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Debt Sustainability Under Uncertainty: a tentative simulation model for Brazil¹

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Abstract

This paper presents a stochastic Monte Carlo simulation model designed for the analysis of debt sustainability under macroeconomic uncertainty. The framework integrates four key features of debt dynamics, including (1) a two-state Markov-switching process for growth; (2) growth rates that respond to domestic and international conditions; (3) interest rate and surplus processes that react to high values of the debt-GDP-ratio, and (4) a shock structure implemented via a Cholesky decomposition that allows for macroeconomic shocks to be intercorrelated. The model is implemented as an interactive tool allowing for extensive scenario and sensitivity analysis.

¹ 2025

1 Introduction

Debt levels have increased in the last few years in most countries, following expenses with the pandemic and efforts to curb inflation. According to the IMF Fiscal Monitor (April 2025), the world's general government debt has increased from 83.8 percent of GDP in 2019 to 92.3 percent in 2024. In advanced countries, the average debt-to-GDP ratios have grown from 103.6 in 2019 to 108.5 percent in 2024, with the United States debt increasing 12.6 percentage points over GDP during the same period. In emerging markets and middle-income economies, the increase has been even more pronounced, with average debt growing 19.6 percentage points over GDP between 2019 and 2024, from 55.2 to 74.8 percent. For Brazil, the IMF's data shows a debt-GDP increase of five percentage points, from 87.1 in 2019 to 92.0 in 2024.

Debt-GDP ratios are monitored by policymakers, along with other macroeconomic indicators, to analyze how solid a country's fiscal condition is and how much fiscal space the government might have to implement fiscal policies. In analyzing the sustainability of fiscal policy and debt-to-GDP levels, it is important to identify risks and a range of macroeconomic circumstances. In the case of countries in the EURO zone, for example, the debt target was established in 60% of GDP, a level that has been hit or overpassed by most member countries after the financial crisis of 2008. For countries that do not have explicit targets for the debt, maintaining it below certain levels might be prudent as borrowing costs tend to rise fast when lenders are not confident about the government's solvency. The slippery slope of borrowing costs usually becomes worse when a government is already facing excessive costs and fiscal retrenchment depends on politically difficult fiscal adjustments.

Traditional deterministic approaches to debt sustainability often fail to capture the profound impact of macroeconomic uncertainty on debt trajectories. Recently the IMF staff has called attention to the benefits of using probabilistic forecast methods to analyze a country's debt trajectory in the mid to long term, and to analyze the chances of compliance to a debt target [IMF 2021]. To address the limitation of deterministic approaches, we present a stochastic simulation model that allows for a probabilistic assessment of debt dynamics and we apply it to Brazilian data.

A probabilistic approach applied to Brazilian data was pioneered by Garcia and Rigobon (2004). The authors estimated a vector autoregression model (VAR) and used the VAR coefficients to generate paths for debt. Tanner and Samake (2008) followed the same methodology and applied to data for Brazil, Mexico and Turkey. Here, we do not use the coefficients of a VAR model to simulate paths for the debt. Instead, our model extends the standard debt accumulation equation by specifying stochastic processes for each of its key drivers: the real interest rate, the real GDP growth rate, and the primary surplus. Modeling each component of the debt equation allows us to incorporate features to enhance the realism of the model. First, the GDP growth process is governed by a two-state Markov-switching model, capturing the distinct dynamics associated with economic expansions and recessions. Second, the process for the interest rate reflects the idea that an increasing debt-income ratio may require a greater risk premium; third, governments usually react to a growing fiscal risk and try to increase its primary surplus. Finally, reflecting the empirical observation that macroeconomic disturbances are not independent events, we assume that variables are affected by correlated shocks.

2 What is a Sustainable Debt

Most of the economic literature defines debt solvency based on the concept of sustainability. The debt is considered sustainable if its current market value (B_t) can be offset (backed) by a sequence of expected primary surpluses (S_{t+j}) of equal present value. That is, the debt is sustainable if

$$B_t = \sum_{j=1}^T E_t S_{t+j}$$

This condition implies that the government is following its intertemporal budget constraint.

There are many empirical tests for a sustainable debt based on the concept of a government that follows its intertemporal budget constraint. The seminal work of Hamilton and Flavin (1986) was followed by a prolific literature that explored statistical properties of the series for debt, revenue and expenditure, that were applied to several

countries.² In particular, the general principle designated by Bohn (1998) and the idea of sustainability based on a fiscal reaction function has become a classical framework, with fiscal reaction functions estimated for virtually every country, both individually and as a group. For Brazil, fiscal reaction functions have been estimated by Luporini (2002), De Mello (2005), Mendonça, Santos and Sachsida (2009), Simonassi (2013), and more recently by Pereira (2020). Notwithstanding its popularity, Bohn's principle suffers from a few shortcomings. First, the estimation of a government's fiscal reaction is based on past data and thus implicitly assumes that future fiscal policy will remain the same on average. In fact, governments tend to revise their fiscal policy from time to time, responding to unexpected changes in economic conditions such as economic growth or borrowing costs, as clearly shown by Mauro, Romeu, Binder and Zaman (2015).

One may argue, as we have done, that this potential problem may be minimized by estimating a time-varying fiscal reaction function [Luporini (2015)]. Yet, there is a second problem. The fiscal reaction is based on a marginal effect, the change of the primary surplus to a change in debt. Although a positive coefficient on the surplus response to a change in debt implies compliance with an intertemporal budget constraint on average, the condition does not impose any constraints on the debt-income level, being equally compatible with a low or a high level of debt. Finally, the concept does not offer operational guidance to the policymaker, and governments have turned to debt-to-GDP ratios as an indicator of a sustainable fiscal policy [Escolano, 2010].

A stable ratio implies a sustainable policy in most cases, including those of dynamic inefficiency, when economic growth is larger than the return on the public debt, and the government's budget constraint is not binding. A stable debt-to-GDP ratio implies not only a positive marginal response of surplus to changes in the debt, but also a response that is "large enough". An important issue related to keeping the debt-to-GDP ratio stable over time however is that a constant debt-income may not be an optimal debt policy. Tax-smoothing considerations, for instance, may require variation in the debt-income ratio to accommodate the economic cycle. Besides, in an uncertain environment, a stable

² For Brazil, first generation tests were applied by Rocha (1995), Tanner (1995), Luporini (2000), Issler and Lima (2000), Giambiagi, F. and M. Ronci (2004).

debt-income level may not lead to a sustainable fiscal policy under all states of nature (in the Euro area, “safe” debts ended up not being safe when shocks hit). Thus, a shortcoming of these empirical tests is not considering risk and uncertainty.

Debrun (2015) states that a government debt is safe “if it is associated with a low probability of reaching levels likely to generate significant economic costs with a given time frame”; for Blanchard (2022), a debt is safe when the risk of a steadily increase in debt-to-GDP ratio is small under existing and future policies.³ Under both definitions, the concept of a safe debt requires a debt level not only fiscally sustainable but also robust to fiscal shocks. Simulations are thus required to account for risk and uncertainty.

3 Model Specification

As argued, deterministic approaches often fail to capture the impact of both systemic risk and macroeconomic uncertainty on debt trajectories. To address these limitations, our model extends the standard debt accumulation equation by specifying stochastic processes for each of its key drivers: interest rate, GDP growth rate, and primary surplus. Furthermore, the model incorporates two advanced features to enhance its forecasting. First, the GDP growth process is governed by a two-state Markov-switching model, capturing the distinct macroeconomic dynamics and risk assessments associated with economic expansions and recessions. Second, uncertainties are incorporated by means of a model of random shocks that are correlated, determined by a Cholesky decomposition, as macroeconomic disturbances are never independent events.

The model is structured around a standard equation for the evolution of the debt-income ratio:

³ In fact, Blanchard (2022) defines an “unsafe” debt “when there is a non-negligible risk that, under existing and likely future policies, the ratio of debt to GDP will steadily increase, leading to default at some point.” (p. 8).

$$b_t = \frac{1+r_t}{1+g_t} b_{t-1} - s_t \quad (1)$$

Where,

b_t is the debt-to-GDP ratio at time t (gross debt)

r_t is the real interest rate on the debt stock

g_t is the growth rate of real GDP

s_t is the government primary surplus as a share of GDP

The fiscal reaction function proposed by Bohn (1998, 2005) that reflects a possible corrective action by the government in response to changes in its debt-income ratio is given by:

$$s_t = \bar{s} + \rho b_{t-1} + \mu_t \quad (2)$$

Where ρ reflects the fiscal response of the primary surplus to the debt-to-GDP ratio (sustainability factor); μ_t is bounded as a share to income and represents other determinants of fiscal surplus, such as cyclical variations due to economic activity; \bar{s} is a conditional mean for the primary result. A strictly positive ρ implies that a negative shock to the debt-income ratio eventually leads to an increase in primary surplus [Bohn (1998)]. Substituting (2) into (1), we obtain:

$$b_t = \left[\frac{1+r_t}{1+g_t} - \rho \right] b_{t-1} - [\bar{s} + \mu_t] \quad (3)$$

Assuming that $\bar{s} + \mu_t$ is a function of s_t , we can write:

$$b_t = \left[\frac{1+r_t}{1+g_t} - \rho \right] b_{t-1} - f(s_t) \quad (4)$$

According to equation (4), the real interest rate, growth and the primary surplus are the main drivers of the debt dynamics, with the sustainability response (ρ) acting as an

attenuation factor.⁴ Stochastic processes for growth, real interest rate and the primary result must be modeled to provide simulations for the debt-ratio trajectories over time. A value for ρ will be assigned based on estimates for the fiscal response of the government.

3.1 Stochastic Processes

The main drivers of the debt-ratio dynamics are assumed to follow some stochastic process. We will add an error term to each process to incorporate uncertainty. Moreover, as macroeconomic shocks are usually interrelated, the error terms for each process are allowed to be correlated.

3.1.1 Interest Rate Process

The real interest rate r_t is assumed to follow a first order autoregressive process with a feedback term from changes in the level of debt. The feedback term allows for a scenario where lenders require a higher return to hold government securities, especially when debt increases too fast casting doubts about its sustainability.

$$r_t = c_r + \phi_r r_{t-1} + \phi_b \max(b_{t-1} - b_{t-2}, 0) + \varepsilon_{rt} \quad (5)$$

In equation (5), c_r is a constant (conditional mean), ϕ_r is the autoregressive coefficient, and ϕ_b captures the possible feedback from changes in the debt-ratio. Specifically, ϕ_b is positive if the debt-ratio has increased in the previous period; otherwise, the parameter is zero.⁵ The idea is that an increasing debt-income ratio implies a greater probability of default and may require a greater risk premium. If a sequence of rising debt is followed by a decrease in the debt-income ratio, the perception of risk may not decline, and the

⁴ Estimations for the sustainability factor result in ρ being strictly positive and smaller than one. For estimations see Luporini (2002, 2015), De Mello (2005), Lima and Simonassi (2005), Mendonça, Santos and Sachside (2009), Simonassi (2013), Campos and Cysne (2019), Pereira (2020).

⁵ As an alternative, the feedback parameter could be activated when the previous increase in the debt-ratio reached a particular threshold, for example, five percentage points. In this case, there would be a discontinuity, with a “jump” in the real interest rate.

current real interest rate will carry the increased premium. Thus, the proposed process for r_t implies an asymmetric effect of an increase or decline in debt on the real interest rate. ε_{rt} is the (correlated) interest rate shock.

3.1.2. Economic Growth: a Markov-Switching Model

The real GDP growth rate, g_t , is assumed to follow a process whose parameters depend on the state of the economy, $SS_t \in \{E, R\}$, *Expansion* and *Recession*, respectively. The state transitions are assumed to follow a first-order Markov chain with a transition probability matrix P:

$$P = \begin{pmatrix} P_{EE} & 1 - P_{EE} \\ 1 - P_{RR} & P_{RR} \end{pmatrix} \quad (6)$$

Where P_{ij} is the probability of transitioning from state i to state j .

We also assume that the growth rate is affected by world economic conditions (W) and by the domestic real interest rate (r). Specifically, the growth equation is specified as:

$$g_t = c_{SS_t} + \phi_g g_{t-1} + \phi_{rg} r_{t-1} + \phi_I W_t + \varepsilon_{g,t} \quad (7)$$

Where c_{SS_t} is the state-dependent intercept taking value c_E in an expansion and c_R in a recession; ϕ_g is the autoregressive coefficient for growth; ϕ_{rg} and ϕ_I capture the effect of domestic and international economic conditions on growth, respectively. $\varepsilon_{g,t}$ is a (correlated) growth shock for period t.

Completing the equation for the growth rate, we need to specify a model to capture world economic conditions. The external index is modeled as a simple first order autoregressive process [AR(1)]:

$$W_t = c_w + \phi_W W_{t-1} + \varepsilon_{W,t} \quad (8)$$

3.1.3 Primary Surplus

The primary surplus, s_t , is modeled as a mean-reverting process with a feedback term from the debt-GDP ratio:

$$s_t = s_0 + \phi_s \max(b_{t-1} - \bar{b}, 0) + \varepsilon_{s,t} \quad (9)$$

Where s_0 is the initial surplus (a constant), and $\phi_s \in [0,1[$ is meant to capture a possible extra fiscal consolidation response activated when the debt exceeds a threshold \bar{b} ; if the debt-ratio does not reach the threshold, the surplus is s_0 plus a correlated shock $\varepsilon_{s,t}$.

The idea is to model the primary surplus as a decision variable that may vary when the debt becomes “too high”. In the simulations, we assume $\phi_s = 0.005$ (or 0.5% over GDP) and it is activated when the debt-GDP ratio exceeds 90%.

3.2 The Shock Structure

One of the main problems with analyzing debt sustainability using deterministic models is that scenarios rarely consider co-movements among the key variables for debt dynamics, such as growth and interest rates, and shocks are assumed to be independent [Celasun, Debrun and Ostry (2007); Blanchard (2022)].

For each variable in the system, it is reasonable to assume that shocks may affect macroeconomic conditions and the resulting evolution of the debt-GDP ratio. These shocks are interrelated by nature and involve not only the variables directly related to the debt equation but also other variables such as the exchange rate and international financial conditions. A simple way to depict the relationship between macroeconomic variables is through a vector autoregression system (VAR). Besides getting estimates of the interaction coefficients between these macroeconomic variables, the main purpose of the VAR is to obtain a matrix of correlations between shocks to these variables.

The ordering of the variables in the VAR is immaterial for the estimated coefficients, forecasts and correlation matrix, but it does matter for a Cholesky decomposition because it imposes a recursive (lower triangular) structure on the contemporary relationship

between the variables. For instance, to get correlated shocks for our estimations, we estimate an unrestricted VAR model with the following variables: index for international conditions, real GDP growth, domestic real interest rate, and the primary surplus, or [W, g, r, s]. The ordering used in our VAR, in a Cholesky setup, implies that a shock to the index for international conditions can have a contemporaneous effect on all other variables of the system; a shock to GDP growth can contemporaneously affect the domestic interest rate, and the primary surplus, but not the international index; a shock to domestic interest rate may affect the contemporaneous primary surplus, but not GDP growth or the international index; finally, a shock to primary surplus may only affect future values of the other variables.

The variance-covariance matrix of the residuals of the VAR characterizes the joint statistical properties of the disturbances that will affect the debt equation. In the associated correlation matrix for the residuals, the variance-covariance matrix is standardized (each covariance is divided by the product of the standard deviations of the corresponding residuals) so that all diagonal elements will equal 1 (representing the correlation of a residual to itself) and the off-diagonal elements will represent the correlations among the shocks of the variables in the VAR.

For the simulations, we define a vector of shocks $\epsilon_t = [\epsilon_{W,t}, \epsilon_{g,t}, \epsilon_{r,t}, \epsilon_{s,t}]'$, a correlation matrix C , and a vector of standard deviations, $\sigma = [\sigma_W, \sigma_g, \sigma_r, \sigma_s]$. The correlated shocks are generated by first drawing a vector of independent standard normal shocks, $z_t \sim N(0,1)$, and then applying the Cholesky decomposition of the correlation matrix such that $C = LL'$. The vector of correlated standard normal shocks is then $\epsilon_t^{std} = LZ_t$.

The final economic shock for each variable is obtained by scaling the corresponding element of ϵ_t^{std} by its standard deviation. This procedure is standard [Celasun, Debrun and Ostry (2007)]. For example, the shock to the growth rate becomes $\epsilon_{g,t} = \sigma_g \cdot \epsilon_t^{std}$. The advantage of this procedure is that we keep the correlations obtained from a VAR and we can calibrate the shocks for the simulation by inputting a particular standard deviation for each variable.

4 Implementing the Simulation Model

The simulation model described above depends on parameters and correlations that must be estimated from data. We used quarterly data from 1996 through 2024. A complete description of the data and sources can be found in the Appendix.

4.1 Data and estimated parameters

The domestic real interest rate was calculated as the ratio between *Selic* nominal rate and the Broad National Consumer Price Index (IPCA), both based on monthly data (series 433 and 4390 from the Brazilian Central Bank (BCB)). The real interest rate for the quarter (r_t) was calculated as the average for the months of the quarter, and annualized. Data for real GDP growth (g_t) is the quarterly change from quarter one year ago, not seasonally adjusted, from *Instituto Brasileiro de Geografia e Estatística* (IBGE). The primary result are flows accumulated in the year for the consolidated public sector as percentage of GDP. This data is reported monthly. As it represents flows accumulated in the year, quarterly data (s_t) are values observed for the last month of the quarter (months 3, 6, 9 and 12).

International conditions are represented by the terms of trade for Brazil, defined as the ratio between prices for exports and imports (PX/PI). The index (average 2018=100) is from *Fundação Centro de Estudos do Comércio Exterior* (FUNCEX). As all variables in the simulations are in rates, the variable for international conditions (W_t) represents the percentage change in the terms of trade index.⁶

Coefficients were estimated for each stochastic process. Results are presented in Table 1.

⁶ We experimented with several variables to capture the effects of international economic conditions on Brazilian growth, such as the real effective exchange rate, the volatility index (VIX), and commodity price indexes (international index IGBEA). Terms of trade was shown to be the most influential for Brazilian growth data.

Most coefficients are statistically significant and, most importantly for our purposes, jointly significant in each equation as indicated by the F tests. Also, residuals do not exhibit serial correlation.

Table 1: Estimated Coefficients for Stochastic Processes

Dependent Var	Real Interest Rate (rt)	Growth Rate of Real GDP (gt)	International Conditions (Wt)
constant (c)	1.0908 [0.7951]	0.7496 [0.4345]*	0.4801 [0.3422]
r (t-1)	0.7295 [0.1120]***	-0.1049 [0.0309]***	
r (t-2)	-0.2750 [0.1293]**	0.0958 [0.0411]**	
r (t-3)	0.2967 [0.1368]**	-0.0620 [0.0371]*	
r (t-4)	0.0812 [0.0909]	0.0989 [0.0368]***	
g (t-1)		0.5540 [0.1105]***	
Wt		0.1086 [0.0380]***	
W(t-1)			1.0278 [0.0891]***
W(t-2)			-0.2559 [0.1293]*
W(t-3)			0.0328 [0.1266]
W(t-4)			-0.1674 [0.0854]*
D20Q2		-9.8174 [0.5181]***	
D21Q4			-14.5754 [3.7600]***
D21Q1			8.7416 [3.7403]**
D09Q1			-6.6521 [3.6488]*
Observations:	116	115	116
Adj. R-squared:	0.6124	0.6499	0.754
F-statistic:	46.429	31.2349	51.3483
Prob(F-stat):	0.0000	0.0000	0.0000
LM Test (Prob values)	0.1241	0.3545	0.2472

Notes: OLS estimations, quarterly data, 1996q1-2024q4 (Eviews 14); D20Q2 is a dummy =1 for year 2020, quarter 2.

HAC robust standard errors values in brackets: *** p < 0.01, ** p < 0.05, * p < 0.1.

Serial Correlation LM Test (Breusch-Godfrey): null of no serial correlation.

The possible change in regime for the growth rate is estimated by a simple two-regime Markov switching regression. Results are presented below in Table 2.

Table 2: Markov Switching Regression for Real GDP Growth

Variable	Coefficient	Std. Error	z-Stat	Prob.
Regime 1	-2.1822	0.7388	-2.9536	0.0031
Regime 2	3.0641	0.2545	12.0410	0.0000
LOG(Sigma)	0.7915	0.0718	11.0292	0.0000
Constant transition probabilities:				
		1	2	
	1	0.8210	0.1790	
	2	0.0321	0.9679	
Constant expected durations:				
		1	2	
		5.59	31.11	

Note: Quarterly data, 1996q1-2024q4. Transition matrix parameters are statistically significant at 5%.

The estimated average annualized growth rate for regime 1 is -2.18% (recession); for regime 2, average growth is estimated at 3.06% (expansion). Each regime is highly persistent, as indicated by the matrix of transition probabilities. A positive quarter is estimated to be followed by another positive one with the probability of 0.968; a negative quarter has a 0.821 probability of being followed by another negative quarter.

We are also interested in the correlations between the shocks hitting the main variables in the debt-GDP equation. As discussed earlier, correlations are estimated by an unrestricted vector autoregression model, also on quarterly data, 1996Q1 – 2024Q4, with the following endogenous variables: international conditions (specifically, the percentage change in the terms of trade), real GDP growth, domestic real interest rate, and the change in primary surplus, or $[W, g, r, Ds]$. Since the information set of holders of government securities also includes other variables such as movements in the exchange rate or the issuance of new debt, we exogenously include the change of the real exchange rate, the American funds rate in real terms, and the change in the real value of total securities issued by the Brazilian government.

The funds rate in real terms is the *Effective Federal Funds Rate*, adjusted for CPI inflation; the real exchange rate is the log of *Real effective exchange rate index* (series 11752 of the Brazilian Central Bank). The real value of government securities represents the total value issued by the Treasury (series 2213 of the Brazilian Central Bank) divided by the IPCA index (2024=100); quarterly values are the last month for the quarter.

The VAR is estimated with six lags for the endogenous variables, 3 exogenous variables, a constant term, and some quarter dummies. This specification can capture the memory of the series and address any unit root concerns [Toda and Yamamoto (1995); Banerjee et al (1993)].

As a result, the estimated VAR presents all roots inside the unit circle, no residual autocorrelation up to lag six, and normally distributed residuals (see details in the Appendix).

The estimated correlation matrix is presented in Table 3. As expected, a positive shock to terms of trade (PX/PM) is positively correlated with growth, while a positive (increase) shock to the real interest rate is negatively correlated to the growth rate. These correlations consider the possible (exogenous) impact of the real exchange rate, the real funds rate and a change in the value of government securities.

Table 3: Correlation of Shocks

	Terms of Trade	Growth	Real Rate	Surplus
Terms of Trade	1.0000	0.1890	0.1182	-0.0108
Growth	0.1890	1.0000	-0.3204	0.0348
Real Rate	0.1182	-0.3204	1.0000	0.0953
Surplus	-0.0108	0.0348	0.0953	1.0000

Note: Shocks from VAR estimated with Eviews14.

Finally, we scaled the shocks by the observed (average) standard deviation of growth, terms of trade and the real rate: 2.9, 7.2, and 7.6, respectively. For the change in surplus, the observed standard deviation for the period is 2.6, which we consider “too high” for the current fiscal situation of the Brazilian accounts. The standard deviation for the

surplus was hardcoded at 0.5, a more reasonable variation in the government's current primary result. Table 4 summarizes all parameters used in the simulations.

Table 4: Parameter Values

c_r	1.0908	c_w	0.4801
ϕ_r	0.8324**	ϕ_I	0.1086
ϕ_b	0.02*	ϕ_W	0.6372**
P_{EE}	0.9679	ϕ_s	0.005*
P_{RR}	0.8210	ρ	0.0094*
c_E	3.0641	σ_W	7.2000
c_R	-2.1822	σ_g	2.9000
ϕ_g	0.5540	σ_r	7.6000
ϕ_{rg}	0.0279**	σ_s	0.5*

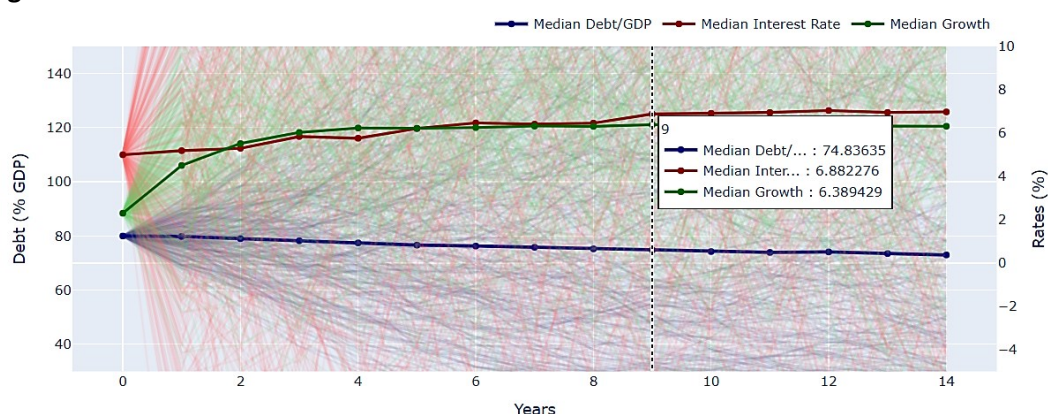
Notes: (*) indicates a hardcoded value; all others have been estimated.

(**) value represents the sum of lagged coefficients, e.g. $0.8324 = \text{sum}(0.7295, -0.2750, 0.2967, 0.0812)$

4.2 Model Implementation

The algorithm provides paths for the debt-to-GDP ratio, the real interest rate, and the growth rate based on the initial conditions provided. Although our main interest is on the paths of the debt-to-GDP ratio, results for growth and the real rate are useful to help us calibrate the model. We start by setting the debt-GDP at 80%, the real interest rate at 5% (natural rate), economic growth at 2.3% (average for period 1993-2023), a primary result (surplus) at 0% of the GDP and international conditions at zero (no variation). The time-horizon is set at 15 years to allow for assessment of medium and long-term risks [ABBAS, S. A. et al. (2021)]. We first simulated the model with the parameter values defined in Table 4. As seen above, the matrix of transition probabilities estimates a probability of 0.96 of the economy remaining in expansion and 0.82 at remaining in recession, with growth at 3.1 and -2.2%, respectively. When these values are jointly used in the simulations, however, median growth rates are not realistic, implying a long-term growth rate consistently higher than 5% per period [Figure A]. Also, medians for the debt-to-GDP ratio seem very benign, most probably underestimating debt growth.

Figure A – Simulation with Estimated Parameters

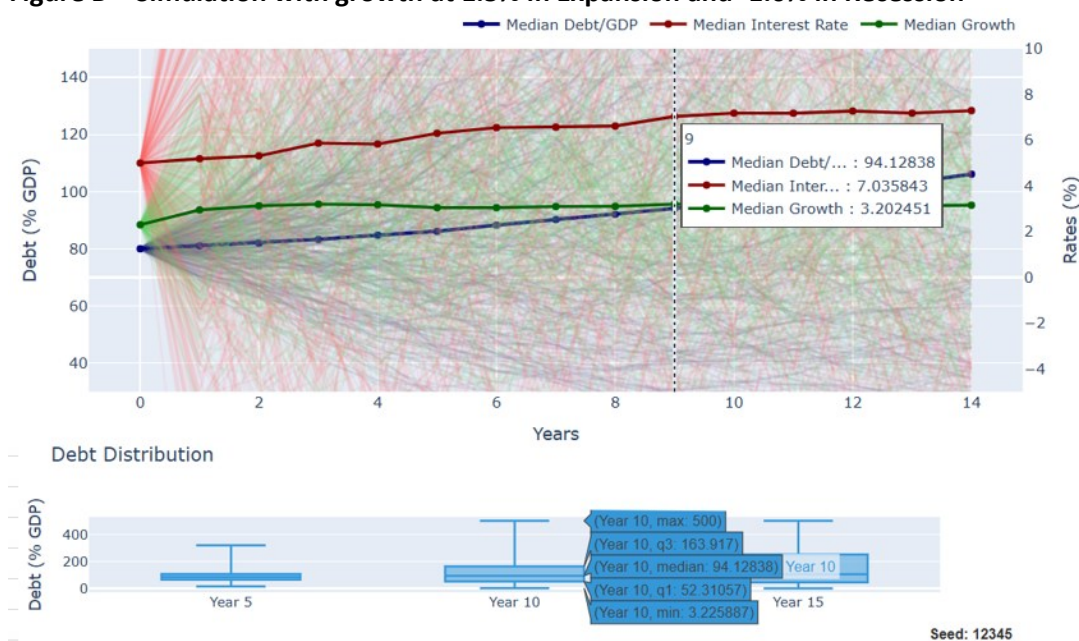


Brazilian real GDP has grown around 2% and estimates for potential GDP has varied from as low as 0.63 (IBRE/FGV) to 1.9 (OECD), or a more optimistic 2.3% (Central Bank) [Monteiro, S. (2024)]. We decided to calibrate the model for average growth of 1.5% in expansions and

-1.0% in recessions, while keeping the estimated probabilities at 0.96 (expansion) and 0.82 (recession). These growth rates represent approximately 1 (one) standard deviation from the mean growth for the period. These values generate a median long-term growth rate of 3.2%, more compatible with Brazil's historical growth [Figure B].

Simulation results for the median debt-to-GDP ratio are in line with those forecasted by market participants and published by Focus (Brazilian Central Bank), at 94.1%. According to Focus research, in May 2025, the median of expectations for debt-GDP ratio 9 years ahead was at 93.6%. Also, in 9 years, we observe in our results a wide distribution of debt paths, an indication that shocks are producing variability in the model.

Figure B – Simulation with growth at 1.5% in Expansion and -1.0% in Recession



Single simulation (10,000 paths with $g = 80\%$, $r = 5\%$, $g = 2.3\%$, $s = 0$, $W = 0$)

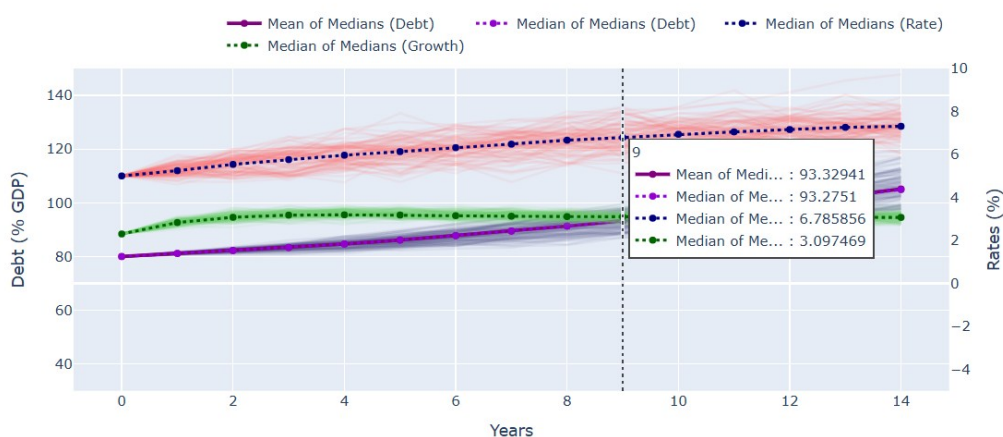
The credibility of a simulation depends, among other things, on the confidence one may have in the information produced by the model and the validity of a simulation model must be tested against its purpose [Sargent (1992)]. The main purpose of our tentative model is to provide future trajectories for the Brazilian debt-GDP-ratio based on different scenarios for growth, real interest rate, and fiscal policy (surplus and responses to changes in debt levels), considering international economic conditions and interrelated macroeconomic shocks that may affect the government budget.

Our simulations are produced by a single run of 10,000 (ten thousand) paths generated by the model, setting a particular seed for random shocks. The seed is kept the same so that results from the simulation may be reproduced, and the estimated median may be directly compared between scenarios. To find out whether our single simulation is typical, we run 1,000 (one thousand) simulations (each with 1,000 paths), with different random seeds and check if trajectories from these “samples” of simulations converge or are “close” to

each other. When paths converge as multiple simulations are run, it is an indication that results are consistent and stable.⁷

Because it is desirable that the seed we used in the single simulations reported be included amongst the multiple shorter simulations, we set the first seed (at 12345) for simulation 1 (of one thousand). The results are depicted in Figure C.

Figure C – Multiples Simulations: growth at 1.5% (Expansion) and -1.0% (Recession)



Multiple simulations (1000 simulations of 1000 paths with $g=80\%$, $r=5\%$, $g=2.3\%$, $s=0$, $W=0$)

We see in Figure C that simulations of our model with the estimated parameters, calibrated with growth rates at 1.5% (expansion) and -1.0% (recession) generate consistent paths, with the mean of medians for the debt-to-GDP ratio at 93.3%. This indicates that our single simulation (seed 12345) might be considered “typical”.

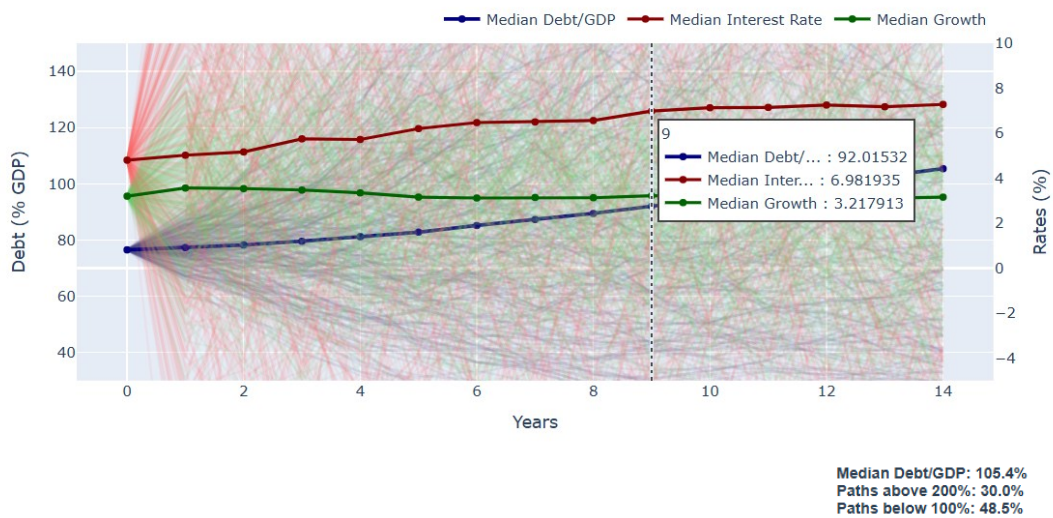
Given the consistency of our simulations, we simulated the model with the initial conditions that prevailed in the last quarter of 2024: growth rate of 3.2%, a primary surplus of -0.4% of GDP, a variation of 1.7 in terms of trade and a real interest rate of 4.8%. For the initial debt-GDP ratio, we first assumed the value of 76.5% as published

⁷ As pointed out by Balci (1998), determining the validation of simulation models may be costly and time-consuming; ideally it should be done by a third party, as the views of the researcher are subjective and influenced by the very process of developing the model. Running multiple simulations is a way to verify stability and consistency of the results.

by the Brazilian Central Bank.⁸ Results indicate a median debt at 92.02% of GDP after 9 years, a figure in line with market predictions. In this simulation, 48.5% of paths have debt levels below 100% of GDP in year 15, and 30.0% of paths present debt levels above 200% of GDP [Figure D].

According to the methodology of the IMF, which includes sovereign debt held by the Central Bank, the Brazilian debt was at 87.3% of GDP in 2024. Using this initial value for the debt, we reach a median debt of 104.3% of GDP in 9 years. The IMF predicts that the debt-ratio for Brazil will reach 99.4% of GDP in 5 years (2030) [Fiscal Monitor, 2025-April]; according to our model, this level of debt would be reached 1 years later, in 2031. In year 15, debt-GDP ratio is below 100% in 44.3% of paths, and above 200% in 33.7% of paths [Figure E]. Table 5 presents a summary of the main results.

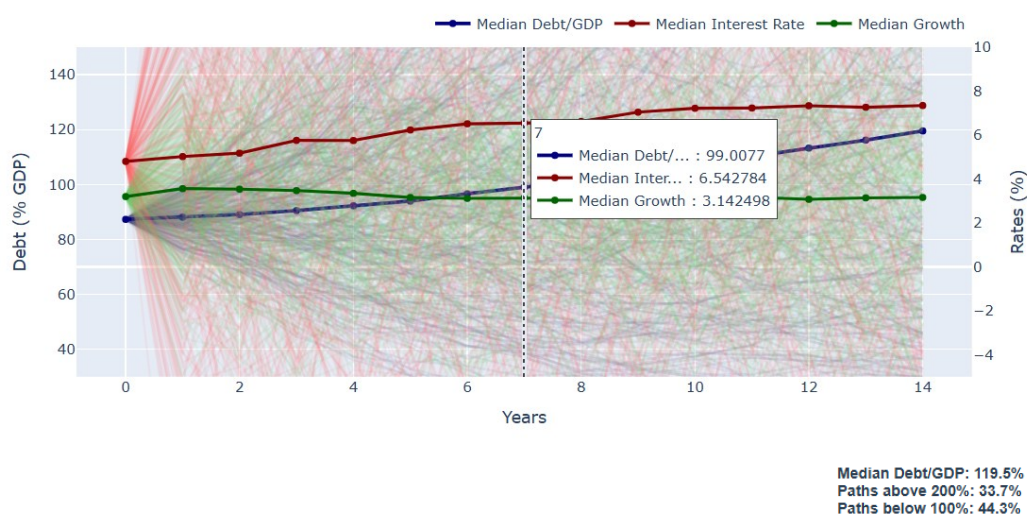
Figure D – Conditions in 2024Q4 with Debt at 76.5% of GDP



[debt at 76.5%, surplus of -0.4%; $r = 4.8\%$, $g = 3.2\%$, $W = 1.7\%$]

⁸ The simulation model was designed to be “user friendly”; frames of the simulator interface is presented in the appendix.

Figure E – Conditions in 2024Q4 with Debt at 87.3% of GDP



[debt at 87.3%, surplus of -0.4%; $r = 4.8\%$, $g = 3.2\%$, $W = 1.7\%$.]

Table 5: Conditions in last quarter of 2024

	BCB (initial debt 76.5%)	IMF (initial debt: 87.3%)
Debt-GDP (%), median		
At Year 5	82.81	93.95
At Year 9	92.02	104.30
Probability of Debt < (or equal) 100% of GDP		
At Year 5	0.746	0.615
Until Year 5 (inclusive)	0.740	0.592

Note: $s = -0.4\%$; $r = 4.8\%$, $g = 3.2\%$, $W = 1.7\%$; time horizon 15 years, 10000 paths.

5 Scenarios

In this section, we will analyze some scenarios considering variations in primary surplus, growth, interest rates, and international conditions.

5.1 Variations in the Primary Result (Surplus)

According to our model, the process for the surplus is governed by s_0 , which reflects a policy decision, shocks, and a response to the debt. We assume that surplus increases by 0.5% of GDP when the debt-GDP ratio exceeds 90%. Also,

the long-run fiscal reaction of the surplus to debt is captured by ρ . This parameter is not a direct result of a particular policy decision but estimated based on data for surplus and debt over several fiscal years. We analyze the effects of varying the initial conditions for the primary surplus by running 10,000 (ten thousand) paths for each scenario. Simulations are presented in Figure 1 and results are summarized in Table 6.

In Scenario A, we assume that the government established a positive surplus of 0.5% of GDP. Results indicate that the median for the debt-to-GDP is expected to be at 89.1% after 9 years. That is five percentage points below the median expected under a scenario of a surplus=0% (94.1, see results in Figure B). The number of debt-GDP paths below 100% in Scenario A is 50.9%, against 48.4% before in year 15. These results suggest that a fiscal effort of 0.5% of GDP could be enough to change the odds towards a more sustainable debt in the next 5 years.

Figure 1 – Scenario A: a surplus of 0.5% of GDP

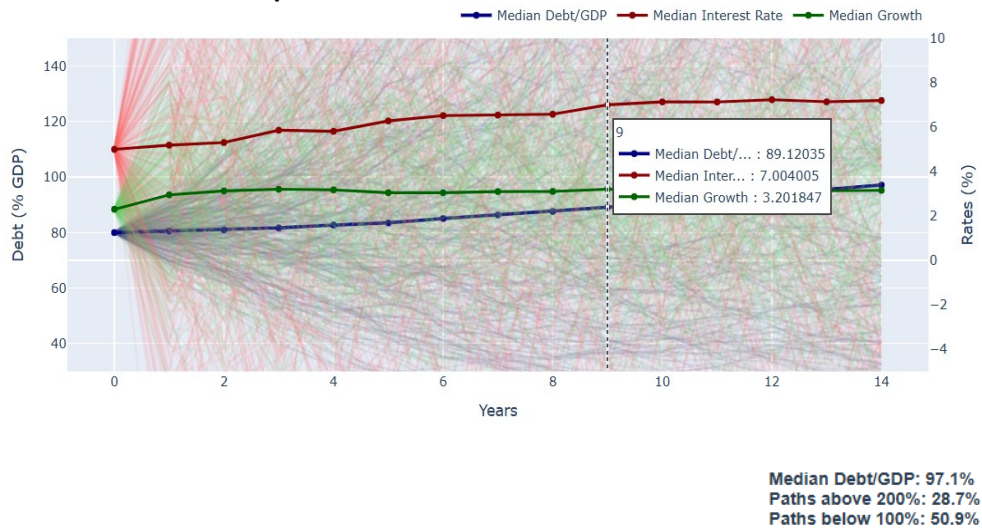


Figure 1- Scenario A (surplus of 0.5% of GDP; $b = 80\%$, $r = 5\%$, $g = 2.3\%$, $W = 0\%$)

In Scenario B, we assume a deficit of 0.5% of GDP. The median for the debt-to-GDP increases accordingly, reaching 99.4% of GDP by year 9, and the percentage of paths below 100% of GDP declines to 45.5%.

Figure 2 – Scenario B, deficit of 0.5% of GDP

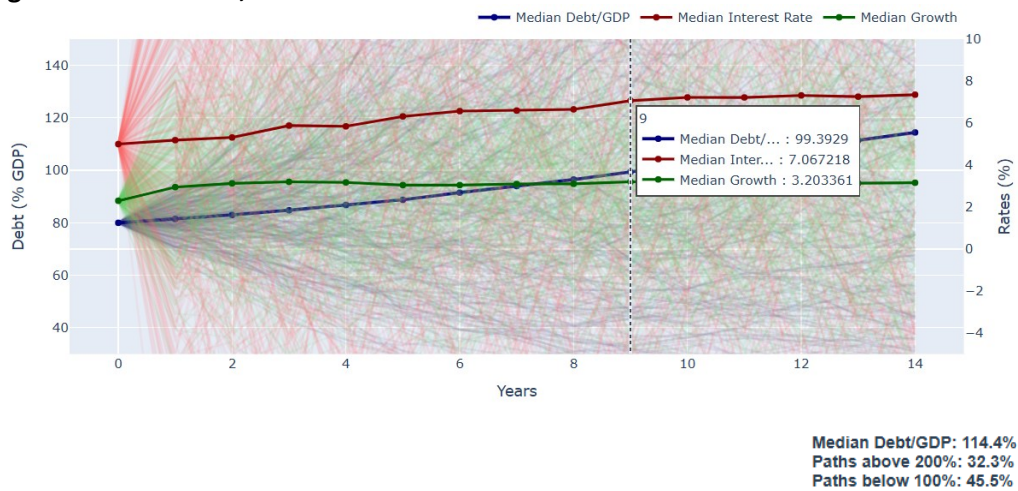


Figure 2- Scenario B (deficit of 0.5% of GDP; $b = 80\%$, $r = 5\%$, $g = 2.3\%$, $W = 0\%$)

Finally, in an optimistic scenario where the primary surplus starts at 1% of GDP (Scenario C), the median for the Debt-GDP increases slowly, implying a debt-GDP of 84.06% (median), after 9 years, and the number of paths below 100% in year 15 increases to 53.6%.

Figure 3 – Scenario C, a surplus of 1% of GDP

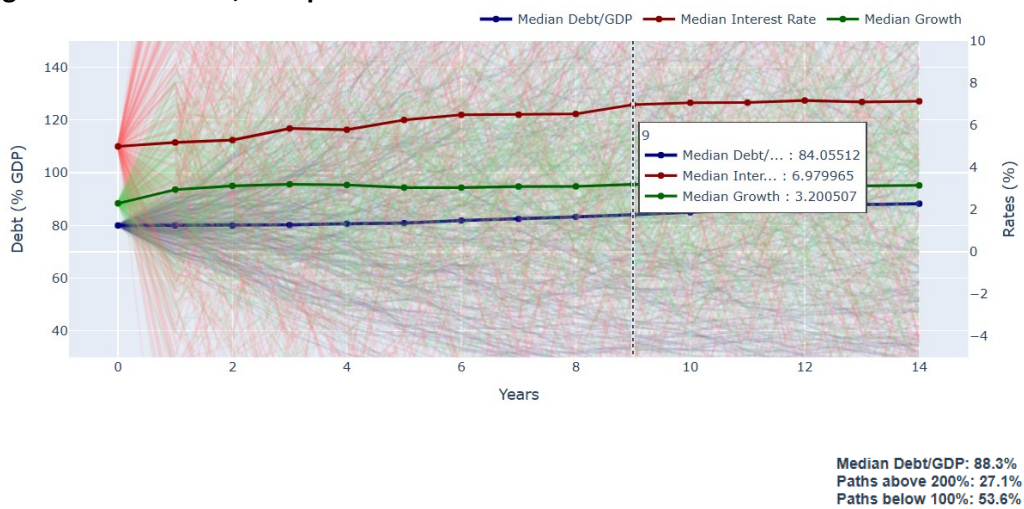


Figure 3- Scenario C (surplus of 1.0% of GDP; ; $b = 80\%$, $r = 5\%$, $g = 2.3\%$, $W = 0\%$)

Table 6 summarizes the results and brings the probability of debt remaining below(or equal) 100% of GDP during the next 5 years under different scenarios for the primary

result. When the surplus is at 0, debt paths remain below 100% of GDP during the next 5 years in 69.5% of cases (not shown). Comparing with other scenarios for surplus, the odds of the debt remaining below 100% of GDP increases by 2 percentage points with a surplus of 0.5% (to 71.6), and by 4 percentage points with a surplus of 1% (to 73.6). In the case of a deficit of 0.5% of GDP, the odds of debt remaining below 100% decreases to 68.3%, a reduction of 1.2 percentage points vis-a-vis a surplus of 0% of GDP. These results indicate a nonlinear response of the debt-GDP ratio to fiscal improvement.

Table 6: Scenarios in Surplus (% of GDP)

	surplus of 0.5	deficit of 0.5	surplus of 1.0
Debt-GDP (%), median			
At Year 5	83.48	88.77	80.89
At Year 9	89.12	99.39	84.06
Probability of Debt < (or equal) 100% of GDP			
At Year 5	0.726	0.683	0.745
Until Year 5 (inclusive)	0.716	0.673	0.736

Note: $b=80$, $r=5$, $g=2.3$, $W=0$; time horizon 15 years, 10000 paths.

5.2 Variations in Growth Rates

We analyzed the effects of variation in growth rates, while maintaining initial conditions at 80% for the debt-GDP ratio, the real interest rate at 5% (natural rate), primary result (surplus) at 0% of the GDP and international conditions at zero (no initial impact). We also assume transition probabilities and growth regime constants the same, at 9.6 (expansion) and 8.2 (recession), with $c_E = 1.5\%$ and $c_R = -1.0\%$. Simulations are presented in Figures 4, 5 and 6. A summary of results is in Table 7.

In Scenario A, we assume the current growth rate of 3.2%. The debt-to-GDP ratio increases slowly, reaching the median of 93.1% of GDP in 9 years. This scenario is also compatible with market expectations reported by Focus [Brazilian Central Bank]. We also analyze a scenario of rapid growth (scenario B, growth at 4.5%) and a recession (scenario C, growth rate of -1.6%,). In Scenario B, as expected, the debt-to-GDP ratio

responds to rapid growth, with the median falling by 1.5 percentage points vis-à-vis a scenario where the growth rate is at 3.2%.

Figure 4 – Scenario A, growth of 3.2%

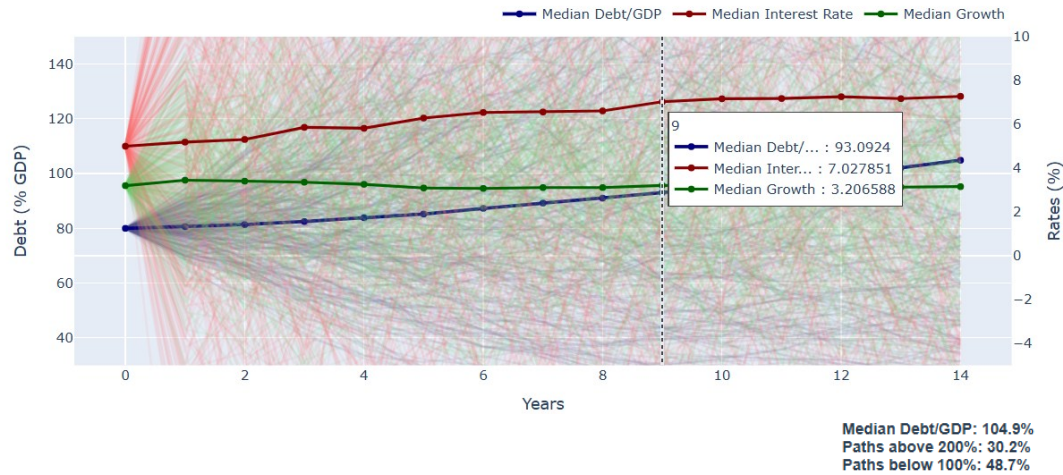


Figure 4- Scenario A (Growth of 3.2%; $b = 80\%$, $r = 5\%$, $s = 0\%$, $W = 0\%$)

Figure 5 – Scenario B, growth of 4.5%

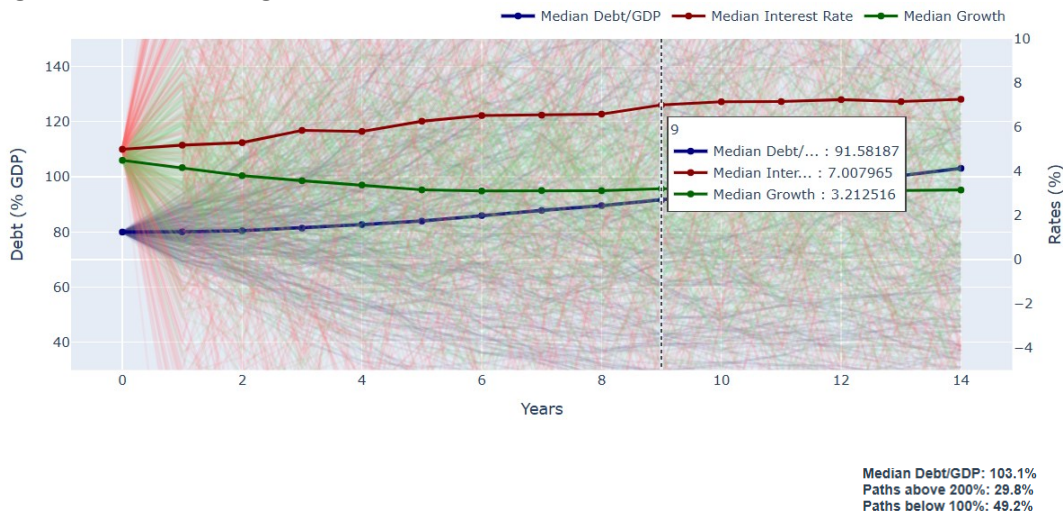


Figure 5- Scenario B (Growth of 4.5%; $b = 80\%$, $r = 5\%$, $s = 0\%$, $W = 0\%$)

In a scenario in which the simulation starts with a recession ($g = -1.6\%$), the median for the debt-to-GDP reaches 98.7% in 9 years. In this scenario, the debt-to-GDP ratio reaches 90% in 5 years; at this point, our model assumes that the government triggers a fiscal response, increasing surplus by 0.5% of GDP.

Figure 6 – Scenario C, recession (g = -1.6%)

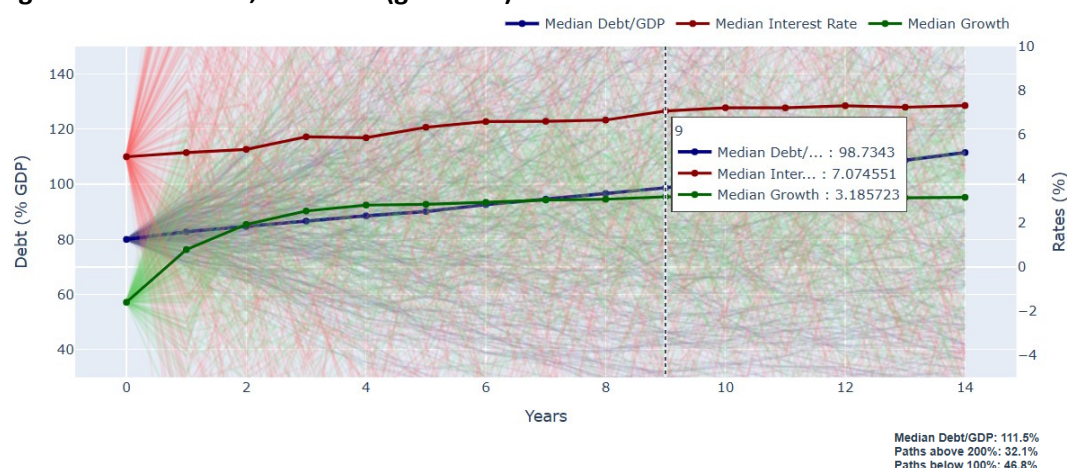


Figure 6- Scenario C (Recession $g = -1.6\%$; $b = 80\%$, $r = 5\%$, $s = 0\%$, $W = 0\%$)

Table 7: Scenarios for Growth Rates

	$g = 3.2$	$g = 4.5$	$g = -1.6$
Debt-GDP (%), median			
At Year 5	85.2	84.0	90.2
At Year 9	93.1	91.6	98.7
Probability of Debt < (or equal) 100% of GDP			
At Year 5	0.715	0.728	0.658
Until Year 5 (inclusive)	0.706	0.719	0.645

Note: $b = 80\%$, $r = 5\%$, $s = 0\%$, $W = 0\%$; time horizon 15 years, 10000 paths.

5.3 Variations in Interest Rates

For the real interest rate, we analyzed the effects of setting the annual rate at 5% (neutral), at a high level (at 7%), and a scenario of low real interest rate, at 3%. Results are depicted below. As already mentioned, the scenario with r at 5% results in a median debt-GDP ratio of 94.1%.

Figure 7 – Interest rate at 5% (Neutral)

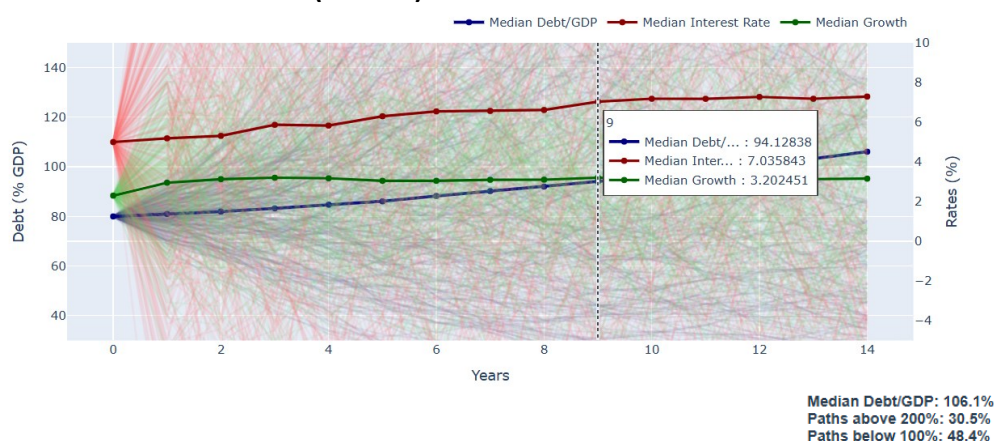


Figure 7 – Scenario A (Interest rate at 5%; $b = 80\%$, $g = 2.3\%$, $s = 0\%$, $W = 0\%$)

If we start with real interest rates at 7%, the debt-GDP ratio surpasses 100% in 9 years, and we observe an important change in the inclination of the debt ratio path depicted by the median values [see Figure 8]. A low interest rate, as expected, brings a scenario with a much slower growth of the debt-GDP ratio, and increases the probability of debt staying below 100% of GDP. Debt continues to rise, however, and reaches 87.7% after 9 years. This scenario suggests that even a low real interest rate would not be enough to stabilize the debt ratio if no fiscal effort is achieved. We present a summary of the results in Table 8.

Figure 8 – Interest rate at 7%

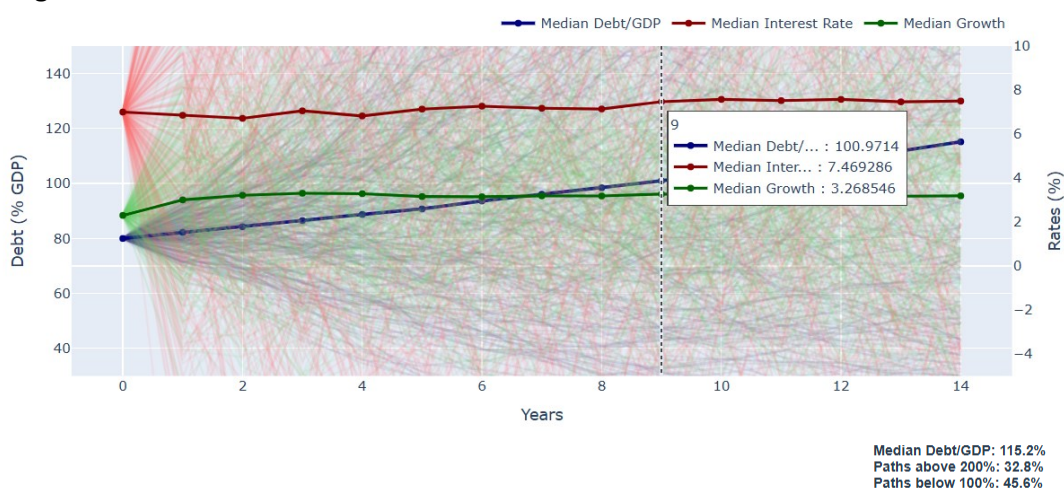


Figure 8 – Scenario B- High rate ($r = 7\%$; $b = 80\%$, $g = 2.3\%$, $s = 0\%$, $W = 0\%$)

Figure 9 – Interest rate at 3%

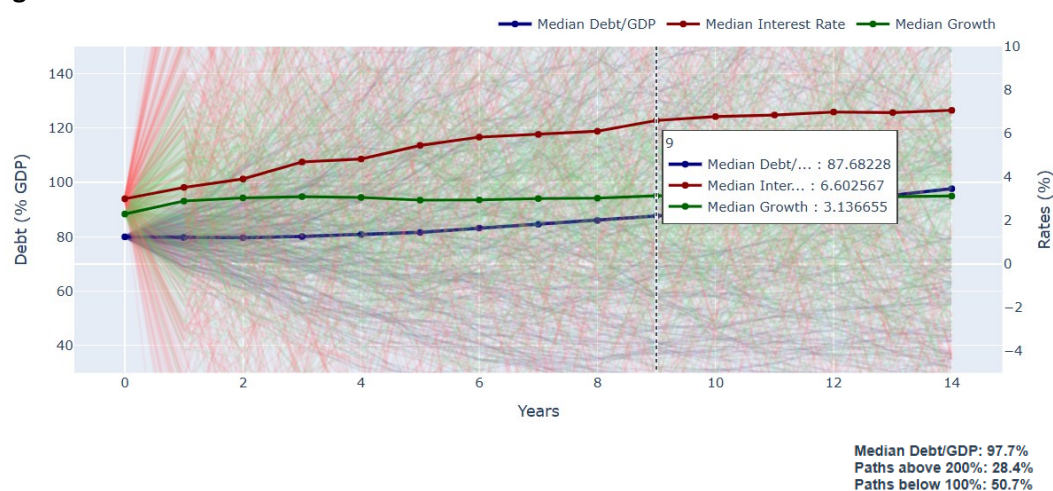


Figure 9 – Scenario C- Low rate ($r = 3\%$; $b = 80\%$, $g = 2.3\%$, $s = 0\%$, $W = 0\%$)

Table 8: Scenarios for Interest Rate

	$r = 5.0$	$r = 7.0$	$r = 3.0$
Debt-GDP (%), median			
At Year 5	86.1	90.8	81.6
At Year 9	94.1	101.0	87.7
Probability of Debt < (or equal) 100% of GDP			
At Year 5	0.74	0.658	0.746
Until Year 5 (inclusive)	0.695	0.646	0.737

Note: $b = 80\%$, $g = 2.3\%$, $s = 0\%$, $W = 0\%$; time horizon 15 years, 10000 paths.

5.4 Variations in International Conditions

In our model, we assume that international conditions are reflected in variations in Brazil’s terms of trade. For simulations, we used percentage variation in terms of trade (TT), and we had to decide which values would represent “good” and “bad” international conditions. As terms of trade follows a general trend, we assume that values above (below) trend can be considered “good” (“bad”). We applied the Hodrick-Prescott filter to the series and calculated the average of values, above and below trend. Accordingly, the starting values for international conditions are set at 5.56% (favorable) and -4.21% (unfavorable). Results are depicted in Figures 10 and 11.

When the simulation starts under favorable international conditions, the median for the debt-to-GDP ratio is simulated at 92.0% in 9 years. Under unfavorable conditions, in 9 years, the median for the debt is 95.7% of GDP, 3.7 percentage points higher. As seen in Table 9, international conditions also seem to affect the probability of the debt remaining under 100% of GDP. Under unfavorable conditions, that is, when changes in terms of trade are below trend, 68.3% of paths have the debt-GDP under 100% of GDP until year 5; under favorable conditions, this probability rises to 0.711, an increase of 2.8 percentage points. This rise in probability is comparable in size to the one obtained under a scenario of a strong primary surplus (3 percentage points increase vis-à-vis the scenario with a surplus of 0%). This suggests that international conditions can be important for fiscal sustainability.

Figure 10 – Favorable international conditions

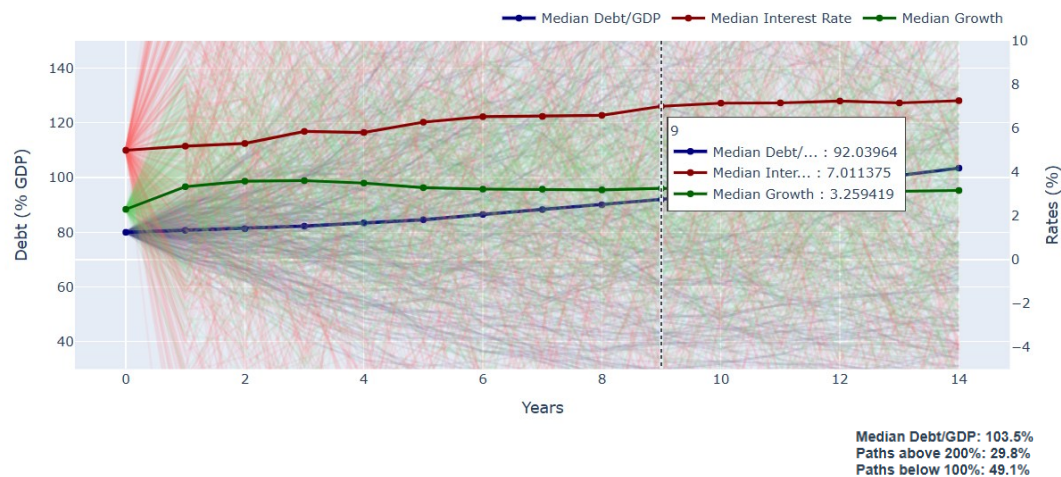


Figure 10 – Good conditions, variation of TT above trend, at 5.56%
($b=80\%$, $g=2.3\%$, $r=5\%$, $s=0\%$)

Figure 11 – Unfavorable international conditions

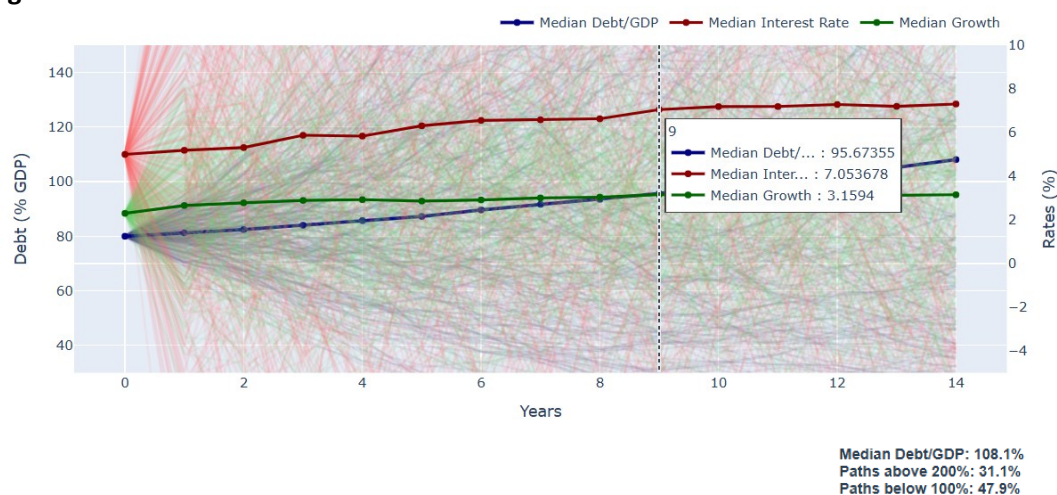


Figure 11 – Bad international conditions, variation of TT below trend, at -4.21%
($b=80\%$, $g= 2.3\%$, $r= 5\%$, $s= 0\%$)

Table 9: Scenarios for International Conditions

	TT above trend	TT below trend
Debt-GDP (%), median		
At Year 5	84.6	87.3
At Year 9	92.0	95.7
Probability of Debt < (or equal) 100% of GDP		
At Year 5	0.721	0.693
Until Year 5 (inclusive)	0.711	0.683

Note: $b = 80\%$, $g = 2.3\%$, $r = 5.0\%$, $s = 0\%$; time horizon 15 years, 10000 paths.

6 Final Remarks

This paper provides simulations for the debt-GDP ratio applied to Brazilian data. The model assumes stochastic processes for the main drivers of the debt-to-GDP ratio, with shocks that are correlated and reflect the observed macroeconomic volatility of the Brazilian economy during the period 2000-2024. We obtain the correlations for the shocks from a vector autoregression model which includes variations in the real exchange rate and international conditions.

We present median debt ratios under various scenarios for growth, interest rates, international conditions and primary surplus. Setting starting values that prevailed in the last quarter of 2024 and debt as measured by international standards, our results indicate a median debt at 99.0% of GDP in 2031. The Fiscal Monitor of April 2025 (IMF) predicts that the Brazilian debt will reach 99.4% of GDP in 2030.

Our results also indicate that although lower real interest rates would bring scenarios with a much slower growth of the debt-GDP ratio, a real rate as low as 3% would not be enough to stabilize the debt ratio if no fiscal effort is achieved. At the same time, our results suggest that debt-to-GDP ratios can be quite sensitive to changes in the primary result, an indication that the Brazilian economy is not under fiscal fatigue. Regarding the possibility of “growing out of the debt”, our simulation indicates that even a growth rate of 4.5% would not stabilize the debt-GDP ratio in the absence of fiscal efforts.

It should be noted that our simulations reflect the model, starting values, and parameters on which they are based. And as such our results must be viewed from this perspective. Future developments involve checking how robust results are to changes in the parameters, for example, using estimations from a shorter but more recent sample of the Brazilian data. Regarding the model, a more detailed specification of the primary result equation could be pursued so that the effects of a change in (permanent) revenues (or expenditures) on debt paths can be analyzed. The model could also be easily adapted to reflect characteristics of other countries and be applied to different sets of data. Stress tests may be analyzed by increasing volatility parameters in the simulations; the effects of different views of fiscal policy on debt-GDP may be analyzed by varying starting values for primary surplus or growth. In fact, simulation models offer a flexible yet powerful framework for the analysis of debt sustainability.

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Appendix

A1 Data

GDP growth (g)	GDP- Real percentage quarterly change from same quarter in previous year (%) . Source: Instituto Brasileiro de Geografia e Estatística, Sistema de Contas Nacionais Trimestrais (IBGE/SCN Trimestral) - SCN104_PIBPBG104 .
Real interest rate (r)	433 - Broad National Consumer Price Index (IPCA) - Monthly % var. 4390 - Interest rate - Selic accumulated in the month - % p.m. Each series is annualized; Real rate (r) computed as $((1+i/100)/(1+inf/100) - 1)*100$ Annualization: $(1+month/100)^{12} - 1$ *100 ; quarter values are averages of the months in the quarter. Source: Brazilian Central Bank (BCB-Demab and BCB-DEPEC)
Primary surplus (s)	Public Sector Borrowing Requirements without exchange devaluations, consolidated public sector (PSBR % of GDP); flows accumulated in the year (series 5507). Primary surplus (s) is the negative of the PSBR, that is, $s = (-1)*series\ 5507$. Data is reported monthly. As it represents flows accumulated in the year, quarterly data (st) are values observed for the last month of the quarter (months 3, 6, 9 and 12). Source: Brazilian Central Bank (BCB-DSAT)
Terms of Trade	Terms of trade - index (average 2018=100). Defined as the ratio of export to import prices (Px/Pm) Data reported monthly. Quarter values are average of months in the quarter. Source: Fundação Centro de Estudos do Comércio Exterior (Funcex) - FUNCEX12_TTR12
Real effective exchange rate	11752 - Real effective exchange rate index (IPCA) - Jun/1994=101 - Index Quarter values are the averages of months in the quarter. Variable is used in logs. Source: BCB-STAT
Federal Funds Rate	Effective Federal Funds Rate (Percent), Level, Percent, Quarterly, Not Seasonally Adjusted Source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis (Link: https://fred.stlouisfed.org)
CPI Inflation	Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Index 1982-1984=100, Monthly, Not Seasonally Adjusted. CPI inflation calculated as $((cpi - cpi(-12))/cpi(-12))*100$ Quarter values are average of months in the quarter Source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis (Link: https://fred.stlouisfed.org)
Government securities	Federal domestic securities (balances) - National Treasury liabilities - Total issued - c.m.u. (million), series 2213. Nominal data; divided by price index (IPCA index, 2024m12=1), resulting in the series at constant values of december 2024. Data reported monthly. Quarter values are quarterly equal last month of quarter. Source: BCB-STAT

A2 Variables - Descriptive Statistics

Descriptive Statistics

	g	W	r	s
Mean	2.22172	0.87442	7.63325	1.66491
Median	2.34000	0.09893	6.99735	2.14500
Maximum	11.79000	22.81997	35.81786	5.83000
Minimum	-9.67000	-14.67007	-9.85348	-11.46000
Std. Dev.	2.94173	7.22889	7.59158	2.94201
Skewness	-0.45698	0.60699	0.88485	-1.82449
Kurtosis	5.15095	3.89691	4.79246	8.81401
Jarque-Bera	26.3992	11.0112	30.6661	227.7360
Probability	0.0000	0.0041	0.0000	0.0000
Observations	116	116	116	116

Note: Calculated in Eviews 14.

A3 Vector Autoregression

Vector Autoregression Estimates				
	W(t)	g(t)	r(t)	D_s(t)
W(t-1)	1.083684 [8.85459]	0.136135 [2.17233]	0.263732 [2.07093]	0.05147 [1.39439]
W(t-2)	-0.235361 [-1.50216]	-0.070706 [-0.88130]	-0.426262 [-2.61455]	-0.085307 [-1.80524]
W(t-3)	0.238542 [1.63964]	0.001941 [0.02605]	0.045165 [0.29835]	0.129275 [2.94623]
W(t-4)	-0.496069 [-3.23787]	0.06834 [0.87112]	0.135887 [0.85238]	-0.014184 [-0.30697]
W(t-5)	0.275301 [1.68626]	-0.025878 [-0.30955]	-0.232244 [-1.36709]	-0.001769 [-0.03593]
W(t-6)	0.023469 [0.21844]	0.053864 [0.97908]	-0.02155 [-0.19276]	-0.001999 [-0.06170]
g (t-1)	0.211665 [0.85131]	0.725567 [5.69910]	-0.031449 [-0.12156]	0.266214 [3.55008]
g (t-2)	-0.492729 [-1.92392]	-0.02683 [-0.20459]	-0.152204 [-0.57114]	-0.124499 [-1.61179]
g (t-3)	0.284778 [1.32032]	0.213155 [1.93000]	0.01392 [0.06202]	-0.288197 [-4.43025]
g (t-4)	-0.1237 [-0.51693]	-0.514807 [-4.20144]	-0.10797 [-0.43362]	0.074246 [1.02874]
g (t-5)	-0.163223 [-0.57553]	0.241071 [1.66003]	0.941309 [3.18973]	0.181677 [2.12398]
g (t-6)	-0.188759 [-0.81410]	-0.085715 [-0.72196]	-0.546836 [-2.26655]	-0.162681 [-2.32634]
r (t-1)	0.245176 [1.88936]	-0.002569 [-0.03866]	0.690815 [5.11609]	0.087548 [2.23693]
r (t-2)	-0.295689 [-2.23193]	0.082332 [1.21367]	-0.363628 [-2.63779]	-0.125454 [-3.13975]
r (t-3)	0.133511 [0.98769]	-0.052196 [-0.75410]	0.308832 [2.19566]	0.066971 [1.64269]
r (t-4)	-0.153188 [-1.14008]	0.096092 [1.39664]	-0.031454 [-0.22497]	-0.087388 [-2.15639]
r (t-5)	0.119112 [0.93083]	-0.066701 [-1.01798]	0.216928 [1.62918]	0.013488 [0.34950]
r (t-6)	0.014869 [0.12783]	-0.008481 [-0.14240]	0.124469 [1.02836]	0.014427 [0.41123]
D_s (t-1)	-0.450603 [-1.47090]	-0.243297 [-1.55101]	0.274377 [0.86075]	0.831785 [9.00259]
D_s (t-2)	-0.459279 [-1.01628]	-0.006454 [-0.02789]	0.471537 [1.00274]	-0.045192 [-0.33156]
D_s (t-3)	-0.180788 [-0.42004]	0.081949 [0.37184]	-0.153053 [-0.34174]	-0.361391 [-2.78395]
D_s (t-4)	0.400672 [1.13084]	-0.173924 [-0.95865]	0.234243 [0.63536]	-0.119531 [-1.11856]
D_s (t-5)	0.400177 [1.14190]	0.398643 [2.22152]	-0.076429 [-0.20959]	0.297309 [2.81288]
D_s (t-6)	-0.481711 [-1.84562]	-0.341659 [-2.55645]	0.233084 [0.85823]	-0.145907 [-1.85352]
r_fundsrate (t-1)	-0.013966 [-0.04493]	0.049785 [0.31277]	-0.586672 [-1.81373]	-0.012685 [-0.13530]
D_exchange rate (t-1)	-4.457225 [-1.19800]	-0.558683 [-0.29326]	-2.093547 [-0.54077]	1.214144 [1.08200]
D_debt securities (t)	-4.04E-06 [-1.25556]	-1.49E-06 [-0.90299]	1.28E-07 [0.03821]	-1.35E-06 [-1.38779]
Adj. R-squared	0.82657	0.75883	0.67511	0.89146
F-statistic	14.31707	9.79150	6.80620	23.94748
Mean dependent	1.33620	2.25042	5.52274	-0.12840
S.D. dependent	7.22113	3.13554	5.48985	2.75292

Note: (1) VAR estimated with a constant, quarterly dummies and controls (D2009q1, D2009q2, D2017q2, D2020q2) in Eviews 14. D_ indicate quarter difference. Sample (adjusted): 2001Q1 2024Q4, t-statistics in [].

VAR Model Statistics

Determinant resid covariance (dof adj.)	141.7056
Determinant resid covariance	23.10051
Number of coefficients	140
Roots of Characteristic Polynomial	all inside unit circle

Residual Serial Correlation LM Test (prob. value) ²	0.2329
Residual Heteroskedasticity Test (prob. value) ³	0.1697
Residual Normality Test Jarque-Bera (prob. value) ⁴	0.4151

Notes: (2) LRF test, null of no serial correlation at lags 1 to h (p-values greater than 5% for h=1, 2,...,7)

(3) Chi-square joint test, null of no heterocedasticity

(4) Jarque-Bera joint test, null hypothesis residuals are multivariate normal

A4 Simulation Interface

Stable Debt Simulator

Initial Conditions

Initial Debt (% GDP): 87.3

Initial Interest Rate (%): 4.8

Initial Growth Rate (%): 3.2

Initial Primary Surplus (% GDP): -0.4

Initial Foreign Index: 1.7

Shock Parameters

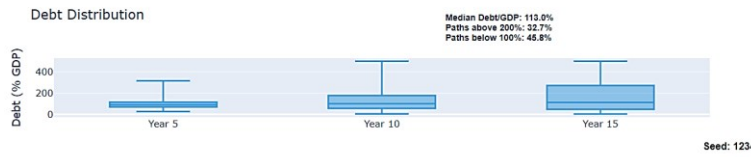
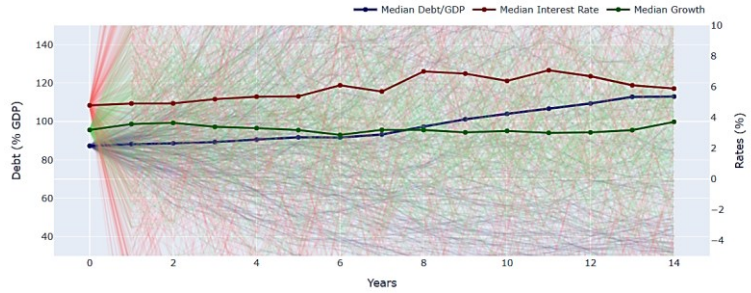
Interest Rate Volatility (% p.a.): 7.6

Growth Rate Volatility (% p.a.): 2.9

Surplus Volatility (% p.a.): 0.5

Foreign Index Volatility (% p.a.): 7.2

Sustainability Factor (ρ): 0.0094



Markov-Switching Parameters

Expansion Growth (%): 1.5

Recession Growth (%): -1

P(Stay in Expansion): 0.9679

P(Stay in Recession): 0.821

Simulation Controls

Time Horizon (years): 15

Number of Paths: 10000

Surplus Truncation ($k\sigma$): 2

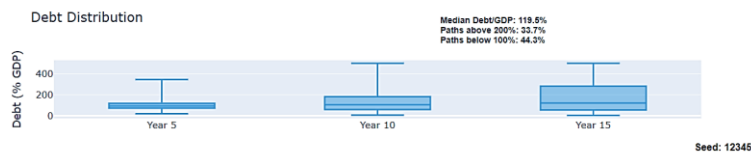
Simulation Mode:

Single Run Multiple Runs Selective Paths

Run Simulation

Random Seed (blank for random): 12345

Save Parameters Load Parameters



```
SIMBRADEBT_0.4_v16_Final.py × SIMBRADEBT_0.4_v17.py
C: > Users > vivia > OneDrive > Documents > A_Box > Doc1 > Projetos-Bolsa > Pos Doc > Paper > SIMBRADEBT_0.4_v16_Final.py
1 import numpy as np
```