that do not create capacity and the closure via the fraction. Section 5 discusses the conditions for the stability of the process of adjusting capacity to the trend of demand in the context of the supermultiplier. Section 6 shows how the model overcomes the Oxbridge dilemma. Section 7 contains very brief final remarks.[4]

[4] For more complete and formal presentations of the Sraffian supermultiplier model see Freitas & Serrano (2015), Serrano, Freitas & Bhering (2019) (concerning the points in sections 2,4 and 5) and Serrano & Freitas (2017) (in what regards the issues dealt in sections 3 and 6).

2 Harrod and the Fundamental Instability of the Warranted Rate

Harrod (1939,p. 43) proposed a model of economic growth based on "a marriage of the 'acceleration principle' and the 'multiplier theory'" That could adequately take into account the *dual character of* investment. The multiplier deals with the effect of current investment on aggregate demand, while the accelerator takes into account the subsequent effect of investment in creating productive capacity (supply).[5] Harrod then sets up the conditions for balanced growth at a steady rate, i.e., under what circumstances the demand and capacity effects of investment can be made consistent, allowing for the maintenance of a balance between aggregate demand and productive capacity over time.

Such conditions come from Harrod's "fundamental equation", which can be derived as follows. From the identity between aggregate investment and savings when the output market is in equilibrium we can obtain, with a few manipulations[6], the following expression:

$$\frac{I}{K} \equiv \frac{S}{Y}\frac{Y^*}{K}\frac{Y}{Y^*} \qquad (1)$$

This truism tells us that the growth rate of the capital stock (*I/K*) is identical to the *average* propensity to save (*S/Y*) multiplied by the inverse of the normal capital-output ratio ($Y^*/K = 1/v$) and by the actual degree of capacity utilization ($Y/Y^* = u$).

5 From now on the term investment shall be used to denote business investment, expenditures that are capable of generating productive capacity for the private sector of the economy, i.e. its purchases of new means of production. We shall here ignore household or residential investment, as well as those of state-owned corporations and the public administration.

6 Simply by dividing both sides of the identity by the capital stock and then tautologically decomposing the right-hand side (*S/K*) into the three parts shown in (1). For the sake of simplicity, all magnitudes are expressed in net terms, not gross. We are also assuming that there is no involuntary change in inventories.

In Harrod's model, the *average* propensity to save is equal to (and totally determined by) the *marginal* propensity to save, *s*, given exogenously to the model (*S/Y* = *s*). This stems from the fact that, in specifying his model, Harrod did not consider the existence of autonomous aggregate consumption[7]. Therefore, the level of output determined by effective demand, is given simply by the ratio between investment (*I*), and the marginal (and average) propensity to save (*s*).

In this model, for a given value of the marginal propensity to save, the actual growth rate of the economy (*g*) is equal to the growth rate of investment (since induced consumption always grows at the same rate as investment). In addition, the growth rate of the capital stock also always follows, with some lag, the growth rate of net investment. As a result, in a longer run, there is a tendency towards equality between these 3 growth rates: output, private investment, and capital stock, as the rate of growth of investment determines the growth of the levels of output and with a lag the rate of growth of the capital stock. Under these assumptions, from (1), we can obtain the following equation:

$$g = \frac{s}{v}u \tag{2}$$

From which we can deduce our version of Harrod's *"fundamental equation"* (1939, p. 17) by making $u = 1$ (denoting the normal, instead of maximum degree of capacity utilization) and getting:

$$g_w = \frac{s}{v} \tag{3}$$

This equation expresses the condition for balanced growth between demand and capacity. Harrod called, g_w, the "warranted rate", the particular rate that allows for this type of

7 In fact, Harrod (1939, 1948) even considered the existence of given levels of autonomous expenditures that do not create capacity for the private sector of the domestic economy, such as government spending and exports. However, Harrod did not consider the hypothesis that these expenditures could grow in the long run at an independently determined rate. The absence of the latter assumption is *crucial* for obtaining the results of his model.

growth with continuous normal utilization of productive capacity. The warranted rate is a positive function of the marginal propensity to save and a negative function of the normal capital-output ratio, which are determined exogenously. Note that the warranted rate only reflects supply conditions, although one of Harrod's aims was to extend some of the Keynesian conclusions originally developed in a short run context to the longer run (when we take into account the effects of changes in productive capacity). In fact, the warranted rate should be understood as a condition for the validity of *Say's Law in* the long run (Serrano (1995a, pp. 68-71) . Note, however, that the warranted rate is not the actual rate of growth of aggregate demand and output, which as we have seen is determined by the growth rate of investment in this model. Nor does the warranted rate represent the rate at which the capital stock and potential output actually grow, since the growth rate of the capital stock, as we saw above, will also tend to grow at the same rate as investment with some lag. In reality, Harrod's warranted rate represents only an upper limit for the growth rate of potential output (with normal capacity utilization), which would only occur if investment was, in every single period, exactly equal to (and determined by) the economy's potential or capacity saving.

Thus, despite the references to the multiplier and accelerator, the balance between capacity and demand described by the growth path given by the warranted rate would only occur if the growth in demand always and continuously matched the previous growth in productive capacity. Since Harrod rejects Say's Law, he correctly concludes that there is no reason for the economy to grow at the warranted rate. Indeed, if for some reason investment and (subsequently) the capital stock grow at an exogenous rate g different from g_w, the degree of utilization will tend to be permanently different from normal ($u \neq 1$). Note that Harrod's warranted rate is the *only* rate at which investment, demand and productive capacity (supply) grow in balance ($u=1$).

As Harrod's warranted rate only represents the limit to growth given by potential saving, it follows from equation (3) above that an actual growth rate of investment (and output) above the warranted rate ($g>g_w$) leads to a situation of overutilization of capacity ($u>1$), and that an investment growth rate below the warranted rate ($g<g_w$) leads to an underutilization of capacity ($u<1$).

Harrod went further and demonstrated that if we add the assumption that investment is induced, in the sense of some version of the accelerator (or the principle of capital stock adjustment (Matthews, 1960)), any rate of growth of investment other than the warranted one actually causes *cumulative disequilibria*. In order to see this, let us now assume that aggregate investment is totally *induced* and sensitive to the degree of capacity utilization. This clearly leads to what Harrod called the principle of fundamental instability. Faced with an initial overutilization of capacity ($u>1$), firms would react by increasing their investments, while when faced with a situation of under-utilization of capacity ($u<1$) they tend to reduce the growth of investment. In both cases, this reasonable reaction of firms would cause the actual rate of growth rate of output to move further and further away from the warranted rate (we would have, respectively, $g>>g_w$ and $g<<g_w$).

Note that although a given rate of growth of investment g leads to a stable degree of capacity utilization, as the capital stock will tend later to grow at the same rate as aggregate demand and investment, each time the growth rate of investment changes, the corresponding actual degree of utilization will change. This occurs because the initial effect of an increase in g is to increase demand more than capacity, since investment is always first an increase in demand and only later, when the new capital goods come into operation, there is an increase in productive capacity. Correspondingly, a fall in g initially causes the rate of growth of demand and output to fall before the rate of growth of the capital stock and capacity growth is reduced, leading to a lower degree of capacity utilization. Thus, in the case of induced investment, each round of reduction (or increase) in the investment growth rate due to a fall (increase) in the degree of capacity utilization would lead to a new, even lower (higher) actual degree of capacity utilization.

This is in a nutshell Harrod's *"fundamental instability problem"*: any divergence between g and g_w, *no matter how small,* tends to widen through the mechanism described above. Harrod's warranted rate of growth is *unstable*, *which* implies that a mismatch between aggregate demand and capacity would be the rule in the long run. Note that the reason Harrod considered his demonstration of instability to be *fundamental* was because the adjustment goes in the *wrong direction*. For example, when the effective rate of growth is below the warranted rate and the degree of utilization is low, restoring growth to the warranted rate would require an increase in investment growth, but market signals (the

low actual degree of utilization) imply that there will be an incentive to reduce the rate of growth of private investment (Serrano, Freitas & Bhering, 2019).

3 Oxbridge

Post-Keynesian theories of growth emerged as a response to some of the issues raised by Harrod from his model discussed above. The main one concerns the problem of fundamental instability. Harrod's model was violently and implausibly unstable, which made it unsuitable for explaining and analyzing real economic growth processes. Furthermore, even if the issue of instability were ignored or somehow circumvented, the fact would remain that in Harrod's model the warranted rate depends on the marginal propensity to save and growth at this rate would imply the validity of Say's Law, which was obviously contrary to any attempt to extend the principle of effective demand to a growth context.

Faced with these problems, the main post-Keynesian strands of growth theory have adopted a *common* working *hypothesis*: the idea that *aggregate investment* is an *autonomous* expenditure[8]. With the hypothesis of autonomous investment, post-Keynesian theories manage to get around the problem of Harrod's fundamental instability because, as we have seen, one of the premises of the mechanism that causes instability in Harrod's model is that investment is totally induced with the aim of adjusting capacity to demand.

On the other hand, consumption continues to be considered an expenditure totally induced by income, as in Harrod's model. Post-Keynesian authors specify equation (3) differently in order to explain the relationship between the propensity to save and income

8 For post-Keynesian authors, the level and growth rate of investment are determined exogenously by financial factors (profit margins, interest rates, availability of credit), psychological factors (strong uncertainty, "*animal spirits*", etc.), factors related to the process of capitalist competition (technological change, etc.) and/or historical and political factors. For a more detailed discussion of the autonomous investment hypothesis, see Serrano (2001).

distribution. In fact, for simplicity's sake, we can assume that wages are totally consumed and that capitalists save an exogenously fixed portion of profits, *P* (s_p = *S/P>0*). From this, we can rearrange the right-hand side of the general equation (1) to get:

$$g = s_p(1 - \omega)Ru \tag{4}$$

Where (1-ω) = *P/Y* is the share of profit in output and *R = 1/v* is the maximum rate of profit.[9] On the other hand, the normal rate of profit is *r* = (1-ω)*R*, which gives us the "Oxbridge" equation:

$$g = s_p ru \tag{5}$$

This equation can be used to study and compare the two main strands of post-Keynesian theory, the Cambridge theories and what we will call (following Serrano (1995a, ch. 3)) the Oxford theories. Note that in both strands, and in line with Harrod, there is no autonomous consumption, then the share of profits saved is determined exogenously by s_p an exogenous marginal propensity to save. Furthermore, as discussed above, in contrast to Harrod, both theories assume the existence of autonomous investment whose growth rate, *g*, is determined exogenously and is also the rate that determines the growth of the capital stock and productive capacity. Given these hypotheses, the question is: which of the two variables, *r* or *u*, in the right-hand side of equation (5) is determined by the theory? Here is where the Cambridge and Oxford post-Keynesian theories differ from each other.

Let's look first at the *Cambridge* theories. Its authors[10] believed that in the longer run there should be a tendency towards a normal degree of utilization (*u=1*). For this to happen, there must be some adjustment between aggregate demand and productive

9 Comparing with equation (1) of the Harrod model, we can see that *s* = s_p (1-ω), where ω = *W/Y*.

10 The main works in this post-Keynesian strand are Kaldor ([1956] 1970), Robinson (1962) and Kahn (1959).

capacity. But due to the assumption of autonomous investment, the exogenous growth rate of investment *g* determines the growth of productive capacity, and the only way for this adjustment to occur, *within the hypotheses of these theories*, is by adjusting aggregate demand to productive capacity. In the Cambridge theory, demand is adjusted to capacity through changes in the *distribution of income* (*r*, in the equation above), which makes aggregate consumption adapt to the size of the productive capacity that is left from what is taken up by investment[11] .

Therefore, if the growth of demand is greater than the growth in capacity, there would be a tendency for the normal rate of profit to rise through demand inflation, which would reduce the share of wages in income through the well-known mechanism of "forced savings". That would cause, through a lower size of the multiplier, a reduction in aggregate consumption and aggregate demand. In the opposite situation, there should be a reduction in the normal rate of profit, as prices should fall relative to wages enough to lead to an increase in worker´s consumption and aggregate demand big enough to restore the normal utilization of productive capacity. Therefore, in the Cambridge theories, the dependent variable in the "Oxbridge" equation above is the normal rate of profit ($r=g/s_p$ and $u=1$). In this case, any increase or decrease in the rate *g* would cause, *ceteris paribus*, an increase or decrease in the normal rate of profit respectively.

On the other hand, the main authors associated with the *Oxford* theory[12] did not believe that, at least in advanced countries, distribution could be an adjustment variable in the long run. This is because, in their view, market structures are predominantly non-

11 It is important to note that, as in Harrod, $R = 1/v$ is given, then changes in the distribution (normal rate of profit) come entirely from changes in the share of profit in the product (1-ω).

12 These authors include Steindl (1952, 1979), Kalecki, Hicks and Harrod himself. The "Oxford" model is presented here in a very simplified way to emphasize the central characteristic that interests us, which is the existence of an autonomous component in the investment function and the totally induced nature of consumption. Serrano and Freitas (2017) show that the more modern versions of this type of model presented by neo-Kaleckians and authors such as Marglin and Bhaduri do not in any way alter the conclusions presented in this section. This is because, if distribution is given, the assumption that all consumption is induced is sufficient to make the marginal propensity to save to determine the share of investment in output , whatever determines and whatever happens to the levels of investment, forcing the actual degree of utilization to be the adjusting variable.

competitive (oligopolistic), which means that the degree of monopoly and, therefore, the profit share is exogenous. It follows that the adjustment mechanism between aggregate demand and productive capacity proposed by the Cambridge theory could not work, *given* the assumptions shared by the two strands. In particular, in the case where demand growth is lower than that of capacity, which is determined independently, there would be a tendency towards chronic underutilization of productive capacity ($u<1$) and not towards a change in distribution. It follows that, in the terms of the "Oxbridge" equation, the endogenous variable would be the actual degree of capacity utilization. Increases or decreases in the rate g would lead, respectively, to increases or decreases in the degree of utilization ($u=g/s_pr$).

We can now critically evaluate the two post-Keynesian theories. With regard to the Cambridge theory, it is extremely doubtful - from a theoretical and empirical point of view - whether the proposed mechanism can work smoothly, especially in the case of overcapacity. Under these circumstances we would have to expect downward flexibility in prices and profit margins even in the long run. It is much more reasonable to assume that firms, even in competitive conditions, preserve a minimum profit margin and reduce output as demand falls, especially in the long run.

The case of permanent excess aggregate demand, on the other hand, is equally implausible. This is because autonomous investment and aggregate demand would be assumed to always grow so fast that the size of productive capacity never reaches the level of effective demand and there is always a chronic long-run excess of aggregate demand. This excess demand is not eliminated by adjusting productive capacity in relation to effective demand, but by the mechanism of forced savings by reducing real wages and the share of wages in output. It is implausible that excess demand does not generate some investment reaction in the sense of increasing productive capacity. The Cambridge theory also implies that there is an inverse relationship between the levels of investment and consumption in the long run, which is not observed empirically in capitalist economies.

Regarding the Oxford theories, the main problem[13] is that it is very implausible that investment remains autonomous in the long run and that investment decisions are not affected by a permanent imbalance between aggregate demand and productive capacity, which is what is postulated if we take the degree of utilization as the adjustment variable in the long run.

Thus, the post-Keynesian theory of growth reaches an *impasse*, which we can call the Oxbridge dilemma. For there to be any adjustment between capacity and demand, *it seems* necessary to accept the implausible Cambridge adjustment mechanism. On the other hand, if you don't accept the latter mechanism, assuming exogenous income distribution - as in Oxford's theory - then the economy has no way of adjusting productive capacity and effective demand, and therefore the degree of utilization never tends its normal level.

4 The Sraffian Supermultiplier

However, this impasse is only real if the two crucial assumptions shared by post-Keynesian theories are not questioned: autonomous investment and the absence of autonomous aggregate consumption (or, more generally, autonomous spending that does not create capacity for the private sector) . In fact, it is the hypothesis of autonomous investment that is really responsible for the impasse in question. It stems from the need for aggregate demand to adjust to productive capacity in order to maintain a normal degree of utilization. On the other hand, this hypothesis of autonomous investment seems to be indispensable for getting rid of Harrod's fundamental instability problem. However, this is not true. The fundamental instability of Harrod's warranted rate does not necessarily imply the fundamental instability of all theories based on a "marriage" of the accelerator (induced investment) and the multiplier as soon as we admit the existence of

13 We will not discuss here the shortcomings of the distribution theory based on the "degree of monopoly". See Pivetti (1991).

autonomous consumption spending (final spending), the growth of which can drive a demand-led growth process.

4.1 Autonomous consumption and the fraction

Let's assume the existence of autonomous aggregate consumption, *Z*, growing at an independent rate, *z*. It follows from this new hypothesis that, contrary to what happens in Harrod's model and post-Keynesian theories, the marginal and average propensities to save are *not, in* general, *equal*. The average propensity to save (*S/Y*) is given by:

$$\frac{S}{Y} = s - \frac{Z}{Y} \tag{6}$$

In (6) it is clear that the two propensities are only equal (*S/Y* = *s*) in the absence of autonomous consumption (*Z* = 0). The marginal propensity only imposes a ceiling on the average propensity which, in general (*Z>0*), will be strictly lower than it.

Note also that although the marginal propensity to save is exogenous, the average propensity to save depends positively on the level of output. An increase in output caused by an increase in investment decreases the relative weight of the "dissavings" represented by autonomous consumption, increasing the ratio between the average propensity to save and the (given) marginal propensity to save.

This becomes clearer if we write the expression for the average propensity to save in terms of the independent variables (*s*, *I* and *Z*). Since *S/Y* = *I/Y* and *Y* = (*I*+*Z*)/*s*, then:

$$\frac{S}{Y} = \frac{I}{I+Z}s$$

$$\frac{S}{Y} = fs \tag{7}$$

Where *f is* what Serrano (1995b) calls "the fraction" that corresponds to the ratio between the average and marginal propensity to save. From equation (7) we can see that the average propensity to save is no longer determined solely by the marginal propensity to save. It also depends on the relative levels of investment and autonomous consumption. Thus, an increase in investment in relation to the increase in autonomous spending *Z* causes an increase in the *level and rate of* savings. It follows that the average propensity to save is an *endogenously* determined variable, for any value below its upper limit *s*. Thus, if there is autonomous unproductive spending, the marginal propensity to save only defines the upper limit and not the actual value of the average propensity to save. Below this limit, it is the (relative) level of investment that determines (through changes in the fraction *f*) the economy's savings rate.

4.2 The Marginal Propensity to Invest and the Supermultiplier

Let's now see what happens, in an economy where there is autonomous consumption, when we additionally assume that long run investment is *induced. In* this case, we can define a given marginal propensity to invest, or investment share, as *h.* The level of output, in the case where there is autonomous spending and productive investment is induced, is given by the supermultiplier, which takes into account both induced consumption and induced investment:

$$Y = \frac{Z}{s - h} \tag{8}$$

In this model, given the marginal propensity to save and the propensity to invest, effective demand and the economy will grow at the rate of growth of spending of autonomous consumption, *z*. Furthermore, in the presence of autonomous consumption spending and induced investment, the average propensity to save is entirely determined by the propensity to invest (for any value of the propensity to invest less than *s*). And we see that in this model the *average* propensity to save is entirely determined by the propensity to invest *h*, despite the *marginal* propensity to save (and distribution) being exogenous.

5 Conditions for the fundamental (or static) and dynamic stability of the adjustment of capacity to the trend of demand

5.1 The fundamental stability of adjusting capacity to demand

Under these circumstances, due to the presence of autonomous spending that does not create capacity and grows at an independent rate *z*, the fact that investment is induced does not lead to fundamental instability as in the Harrod's model. On the contrary, this supermultiplier model with autonomous consumption is fundamentally stable because, unlike Harrod's model, the adjustment takes place in the "right" direction.

In Harrod's case, if initially the growth rate of investment is above the warranted rate given by *s/v*, the degree of utilization will be above normal and if the growth rate of investment is below the warranted rate, the degree of utilization will be below normal. If investment is induced, the adjustment is going in the wrong direction from the outset, as we saw in section 2 above. In the case of the Sraffian supermultiplier, growth at the Harrod's warranted rate will continue to be unstable, as it only corresponds to an upper limit of feasible growth rates (with normal capacity utilization). However, in the supermultiplier, where the growth rate of demand will be given by the growth rate of autonomous spending *z*, the growth of the economy at this rate will be fundamentally stable.

For example, suppose that, starting from a situation in which the degree of capacity utilization is equal to normal, the growth rate *z of* autonomous consumption is permanently reduced. This reduction will reduce the growth rate of output *g to the* same extent for the given marginal propensities to consume and invest.

The actual degree of capacity utilization will fall ($u<1$), because initially aggregate demand will start to grow less and only then will the growth rate of productive capacity and the capital stock fall to this lower rate. The slower growth of capacity will occur when the capacity effect of the slower growth rate of the level of investment materializes,

which, for a given rate of induced investment *h*, will grow at the same lower rate of growth of autonomous spending, also reducing the growth rate of the capital stock. When the growth rate of the capital stock adapts to this lower growth rate of output, the degree of utilization will stabilize at a lower level than normal.

However, it is reasonable to assume that, over time, the rate of induced investment *h* itself will reduce in response to the underutilization of productive capacity and/or the reduction in the rate of growth of demand. Let's assume that the rate of investment is gradually reduced due to the lower rate of growth in demand[14] . This reduction in the marginal propensity to invest will have two effects. Initially, it will further reduce aggregate demand and output, further reducing the degree of capacity utilization. Subsequently, the reduction in the investment rate will reduce the growth rate of the capital stock and productive capacity.

However, the presence of autonomous spending growing at an exogenous rate implies that the growth rate of aggregate demand and output is reduced less than proportionally to the fall in the growth rate of investment (otherwise the investment rate would not have been reduced), while the subsequent fall in the growth of the capital stock will be equal to the reduction in investment growth. And this means that the degree of utilization will eventually start to rise again, because although aggregate demand is growing even less, the final reduction in the growth of the capital stock is even greater (which would be impossible if there were no autonomous consumption component).

The process described above will continue as long as the degree of utilization is below its normal level and will only end when the rate of induced investment has been reduced

[14] In models with induced investment, the central idea is that investors try to adjust the size of the capital stock to the trend in demand. This implies that investment will respond to variations in expected demand and the actual degree of capacity utilization. There are several ways to represent this process in very simplified formal terms in a supermultiplier model. One option is to assume that the investment share reacts linearly to deviations in the degree of utilization from the normal level. Another, simpler option is to assume that the investment rate reacts linearly to changes in the expected growth rate of demand. Freitas and Serrano (2015) use the first option above. In this text we use the second. For other specifications for the induced investment function , see Fagundes and Freitas (2018) and Fazzari et al (2020).

sufficiently to the level that allows the growth rate of the capital stock to adapt to the lower growth rate of autonomous spending at the normal degree of utilization.

The same process will occur symmetrically in the opposite direction in the case of an increase in the growth rate of autonomous spending *z*. We would then have an initial overutilization of capacity and gradual increases in the rate of investment *h* induced by the increase in the growth of aggregate demand, which would initially increase the overutilization of capacity even more. Subsequently, the acceleration of capacity growth in relation to that of aggregate demand resulting from a higher rate of investment would gradually bring the degree of utilization back to its normal level with the level and rate of growth of the economy's productive capacity adapted to the higher rate of growth of autonomous spending *z*.

Growth led by the expansion of autonomous spending *Z* is *fundamentally* stable because the reaction of induced investment to the imbalance between aggregate demand and productive capacity generates a greater reduction in capacity growth in relation to demand in the case of falling demand growth and underutilization and a greater increase in capacity expansion than demand in the case of rising demand growth and overutilization of capacity. The economy is therefore heading in the "right direction".

In Harrod's model, this reaction caused instability because, since there was no autonomous consumption ($Z = 0$), the growth in demand always increased (or decreased) at the same rate as the subsequent growth of the capital stock and productive capacity. The absence of autonomous consumption makes it impossible for changes in the level of investment to change the investment share , as that is univocally determined by the marginal propensity to save. In the Sraffian supermultiplier model, the average propensity to save depends entirely on the propensity to *invest*. As the latter increases in response to overutilization of capacity, so does the average propensity to save, which ends up adjusting to the rate of induced investment required to *adjust capacity to aggregate demand*.

In equation (7), given *s* and *v*, variations in the investment share *h* alter the "fraction" $f = [I/(I+Z)]$, the amount necessary for the economy to endogenously generate the savings rate required by the expansion of aggregate demand and investment, causing the degree

of utilization to tend towards 1. In this model, in Harrod's language, the effective rate of growth is given by the expansion of autonomous spending on consumption, and, in a way, it is the "warranted rate" that adjusts to the effective rate via variations in the *average* (but not marginal) propensity to save, caused by induced variations in the rate of investment. The conclusion is that the "marriage" between the "accelerator" (induced investment) and the multiplier (induced consumption) proposed by Harrod works in the end, but only in the presence of a third element, the autonomous expenditures that do not create capacity.

5.2 Dynamic stability and limits of demand-led growth

In the discussion above, reference was made to the *gradual* adjustment of the marginal propensity to invest in relation to discrepancies between the actual degree of capacity utilization u and its normal or planned level ($u=1$). The point is that the fundamental stability of the adjustment of capacity to demand described above is certainly a *necessary* but not *sufficient* condition for the demand-led growth regime described by the Sraffian supermultiplier model. It is the partial or gradual adjustment of the investment share that provides a sufficient condition.

If, for example, faced with an increase in the rate of growth of autonomous expenditures z and the consequent increase in the degree of actual utilization of capacity, the rate of induced investment reacts strongly and increases rapidly, it is possible that the process of adjusting capacity to demand becomes dynamically unstable. This could happen because although the adjustment is going in the right *direction*, its *intensity* may be excessive and the growth (or fall) of induced investment may be excessive (there is an overshooting). If the increase in the induced investment share is very large over a short period, it is quite possible that the growth in aggregate demand will be so high that it will be impossible for supply (production and capacity) to keep up with it. If the induced investment rate plus the marginal propensity to consume becomes an aggregate marginal propensity to spend greater than one, any positive autonomous spending induces infinite total aggregate demand. The dynamic stability of the supermultiplier requires that this does not happen. This is why the model requires the additional assumption that the changes in the

investment rate induced by changes in the effective growth rates of the economy are gradual.

The idea is that the investment share does not depend on the actual growth rate of output in a given period *g but on* the expected long-run growth trend of the economy g^e. When the actual growth rate of output changes, the expected trend of the long run growth rate g^e will be revised but only partially and gradually because firms understand that demand fluctuates and not every variation in demand is permanent and also because in an economy that uses fixed capital, the aim of firms is to adjust productive capacity to demand during the useful life of the equipment and not every moment in time. This type of gradual adjustment of long run expectations of demand is known as "flexible accelerator" as opposed to the so-called "rigid acceleratior" in which firms continuously try to adjust capacity to demand and take any variation in demand as permanent.

Thus, the expected rate of growth of demand in any given period is determined by what was expected in the previous period, plus a term for the correction of previous expectation mistakes. The latter term is given by the difference between the previously expected and the recently observed rates of growth one multiplied by some coefficient, *b*, that represents the intensity of the adjustment. Let us assume that *b* is a sufficiently small coefficient (*b=0* would mean that the investment share is exogenous; *b*=1 would be the case of the rigid accelerator; positive and relatively small *b* is the case of the "flexible accelerator").

Formally, a sufficient condition for the dynamic stability of the process would be that the aggregate marginal propensity to spend on both consumption and induced investment in the vicinity of the position where capacity is adjusted to demand must be lower than one. For this to happen, and for capacity to be able to adjust to demand, the marginal propensity to invest both that required by the economy's expansion trend *vz* and that which arises in response to deviations in the degree of capacity utilization *vb* has to be lower than the marginal propensity to save *s.*

The hypothesis of gradual adjustment of the investment share is quite realistic given the empirical success of flexible accelerator models that imply partial adjustment compared to rigid accelerator models where adjustment is rapid. But in any case, the need to

guarantee the dynamic stability of the process shows that there is a well-defined limit to what can be characterized as a demand-led growth process. This limit shows that the economy is always in a regime of demand-led growth if the growth rate of autonomous spending *z* is not "too high", i.e.:

$$z < \frac{s}{v} - b \tag{9}$$

In the case where we can assume that the supermultiplier model is dynamically stable, there will be a tendency for the investment share to adjust to the growth rate required by the trend in demand growth, which in turn will tend to be equal to the growth rate of spending on autonomous consumption *z* ($g^e = z$). Furthermore, as in the dynamically stable case, the level of productive capacity also tends to adjust to the trend level of output, we will also have that:

$$Y^* = KR = \frac{Z}{s - vz} \tag{10}$$

Therefore, there is a tendency for the economy's potential output to follow the trend in the evolution of effective demand and for this to grow at the rate of growth of autonomous spending that does not create capacity for the private sector (autonomous consumption in the case of this version of the model).

6 Neither Oxford nor Cambridge

We can extend this argument to a critical analysis of the post-Keynesian theories of growth discussed above. To do this, it is enough to remember that in the two post-Keynesian strands the share of savings in profits, $s_p = S/P$, was *given* precisely because there was no autonomous consumption. It follows that if we admit the existence of

autonomous capitalist consumption and assume that the *marginal* propensity to save from profits is equal to one[15] , then s_p becomes endogenous.

In this way, the share of savings in profits is a function of the levels of investment and autonomous consumption, and therefore of the investment share. We can then use the "Oxbridge" equation to determine the value of s_p assuming: (i) that $g=z$; (ii) that the distribution of income (r) is determined exogenously; and (iii) that it tends, as described above, towards normal capacity utilization ($u=1$). In this case, from (5) we obtain:

$$s_p = \frac{z}{r} \tag{11}$$

A reduction in z, for example, would initially lead to a situation of underutilization of productive capacity ($u<1$). However, contrary to what happens in Oxford's theory, this situation would not tend to last. Eventually, firms would react to this situation by reducing the economy's investment share. That would both ultimately allow productive capacity to adjust to aggregate demand and not the other way around, as in the adjustment via distribution advocated by the Cambridge theory, and would also proportionally reduce the *average* share of profits saved s_p .[16]

In the supermultiplier model, the rate of capital accumulation follows the expansion of effective demand and autonomous spending, and the normal rate of profit is determined independently by the forces discussed by Sraffian economists (see Summa, Serrano & Freitas (2024)). Thus, there is no necessary link between economic growth and income distribution as in Cambridge theory, even when the degree of utilization has adjusted to

15 This is a simplifying hypothesis which, if relaxed, would not affect the conclusions of the argument, it is enough that there is *some* autonomous component Z in capitalist consumption (cf. SERRANO, 1995, p. 112, n. 23).

16 The same reasoning could be developed for the case in which g increases and we initially have $u>1$.

normal, since the proportion saved out of profits s_p, is the only endogenous variable in the "Oxbridge equation".[17]

Therefore, based on the assumptions made above, using the Sraffian supermultiplier, it is possible to derive an endogenous tendency to normal capacity utilization, without using the adjustment mechanism proposed by Cambridge theory. Growth in the supermultiplier model is demand-led and productive capacity adjusts to the expansion of autonomous spending and effective demand, *contrary to the* Cambridge view.

Moreover , in the Sraffian supermultiplier the levels of output and productive capacity are “wage led”, as the marginal propensity to save out of profits (set to one here for simplicity) is assumed to be greater than the marginal propensity to save out of wages (set to zero here, again for the sake of simplicity). As can be seen in equation below, a single reduction of the profit share (within the range for which the model is stable) will lead to permanently higher levels of both output and productive capacity, even in the fully adjusted positions in which capacity has fully adjusted to the trend of demand.

$$Y^* = KR = Y = \frac{Z}{[(1-w) - vz]} \tag{12}$$

This however will be just a level effect, as the assumption that business investment is induced and the investment share follows a flexible accelerator process means that investment the stock of capital will grow at the rate of growth of autonomous demand and hence investment and growth will not permanently “wage led”. On the other hand, assumption that investment is a derived magnitude fully induced by effective demand also entails that the investment and capital accumulation will not be profit-led at all, as a single higher share of profits will in fact decrease induced consumption and the levels of output and productive capacity. In this view firm owners probably will be quite happy with a higher rate of profits at normal capacity utilization, but there is no reason why this would make them increase investment and productive capacity if the market is not only

17 Furthermore, it is possible to show that there is no inverse relationship between consumption and investment levels, also contrary to what is supposed in Cambridge theory. See Serrano (1995a, chapter 3).

not growing any faster(and in fact its level will actually be smaller relative to what it was before the change in distribution).[18]This does not mean that changes in the profit share and the normal rate of profits will have no impact on the process of demand led growth, but its impact will be indirectly as distributive conflicts may play a decisive role in the policy and political constraints that ultimately affect the rhythm at which autonomous demand is allowed by the State to expand due to such constraints (for an analysis of these indirect political and policy effects of distribution on demand led growth see Summa, Serrano & Freitas (2024)).

7 Final remarks

In this chapter we have shown that Harrod's principle of fundamental instability of the warranted rate *s/v* is valid under very general hypotheses because the adjustment takes place in the opposite *direction to* the equilibrium. We also saw that this led the post-Keynesian growth model literature to move away from the hypothesis that long run investment should be induced by the principle of capital stock adjustment. The consequent adoption of the notion of autonomous investment in the long run created what we called the Oxbridge dilemma, which consists of having to choose between the trend towards normal utilization (and even then, at the cost of making aggregate demand adjust to capacity) and exogenous distribution. Subsequently, we saw that the introduction of autonomous expenditures that do not create capacity for the private sector[19] allows the economy to grow at the long run growth rate of these expenditures and that this trajectory is fundamentally stable, unlike Harrod's warranted growth rate, as the adjustment between capacity and demand now goes in the "right direction". In addition, the Sraffian supermultiplier model overcomes the dilemma of the Oxbridge models and allows exogenous distribution and the trend towards a normal degree of utilization to coexist in the same model. We also show that under realistic conditions of gradual adjustment of

[18] On this see Serrano & Freitas (2017)

[19] Which in this version of the model appears as autonomous consumption, but in reality, can also include residential investment, all government spending and exports.

the capital stock (or flexible accelerator) the adjustment mechanism of this model, which occurs through endogenous changes in the ratio between the average and marginal propensity to save (the fraction) through changes in the marginal propensity to invest (induced investment rate) can also be dynamically stable[20] .

[20] Note that many Sraffian authors do not adopt this supermultiplier model for methodological reasons. For opposing views on this controversy, see Cesaratto (2015) and Trezzini and Palumbo (2016). On the other hand, nowadays, other heterodox economists have adopted and developed their own versions of this demand-led supermultiplier (Allain, 2015, Lavoie, 2016, Deleidi and Mazzucato, 2019, Dutt, 2019, Palley, 2019, Fazzari et al., 2020).

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